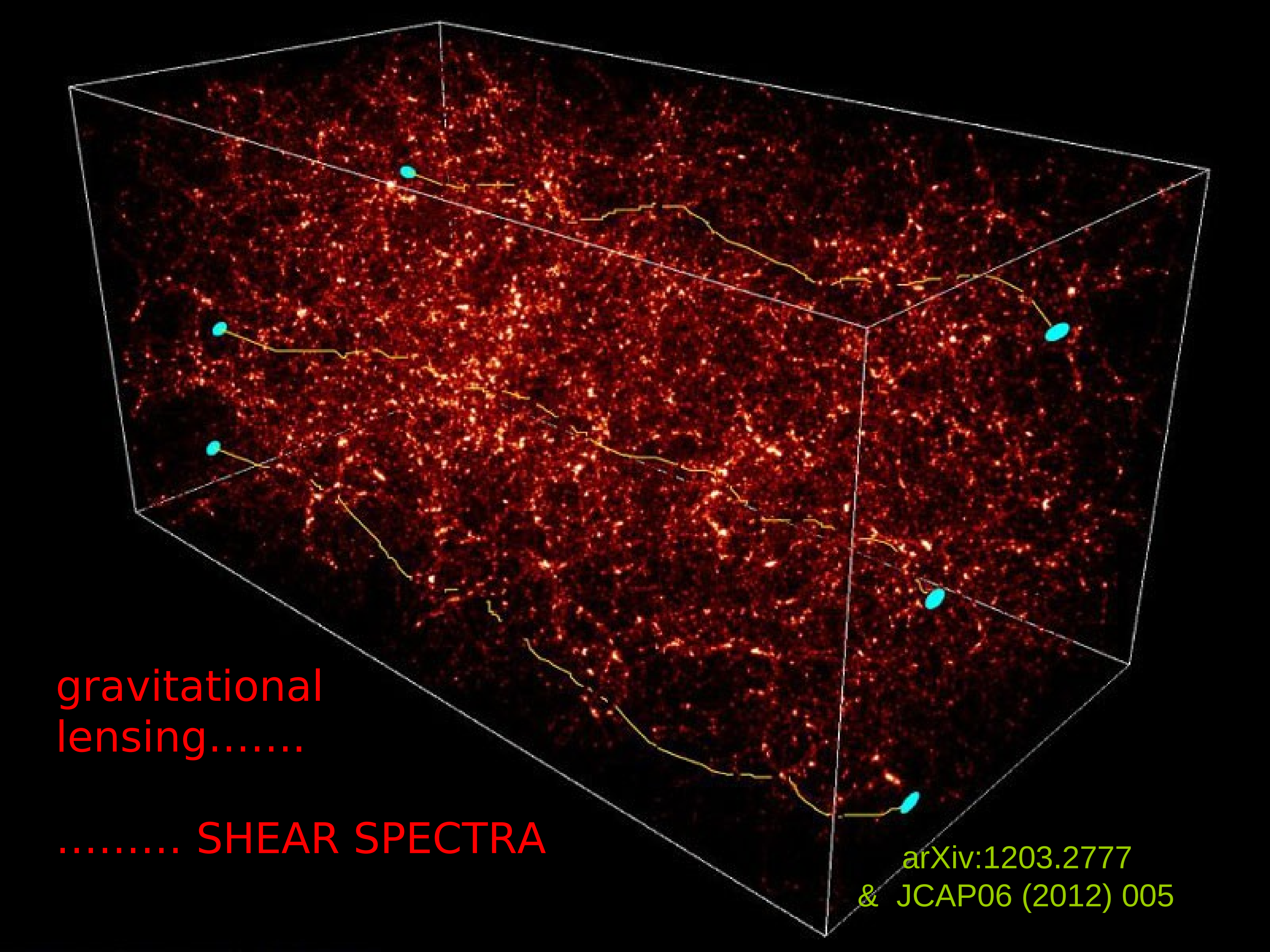
Direct recovery of density fluctuation spectra from tomographic shear spectra

Silvio Bonometto – Physics Dep., Trieste University,
Via Tiepolo 11, 34143 Trieste (Italy)
& C.I.F.S. – Via Pietro Giuria 1, 10125 Torino (Italy)

bonometto@oats.inaf.it

in collaboration with Marino Mezzetti, Luciano Casarini, Giuseppe Murante

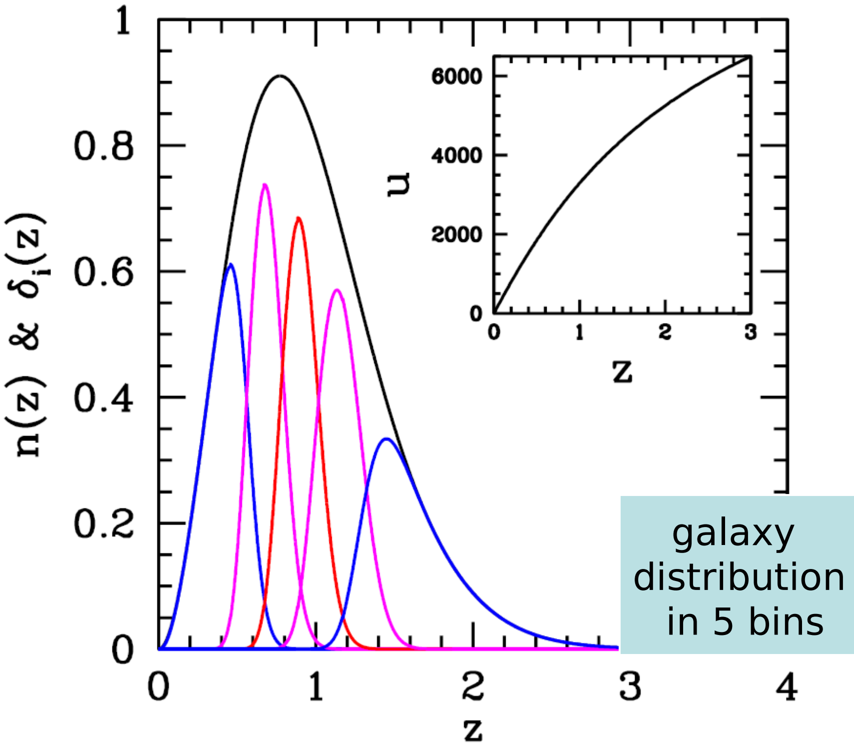
See also: JCAP06 (2012) 005


gravitational
lensing.......
.... SHEAR SPECTRA
arXiv:1203.2777
& JCAP06 (2012) 005

OUTLINE

- From fluctuation to shear spectra (3D→2D)
- Formal inversion (2D→3D, by using tomography)
- Problems in a real use of formal inversion
  (matrix quasi-singularity)
- The SVD technique to gauge singularity level
  (so selecting viable options)
- Technique applied to H.F. and simulations
- Technique applied to “true” shear spectra (problems)
- Renormalizing (by using linear predictions)
- Extended use of SVD: normalization not needed?
- Fluct. spectra & spectral evolution recovered ?

preliminary results



$$n(z) = \frac{d^2N}{d\Omega\, dz} = \mathcal{C}\left(\frac{z}{z_0}\right)^A \exp\left[-\left(\frac{z}{z_0}\right)^B\right]$$

$$\mathcal{C} = \frac{B}{\left[z_0 \Gamma\left(\frac{A+1}{B}\right)\right]}$$

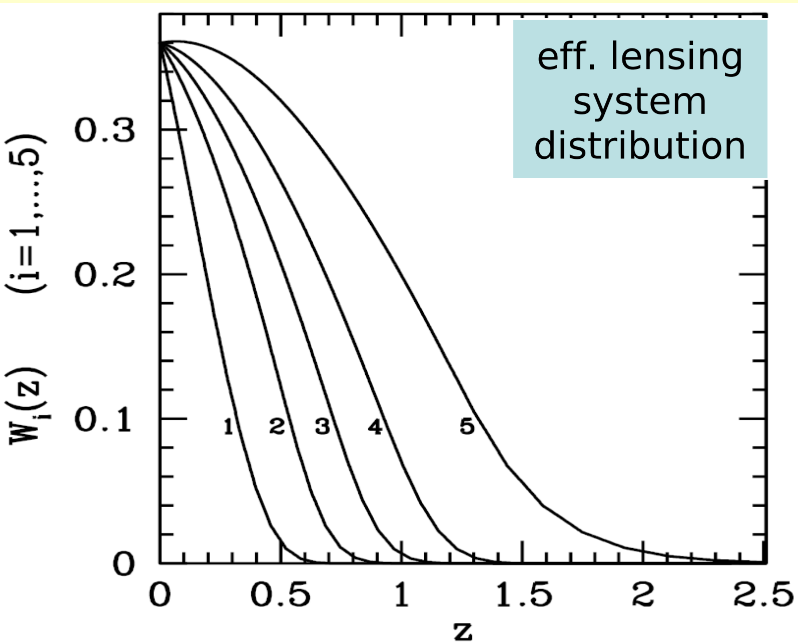
$A = 2$, $B = 1.5$, so that $\mathcal{C} = 1.5/z_0$ ($z_0 = z_m/1.412$, from the median redshift $z_m = 0.9$)

$$\delta_i(z) = \frac{D_i(z)}{\int_0^\infty D_i(z')dz'} \qquad D_i(z) = n(z)\Pi_i(z)$$

$$\Pi_i(z) = \int_{z_{ph,i}}^{z_{ph,i+1}} dz' \, \frac{1}{\sqrt{2\pi}\ \sigma(z)} \exp\left(-\frac{(z-z')^2}{2\sigma^2(z)}\right) =$$

$$= \frac{1}{2}\left[\mathrm{Erf}\left(\frac{z_{ph,i+1}-z}{\sqrt{2}\sigma(z)}\right) - \mathrm{Erf}\left(\frac{z_{ph,i}-z}{\sqrt{2}\sigma(z)}\right)\right]$$

with $\sigma(z) = 0.05\ (1+z)$ (Euclid)



$$ds^2 = a^2(\tau)\left(d\tau^2 - d\ell^2\right)$$

$$\tau_0 - \tau = u(z)$$

LCDM
Ω_m=0.24
h=0.73

$$F_i(z) = \int_{\Delta z_i} dz' \ \delta_i(z') \left[1 - \frac{u(z)}{u(z')}\right]$$

$$W_i(z) = \frac{3}{2}\Omega_m F_i(z)(1+z)$$

$$ds^2 = a^2(\tau)(d\tau^2 - d\lambda^2) \qquad \tau = \tau_0 - u$$

spectra: $P(k,z) = P[\ell/u, z(u)] \rightarrow P(\ell/u, u)$

$$C_{ij}(\ell) H_o^{-4} = \int_0^{\tau_0} du \, W_i(u) W_j(u) \, P(\ell/u, u)$$

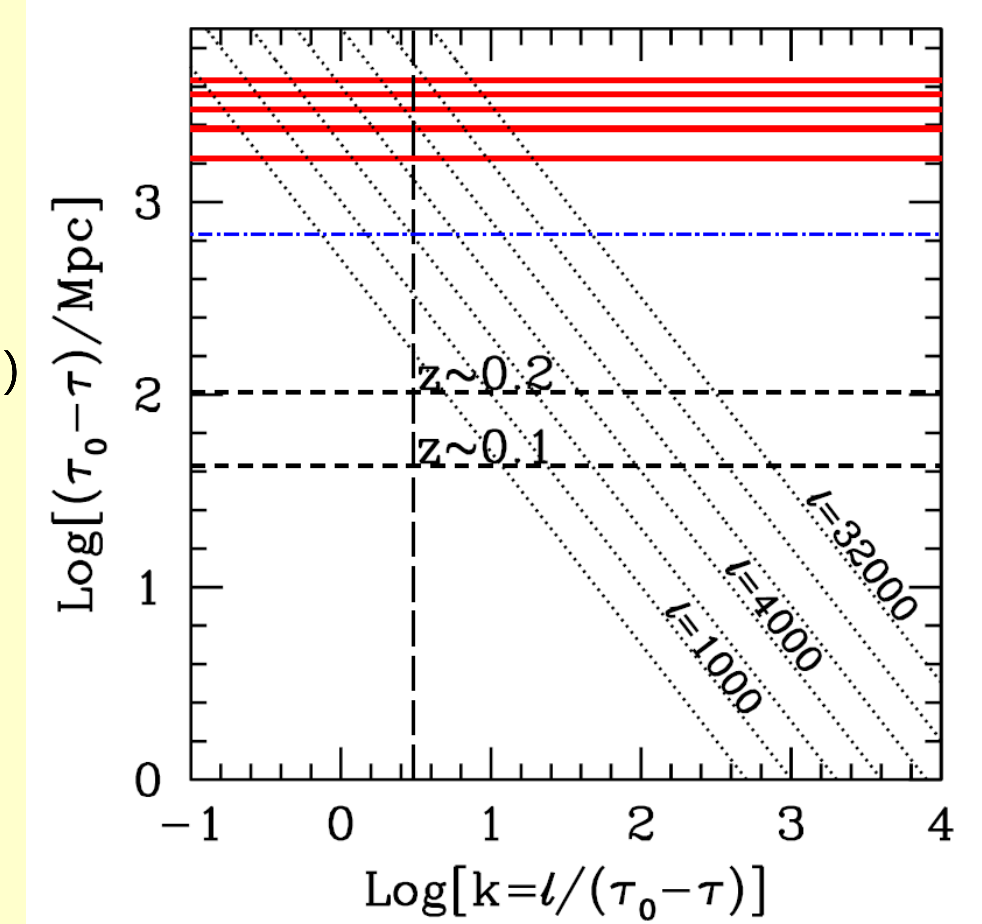
$$c_{ij}(\ell) = \int_0^{\tau_0 \, (\sim\infty)} du \, e^{-(u/\bar{u})^\beta} S_{ij}(u) \, p(\ell, u)$$

Gauss–Laguerre integration $\qquad x = (u/\bar{u})^\beta$

$$c_{ij}(\ell) = \sum_{r=1}^{N} w_r \, s_{ij}(x_r) \, p(\ell, x_r)$$

$$c_A(\ell) = \sum_{r=1}^{N} \mathcal{M}_{Ar} \, p_r(\ell) \qquad ij \equiv A$$

$$P[k = l/u(x_r)] = p(\ell, x_r) = \sum_{A=1}^{N'} \mathcal{M}^{-1}_{rA} \, c_A(\ell)$$



for low A (i.e.: i,j) & high

M(A,r) close to

ero

if M close to singu

hard to inv

x_r

$\rightarrow u_r \rightarrow$

z_r

2 Gauss-Laguerre options vs. Riemann

.. integrals must approach
well” exact integration
emann intgrl 10000pnts, but this is not enough...)

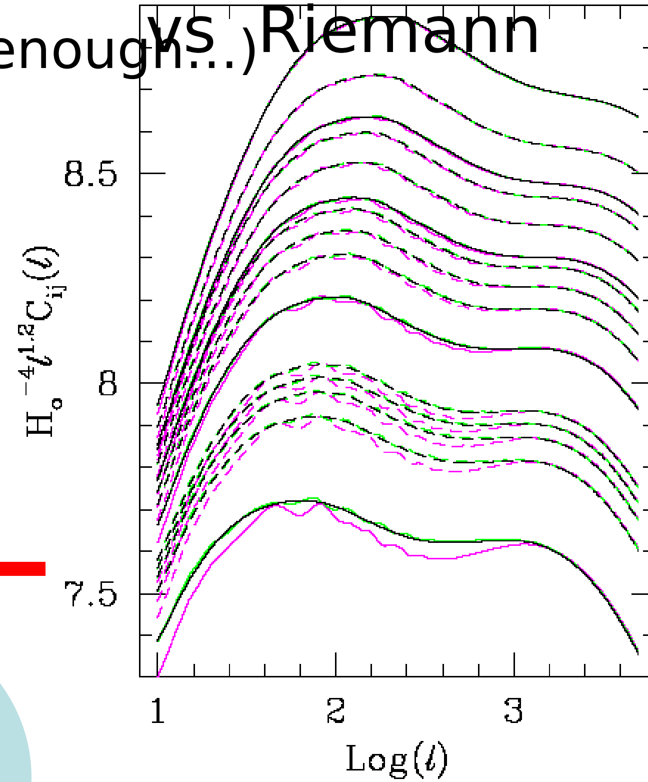
$$x = (u/\bar{u})^\beta$$

$\bar{u} = 600,\ \beta = 1.9 \rightarrow z_r$ values

.0466	.1152	.1886	.2664	.3499	.4402
.5386	.6473	.7700	.9122	1.0850	1.3162

$\bar{u} = 1200,\ \beta = 2.9 \rightarrow z_r$ values

.1234	.2338	.3338	.4304	.5263	.6237
.7245	.8307	.9450	1.0714	1.2171	1.4004



P(k,z) from H.F. exprs.

SVD decomposition

$$\begin{vmatrix} \mathcal{M}_{11} & .. & \mathcal{M}_{N_c1} \\ .. & .. & .. \\ .. & .. & .. \\ \mathcal{M}_{1N_r} & .. & \mathcal{M}_{N_cN_r} \end{vmatrix} =$$

$$= \begin{vmatrix} \mathcal{U}_{11} & .. & \mathcal{U}_{N_c1} \\ .. & .. & .. \\ .. & .. & .. \\ \mathcal{U}_{1N_r} & .. & \mathcal{U}_{N_cN_r} \end{vmatrix} \begin{vmatrix} s_1 & 0 & 0 \\ 0 & .. & 0 \\ 0 & 0 & s_{N_c} \end{vmatrix} \begin{vmatrix} \mathcal{V}^T_{11} & .. & \mathcal{V}^T_{N_c1} \\ .. & .. & .. \\ \mathcal{V}^T_{1N_c} & .. & \mathcal{V}^T_{N_cN_c} \end{vmatrix}$$

Evaluation of singularity level

Test ratio btwn max & min si
Fair inversion requires
No more than 6 o.o.m.

Need u and β allowing
(i) Good integration
(ii) Low singularity level

Inversion of
Gau.Lag. integral

$\bar{u} = 600, \ \beta = 1.9 \to z_r$ values

.0466	.1152	.1886	.2664	.3499	.4402
.5386	.6473	.7700	.9122	1.0850	1.3162

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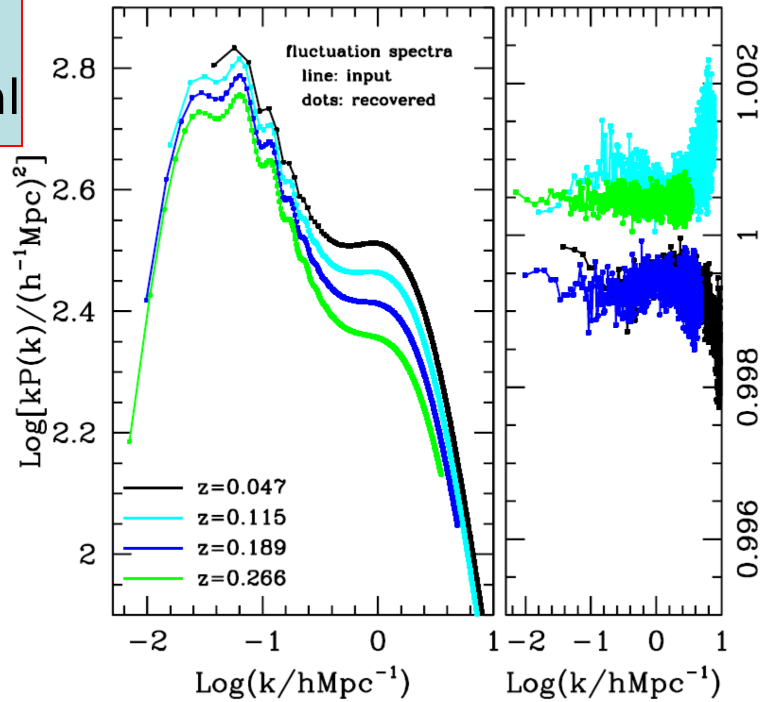
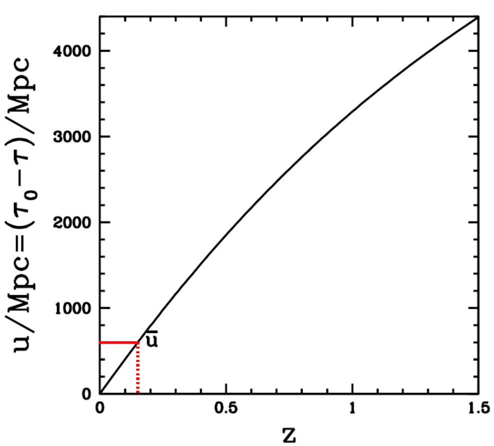
$$= \begin{vmatrix} \mathcal{U}_{11} & .. & \mathcal{U}_{N_c1} \\ .. & .. & .. \\ .. & .. & .. \\ \mathcal{U}_{1N_r} & .. & \mathcal{U}_{N_cN_r} \end{vmatrix} \begin{vmatrix} s_1 & 0 & 0 \\ 0 & .. & 0 \\ 0 & 0 & s_{N_c} \end{vmatrix} \begin{vmatrix} \mathcal{V}^T_{11} & .. & \mathcal{V}^T_{N_c1} \\ .. & .. & .. \\ \mathcal{V}^T_{1N_c} & .. & \mathcal{V}^T_{N_cN_c} \end{vmatrix}$$

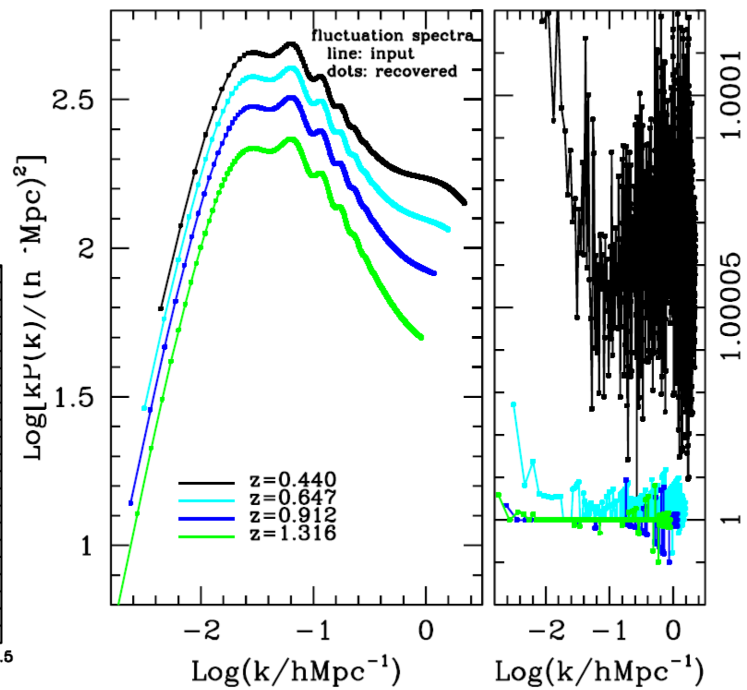


Best results:
12 point integral (600-1.9 case)

yields 15 eqs wit
12 unknown

Redundancy OK

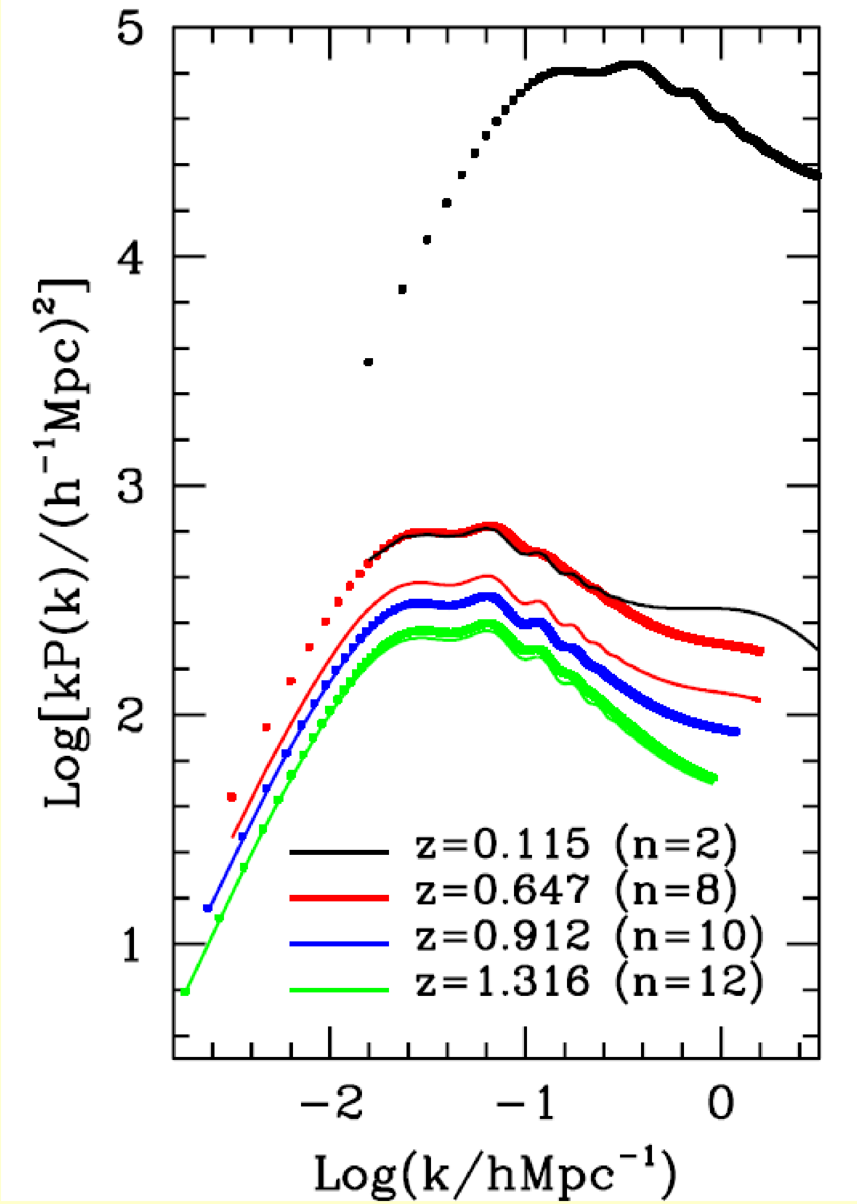




ame inversion matrix, if
pplied to Riemann intgrl

Some high-z fluct.
spectrum is recovered

at most z’s we fail



but there is a way out....

as well as fair
perspectives.....

failure at some z does not
prevent
other z’s to be well

Application to fluctuation spectra from hydro simulation

Cosmology:

ΛCDM with
$\Omega_m = 0.24$, $\Omega_b = 4.13 \times 10^{-2}$, $h = 0.73$, $n_s = 0.96$.
linear $\sigma_8 = 0.8$ $z_{in} = 41$

Simulation features:

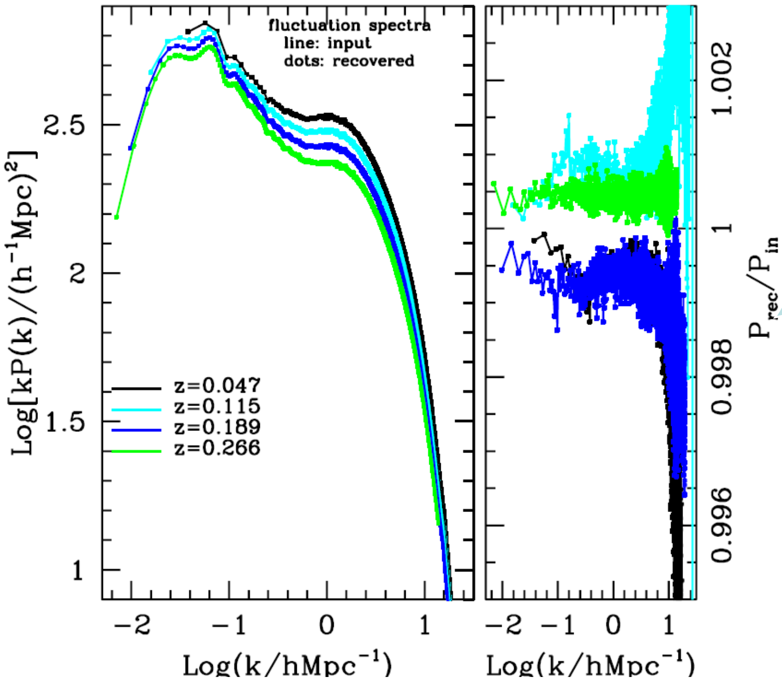
$L = 410\, h^{-1}$Mpc
$(2\times)1024^3$ particles:
$m_c \simeq 1.89 \times 10^9\, h^{-1} M_\odot$ $m_b \simeq 3.93 \times 10^8\, h^{-1} M_\odot$.
$\epsilon_{Pl} = 7.5\, h^{-1}$kpc (physical) from $z = 0$ to $z = 2$
fixed in comoving units at higher redshift

TreePM-SPH GADGET-3 code (Springel 2005).
Initial Zeldovich displacements generated at $z_{in} = 41$

Simulation features

L. Casarini, S.A. Bonometto, S. Borgani, K. Dolag, G. Murante, M. Mezzetti, L. Tornatore, G. La Vacca, *Tomographic weak lensing shear spectra from large N-body and hydrodynamical simulations*, arXiv:1203.5251, A&A (in press)

hen using simulations
ersion algorithm appears more effective



Ga.Lag. integrls inverted
in the simulation case

what about Rie. intgrls?

A tentative option:
Use linear expectns to
normalize

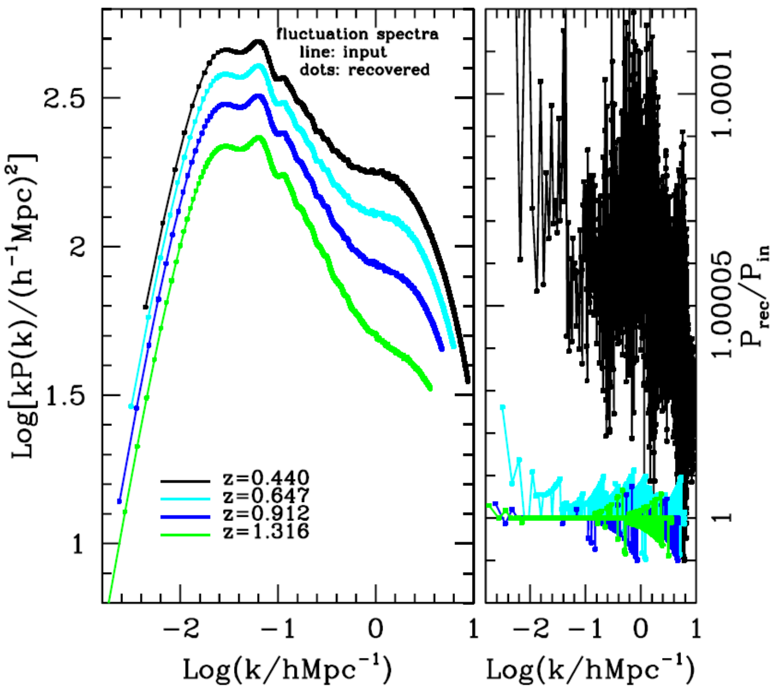
$$\mathcal{N}(k,z) = \frac{\tilde{P}^{(c,lin)}(k,z)}{P^{(c,lin)}(k,z)},$$

Need just “background cosmolog

Ω_m & h in LCDM case

(spectral index irrelevant)

$$\bar{P}^R(k,z) = \mathcal{N}(k,z)\bar{P}(k,z) \ .$$



Counter-indication:
spectra distorted by massive neutr

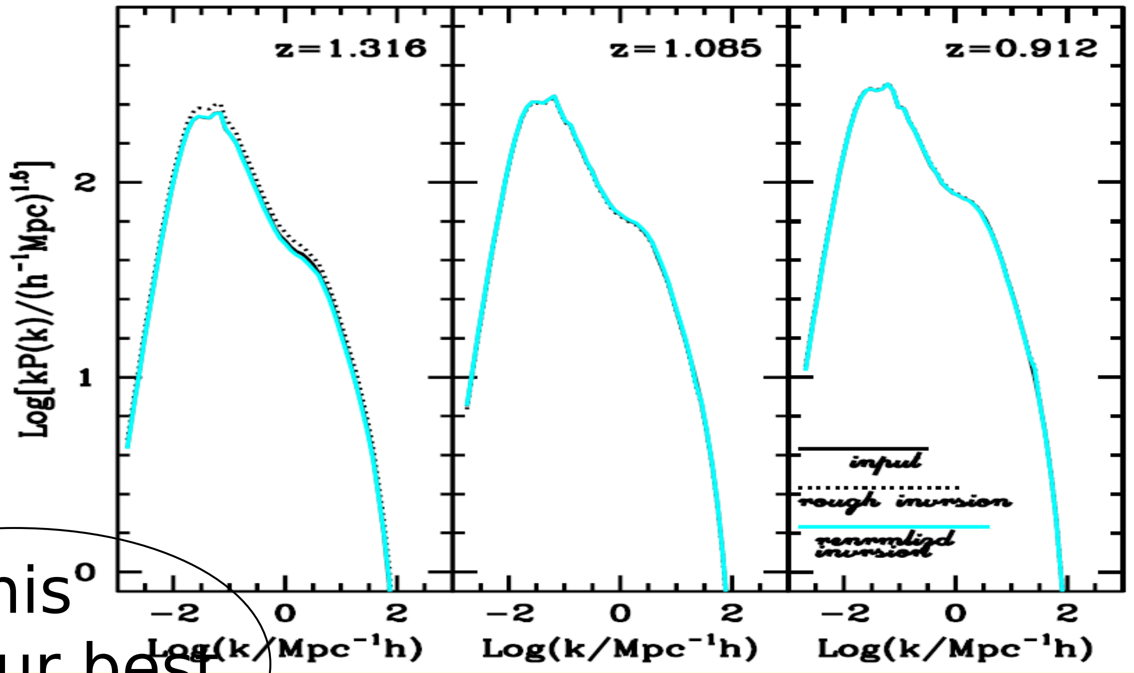
Solid black:
Simul. fluctuation spectra

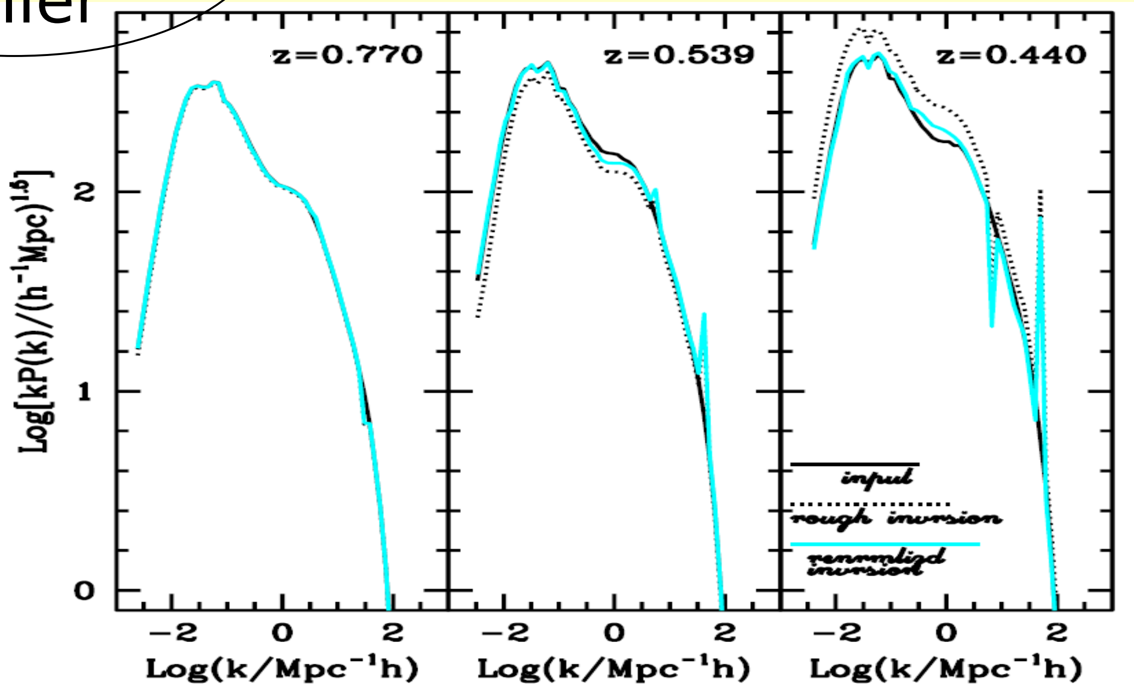
Dotted black:
Ga.La. inv. matrix acting
on Rie. intgrls

Solid cyan:
After renormalization

MUCH BETTER
INVERSION FROM

this
was our best
seller





Once knowing
'geometrical parameters'
(h & Ω_m in this case)
fluct spectra worked out

Allows to inspect,
e.g., all dependence
on baryon physics.

Howerver

Why 5 bins only ?

With N bins N(N+1)/2 possible integratior

up to the same number of lir

$$C_{ij}^{(obs)}(\ell) = C_{ij}(\ell) + \delta_{ij}\frac{\langle\gamma_{int}^2\rangle}{n_i}$$

$\langle\gamma_{int}^2\rangle^{1/2} \simeq 0.22$: r.m.s. intrinsic shear

$n_i = (\#gal/steradian)_{i-th\ bin}$

e.g. 7 bins → 28 pnts/eqs. exp. discrepancies reduced by >6.e-2

10 bins → 55 pnts/eqs. exp. discrepancies reduce by >1.e-3

Useless going too far beyond expcted data precition

In general discrepancies btwn G.L. and exact integral circa 2^{-2n}

but choice of u-bar and β critical for invert risk 10—12 o.o.m. between top and bottom eler in matrix s

In principle, just need quadruple preci SVD technique to be fully exploited

Strengthen bin dependence Spectroscopic z's ?

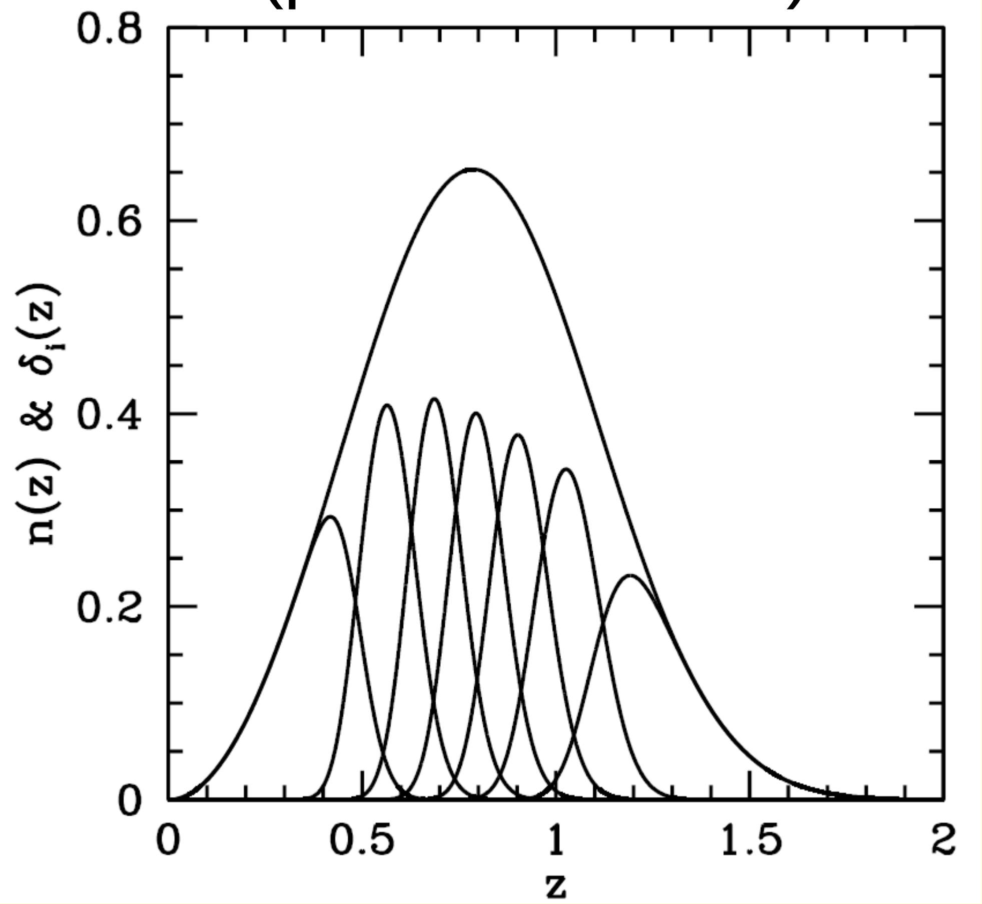
Photometric redshifts: cause confusion betwe

Shot noise: covers signal if not enough differ

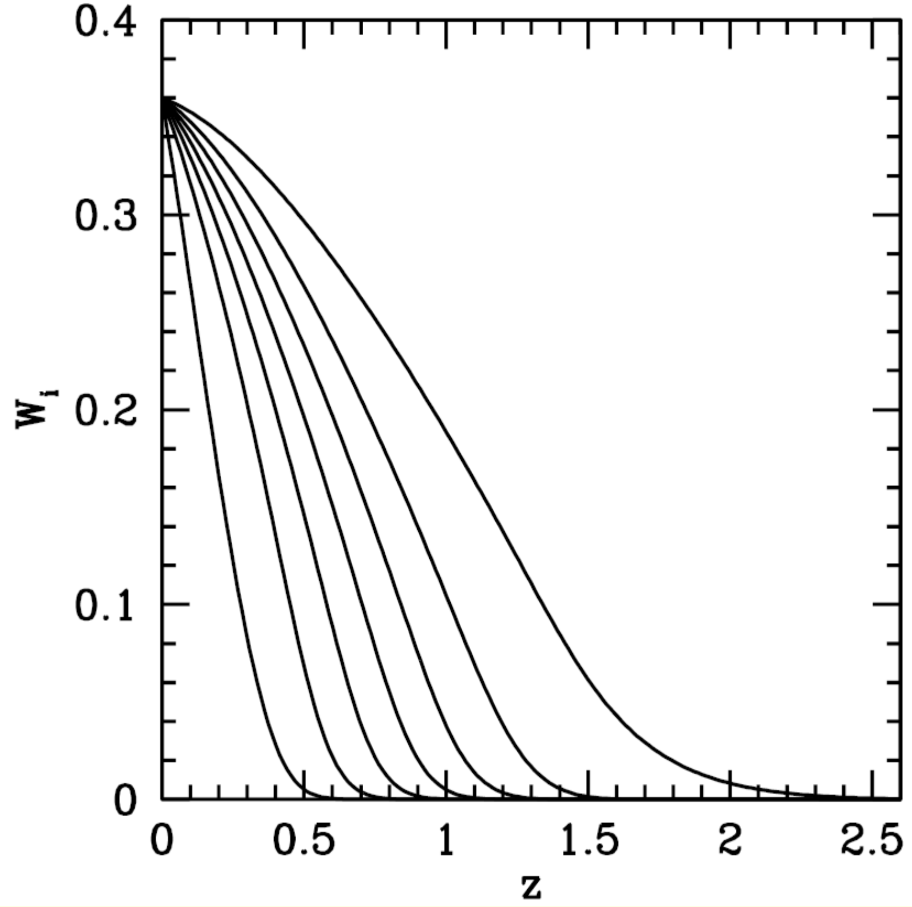
Shot noise ? Adds to C_{ij}(l)
killing bin dependence :
3 or 5 bins same signal with available smpls (10000gls)
Euclid: 1.5 mill gal,
forget shot noise

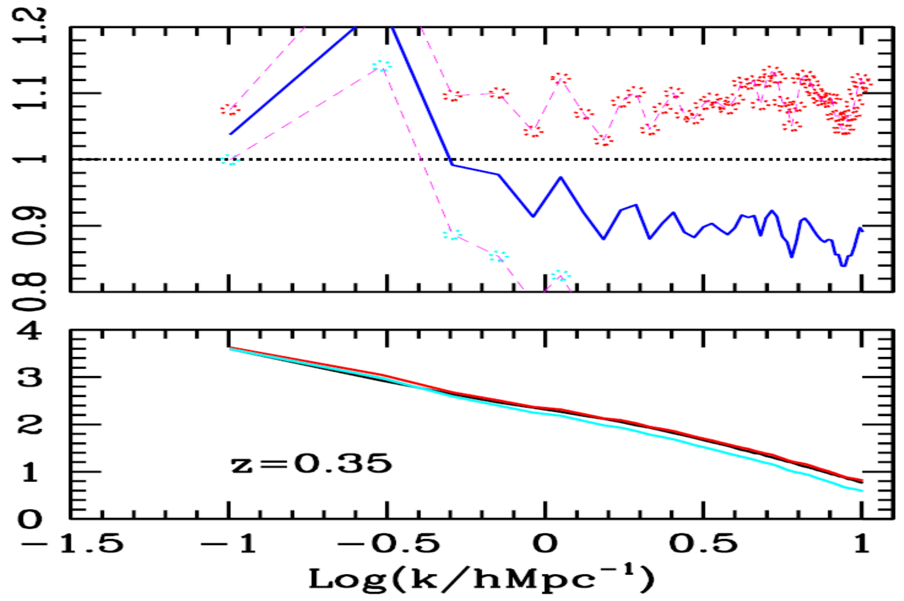
7 bins window functions

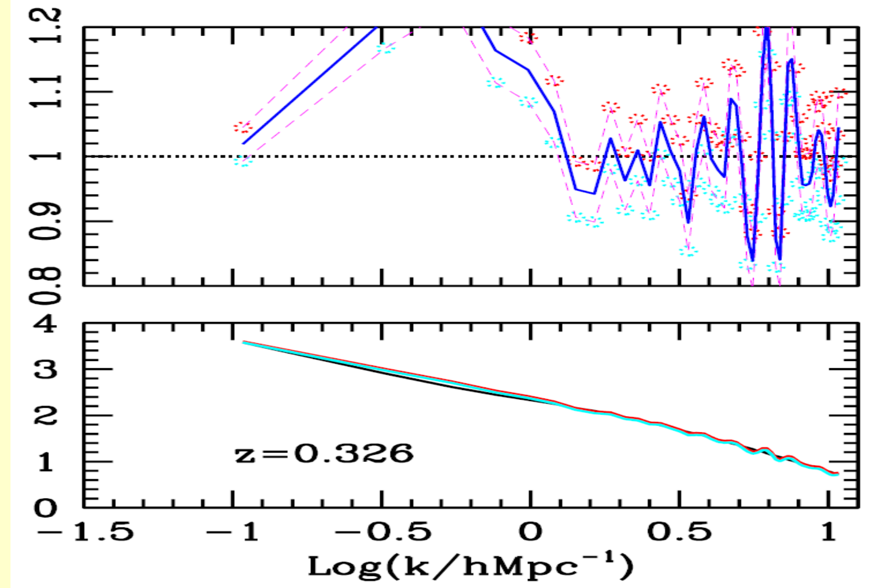
galaxy distribution
(photometric z's)



lensing system
effective distribution



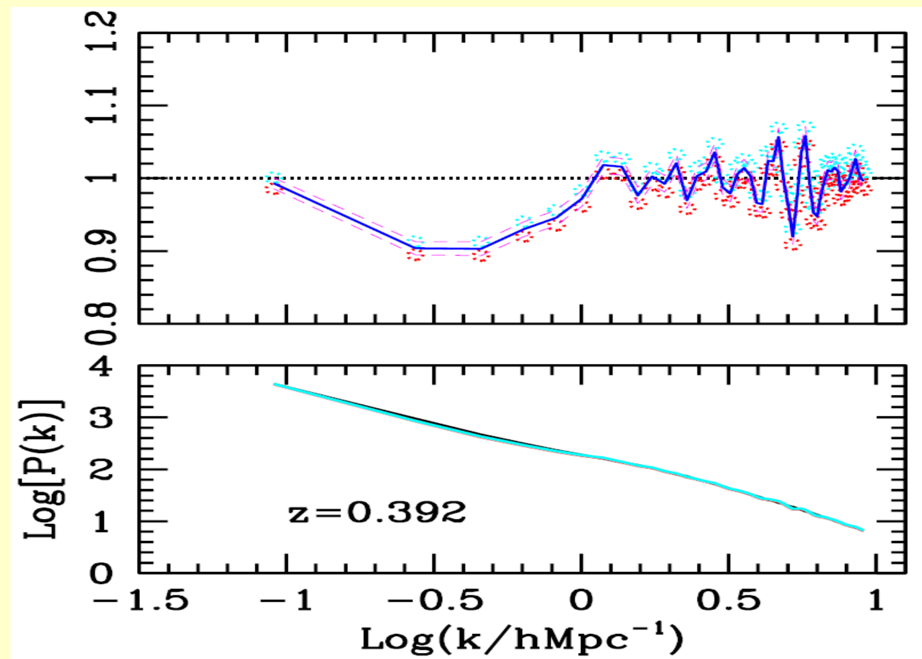




5 bins

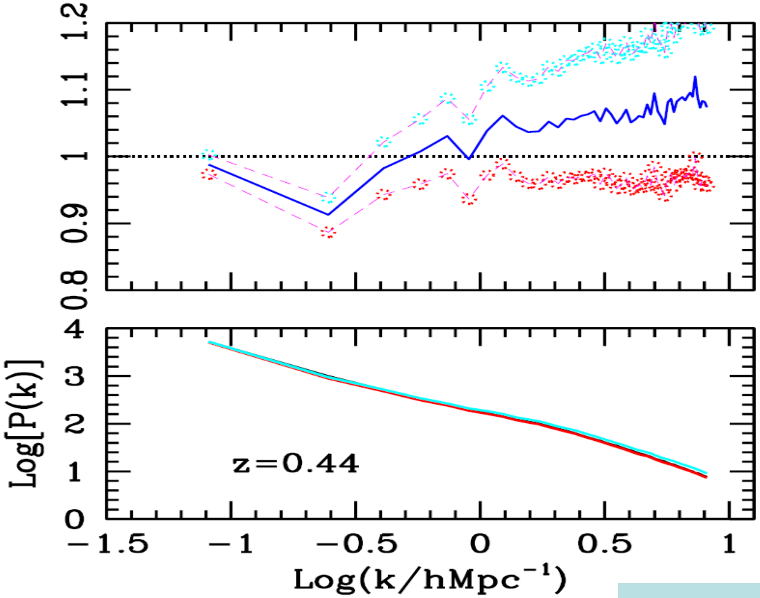
$\bar{u} = 600 \qquad \beta = 1.9$ anywh ere

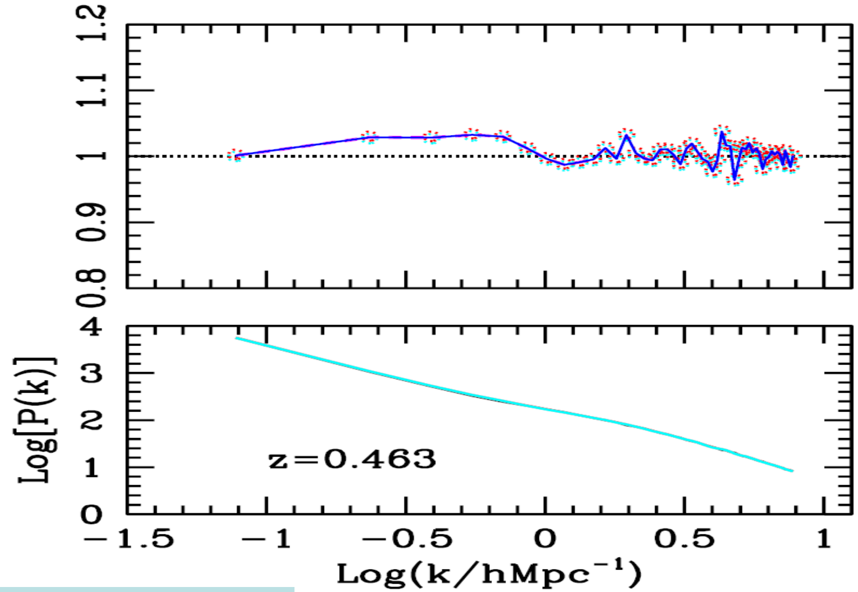
updated operational recipe:
best recovery from average between
simply inverted & renormalized



ordinate range: +/- 20%

7 bins

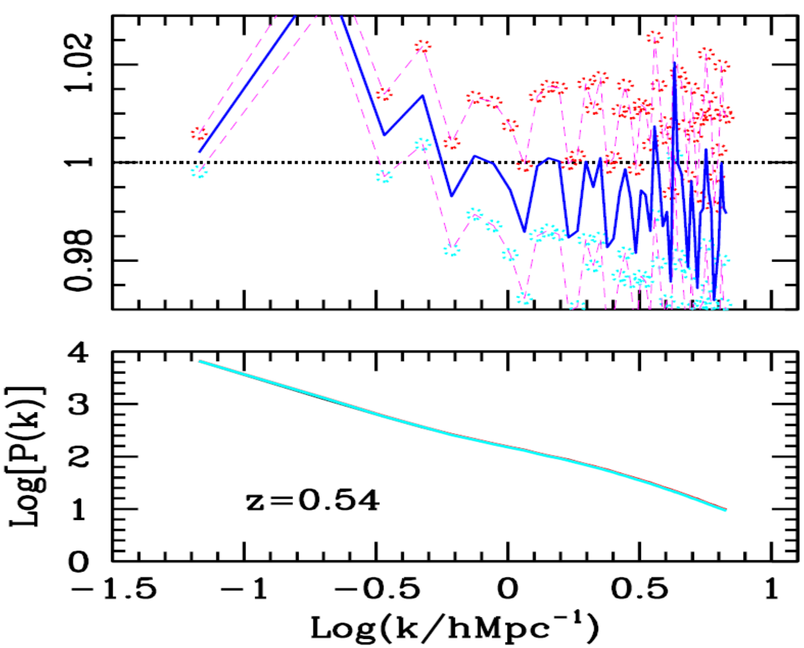


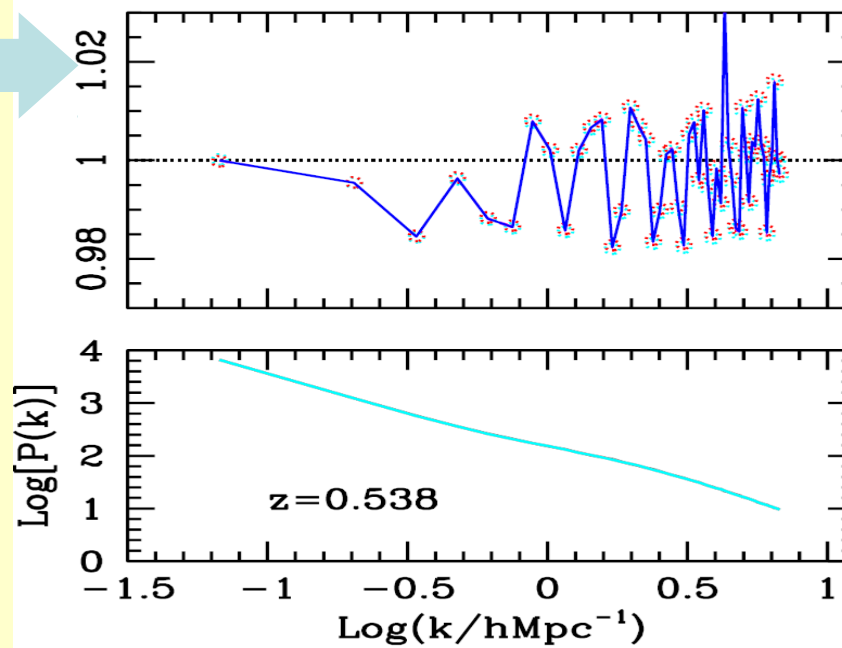


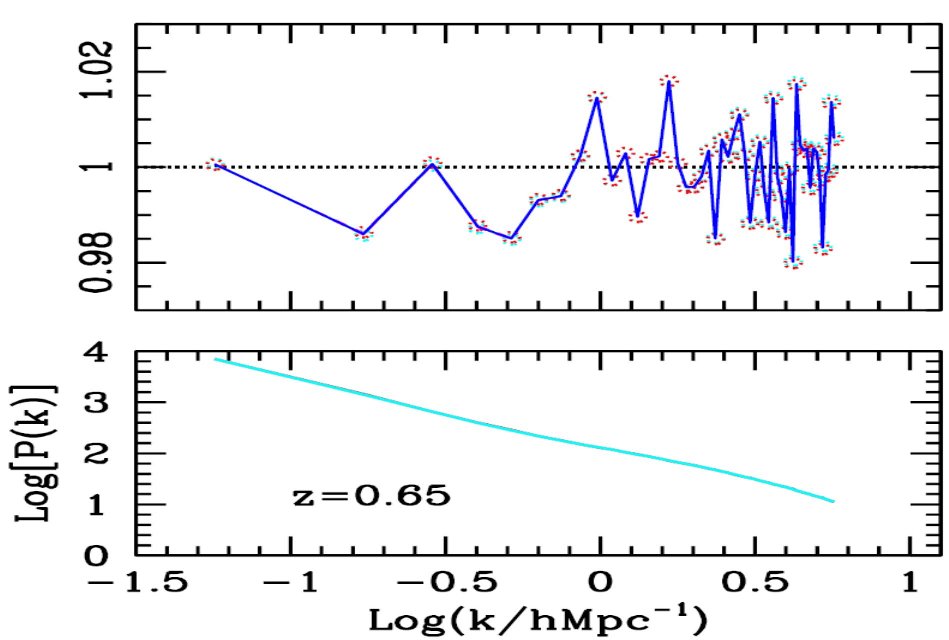
5 bins

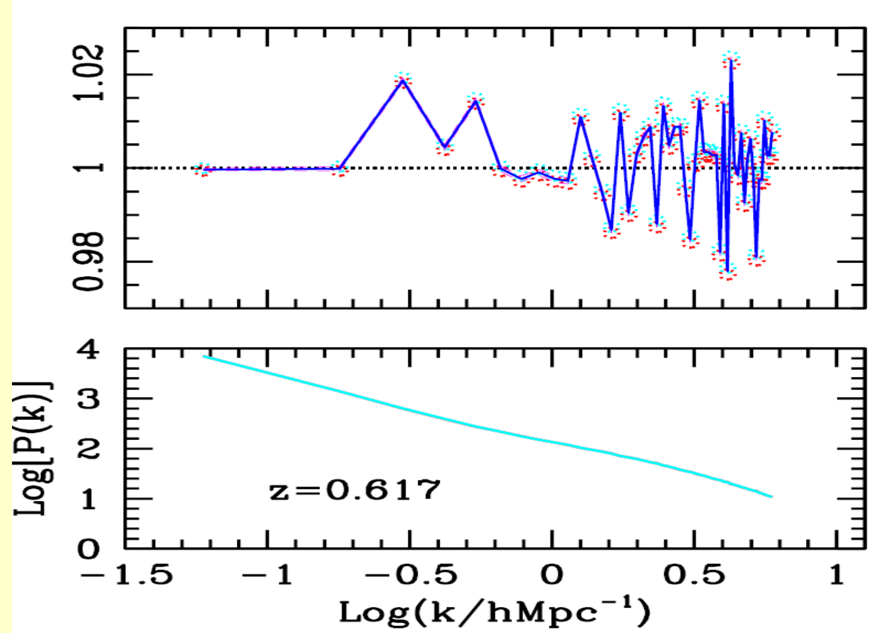
ordinate range: +/- 3%

7 bins



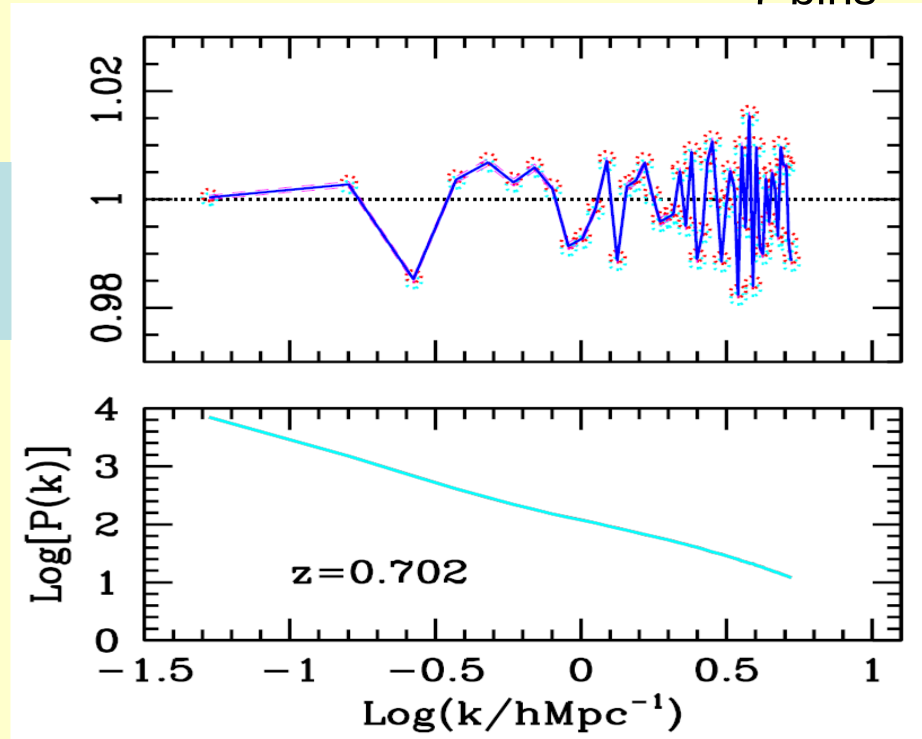




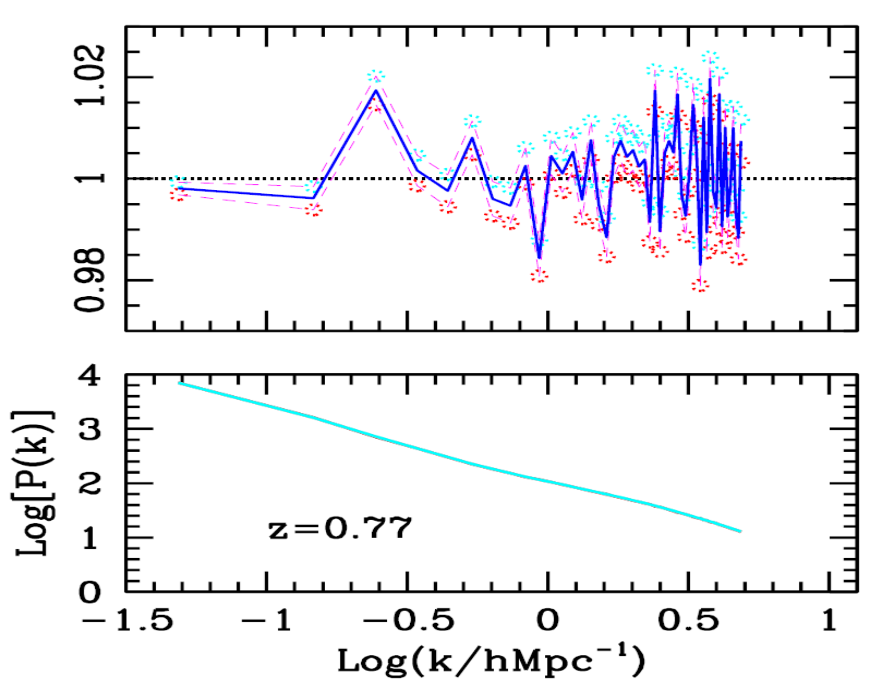


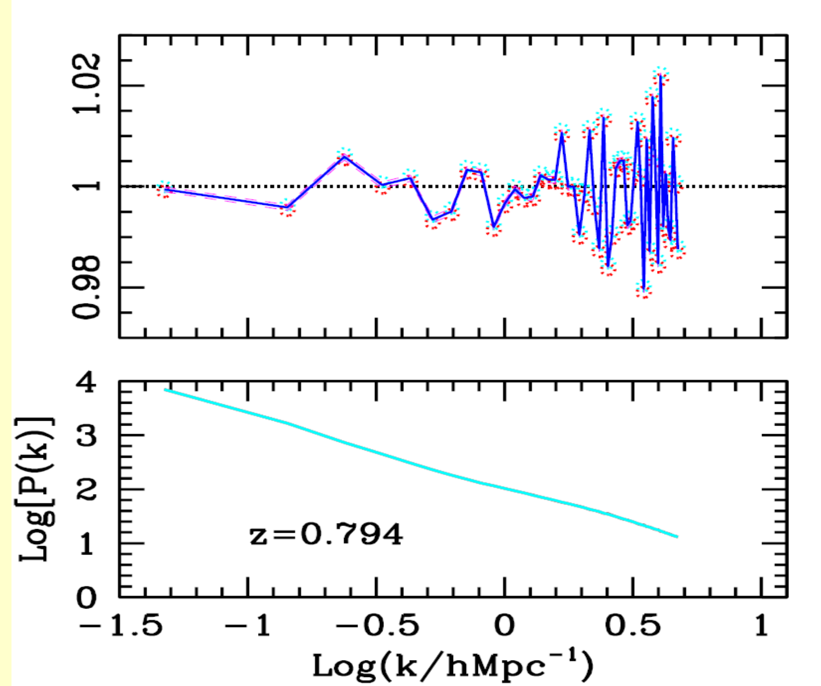
5 bins

7 bins



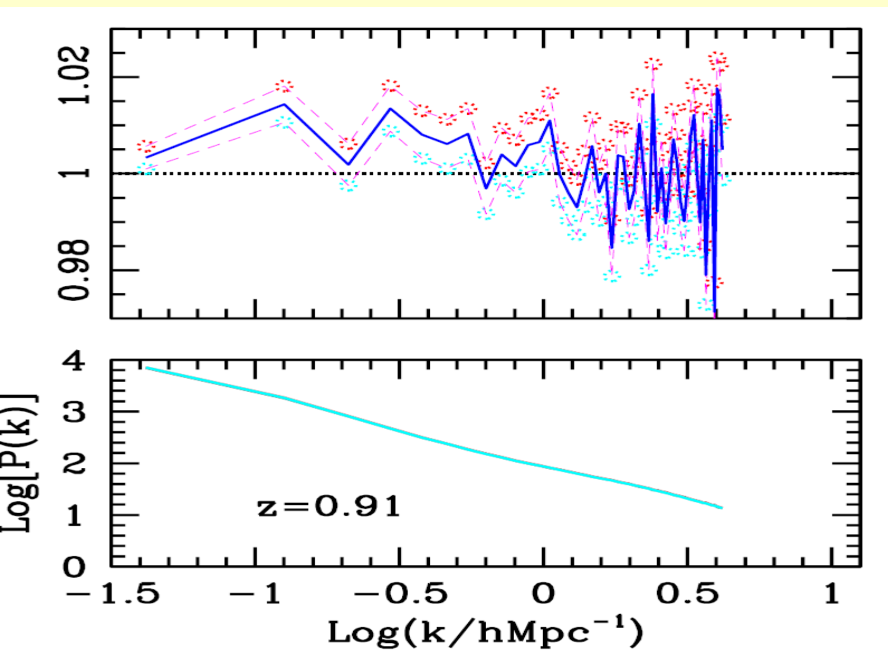
hard to recover spectra with precision > 2%

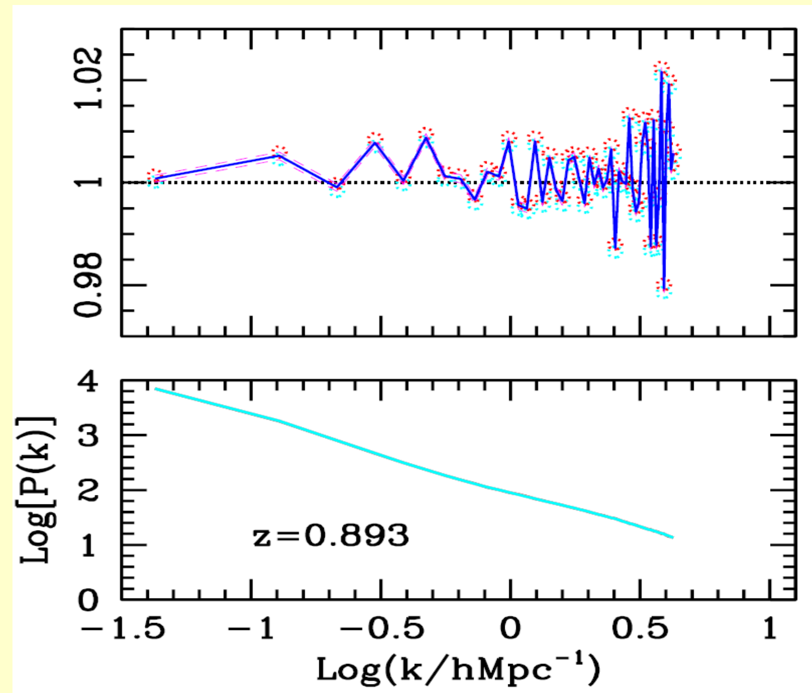

1.02
1
0.98
Log[P(k)]
z=0.77
Log(k/hMpc^{-1})

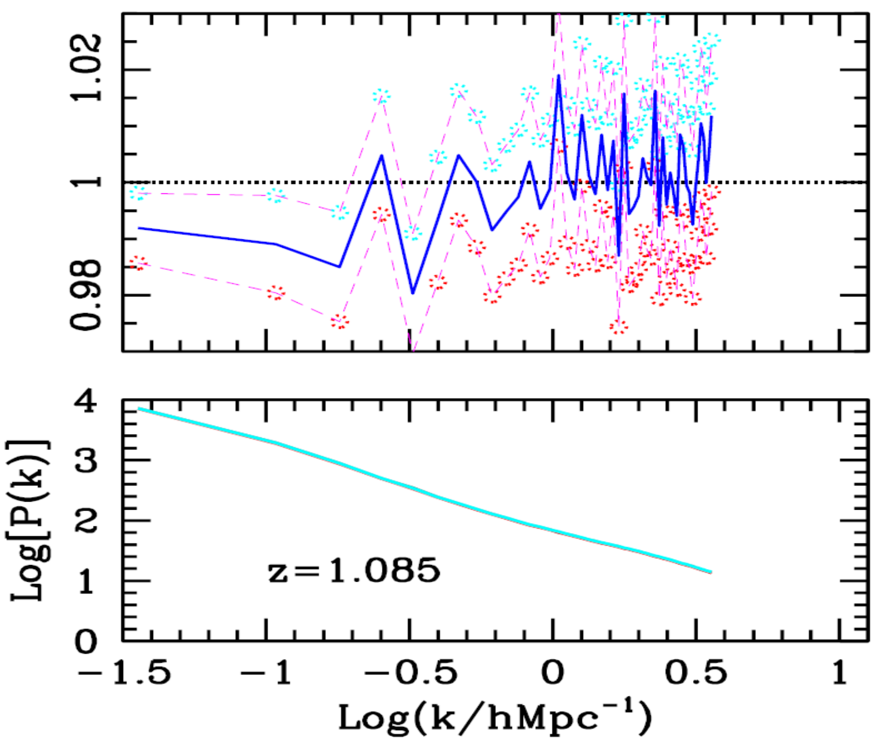

1.02
1
0.98
Log[P(k)]
z=0.794
Log(k/hMpc^{-1})

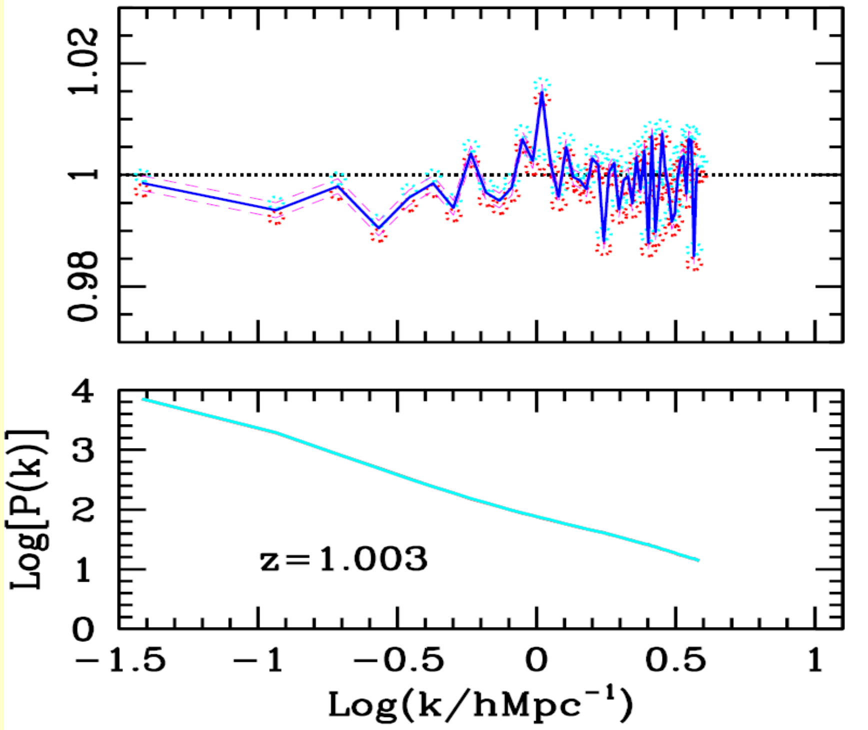
5 bins

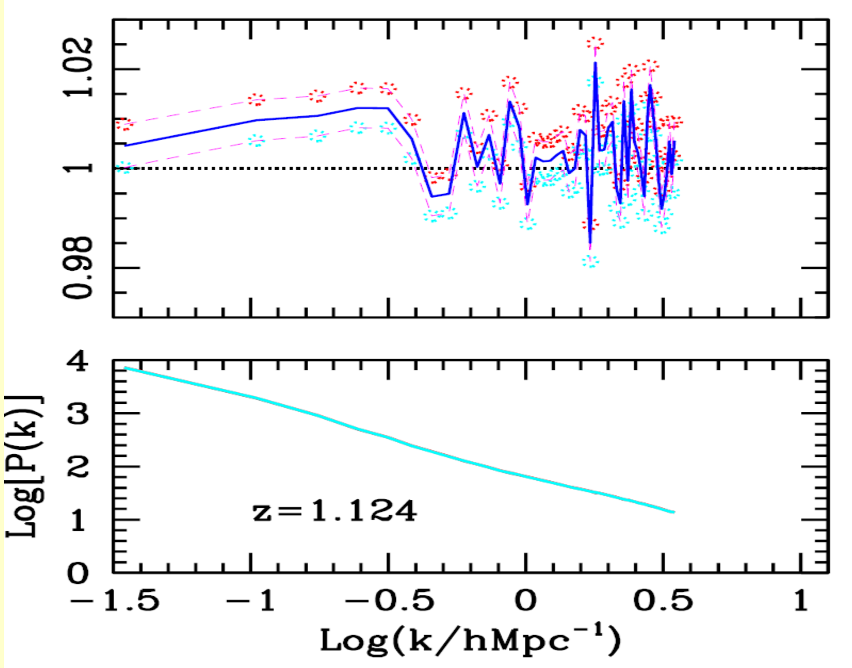
7 bins

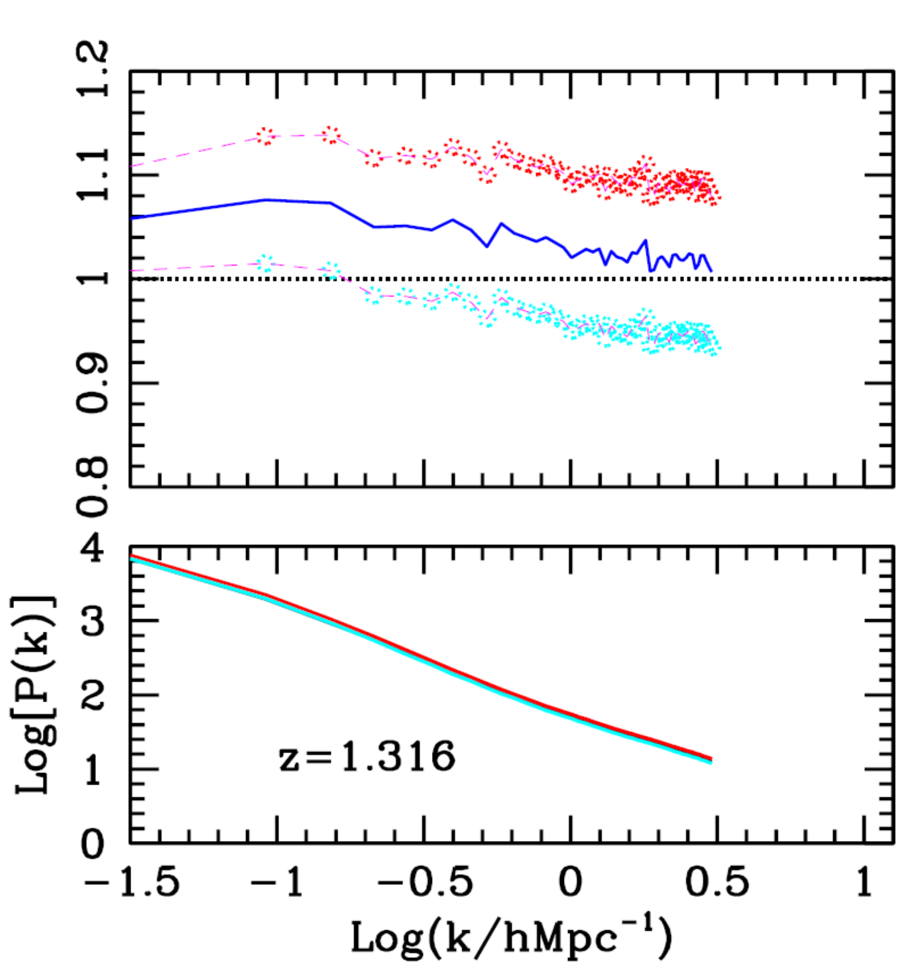

1.02
1
0.98
Log[P(k)]
z=0.91
Log(k/hMpc^{-1})

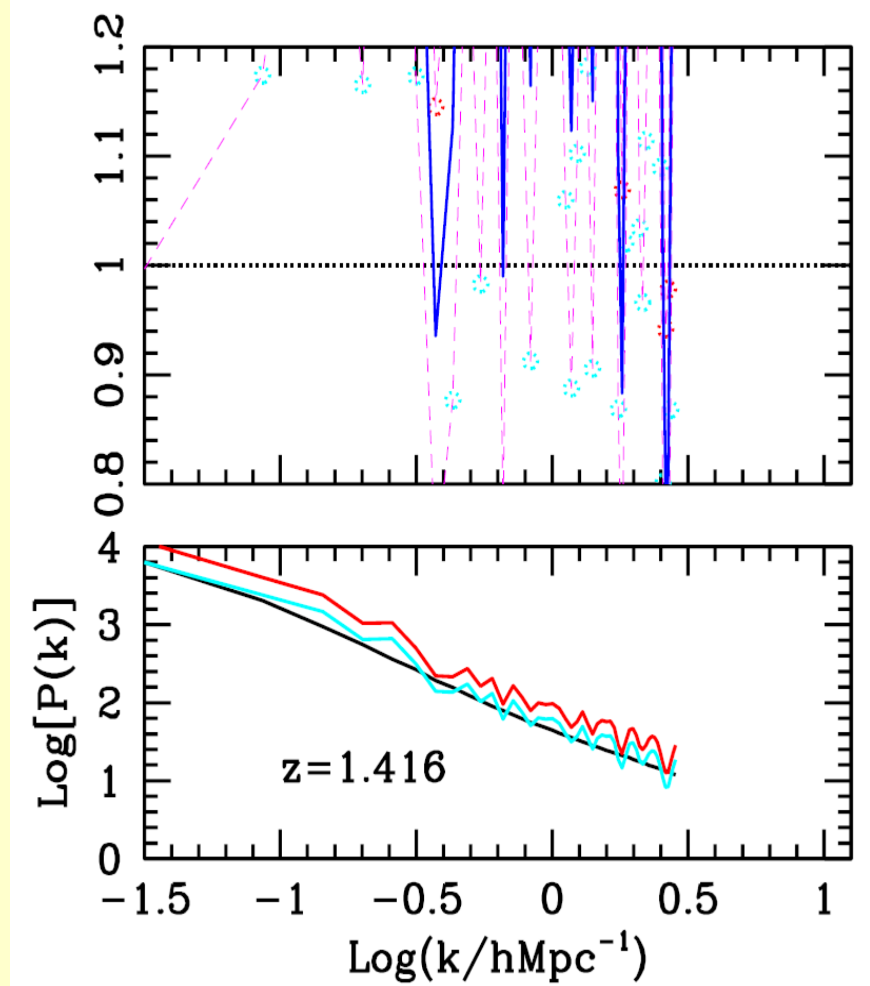

1.02
1
0.98
Log[P(k)]
z=0.893
Log(k/hMpc^{-1})


1.02
1
0.98
Log[P(k)]
4
3
2
1
0
z=1.085
−1.5
−1
−0.5
0
0.5
1
Log(k/hMpc⁻¹)


1.02
1
0.98
Log[P(k)]
4
3
2
1
0
z=1.003
−1.5
−1
−0.5
0
0.5
1
Log(k/hMpc⁻¹)


1.02
1
0.98
Log[P(k)]
4
3
2
1
0
z=1.124
−1.5
−1
−0.5
0
0.5
1
Log(k/hMpc⁻¹)



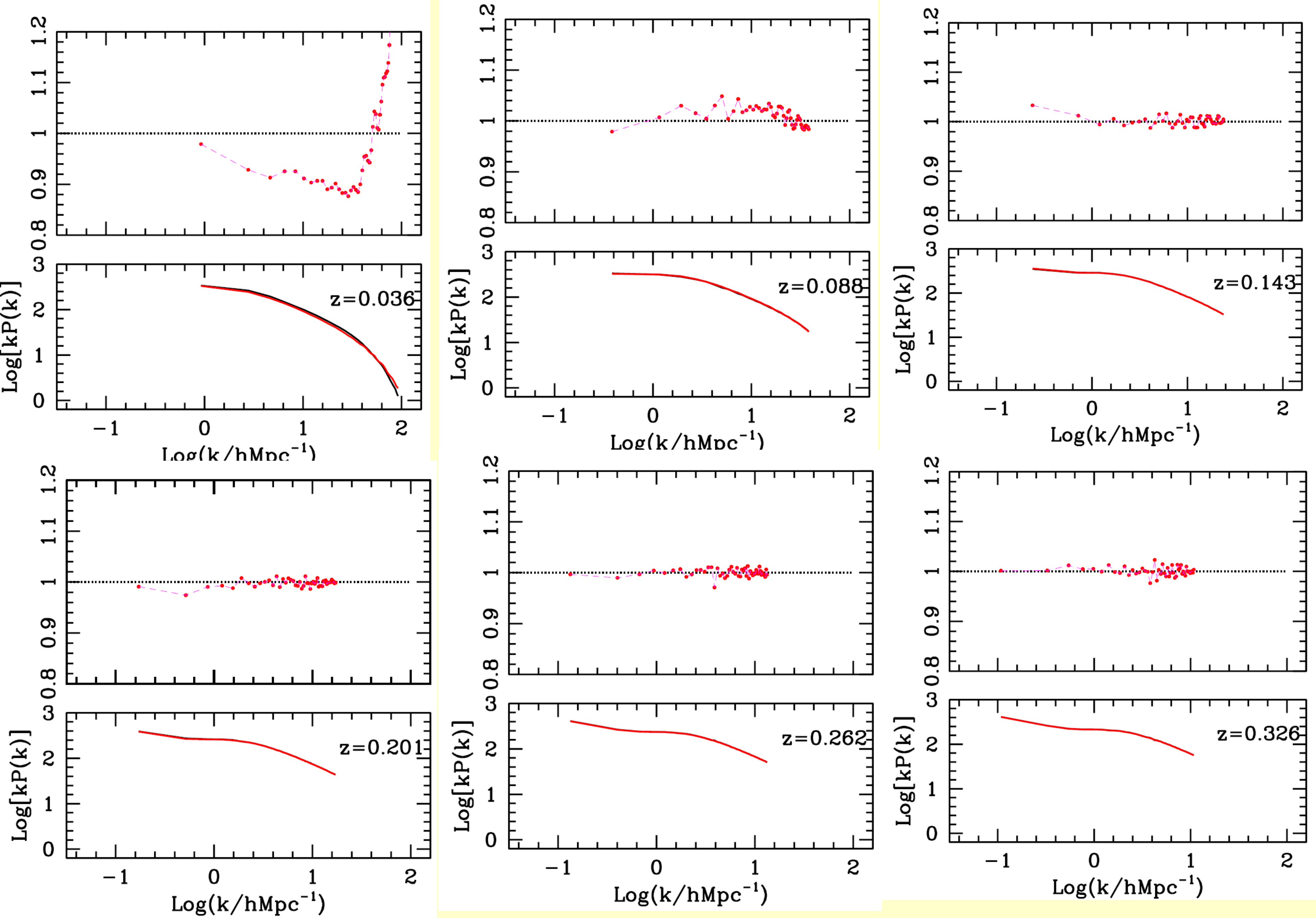


5 bins

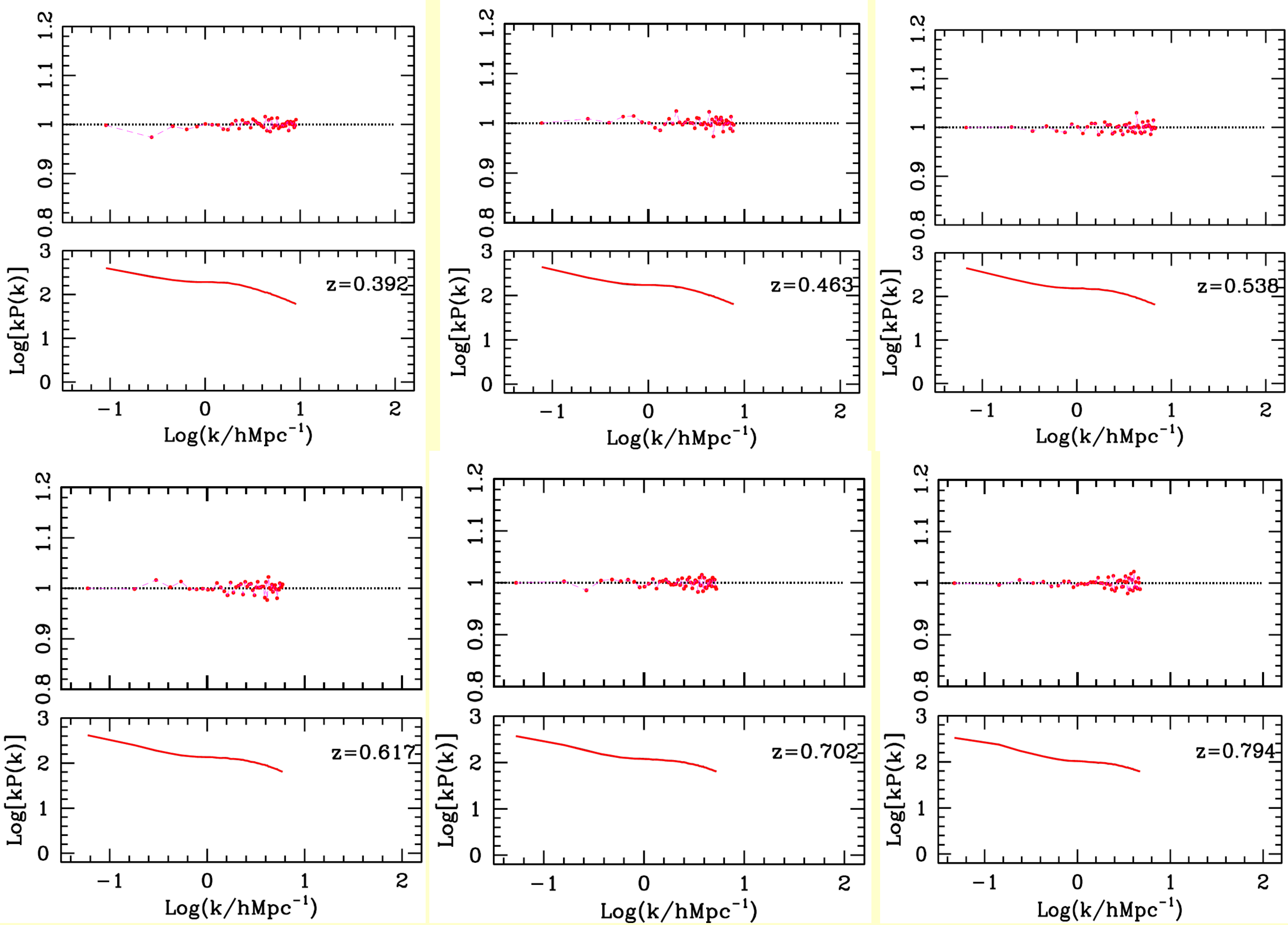
7 bins

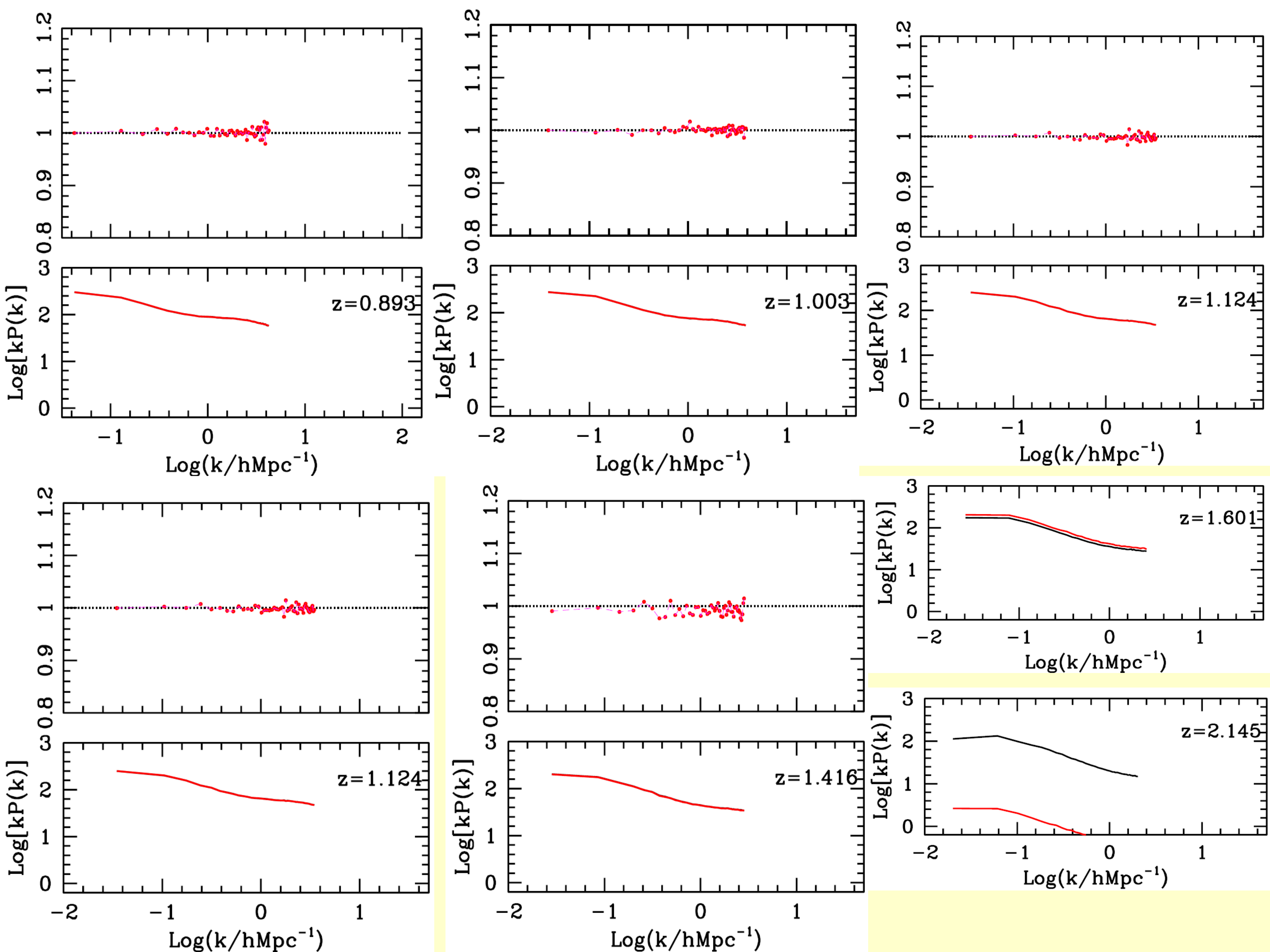
no significant spectral detection at z>1.35

AND 2% recovery only between z=0.54 & z=1.12



ersion – SVD technque exploit.: 3 s_i elemnts set to 0, amng those lasting (17) 6 c


z=0.392
z=0.463
z=0.538
z=0.617
z=0.702
z=0.794
Log[kP(k)]
Log(k/hMpc^{-1})
0.8
0.9
1
1.1
1.2
0
1
2
3
−1
0
1
2



z=0.893
z=1.003
z=1.124
z=1.124
z=1.416
z=1.601
z=2.145
Log[kP(k)]
Log(k/hMpc^{-1})

Inversion requires "geometrical" parameters : Ω_m , h if LCDM

Passing from 5 to 7 bins yields just "some" improvement

unless SVD technique is really exploited

Then: no need to renormalize

Recovered fluctuation spectra can constrain :

- spectral index n
- spectral normalization σ_8
- neutrino spectral effects
- baryon fraction Ω_b
- baryon physics (high k spectral components)

2% interval:
from 7 z's: 0.54 - 1.12, to
16 z's: 0.14(0.09) - 1.42

Open question: Why inversion improves wh

using simulation ?

Method sensitive to using spectra produced

correct dynamics ?

disentangling
baryon physics from
cosmol. parms?

Many thanks

for your