Non-linear PS from resummed PT: the BAO scale and beyond

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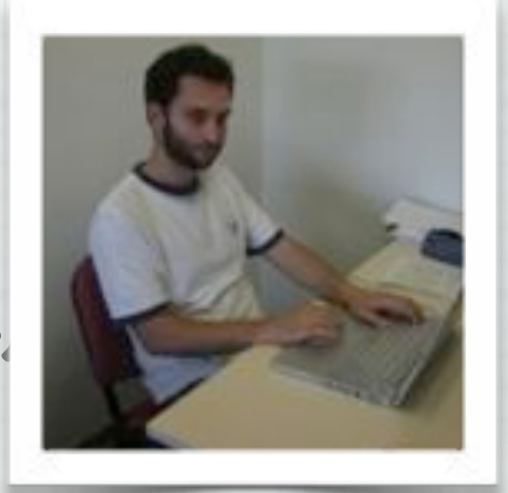


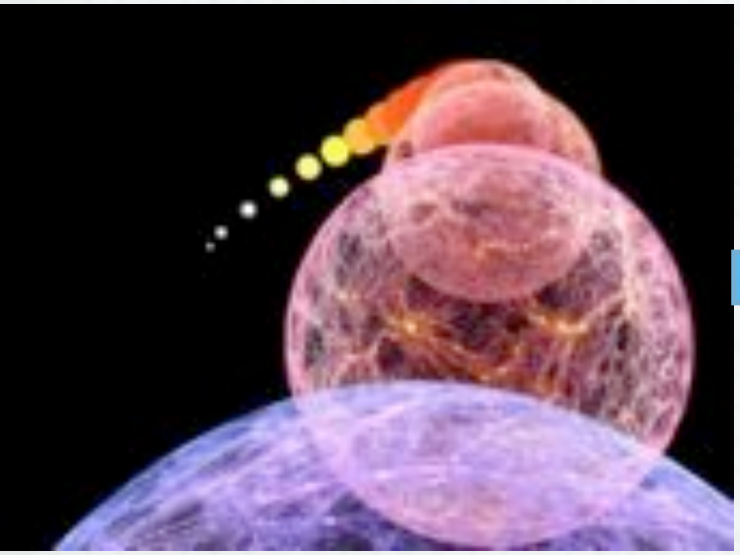
Ringberg Castle
24-29 june 2012

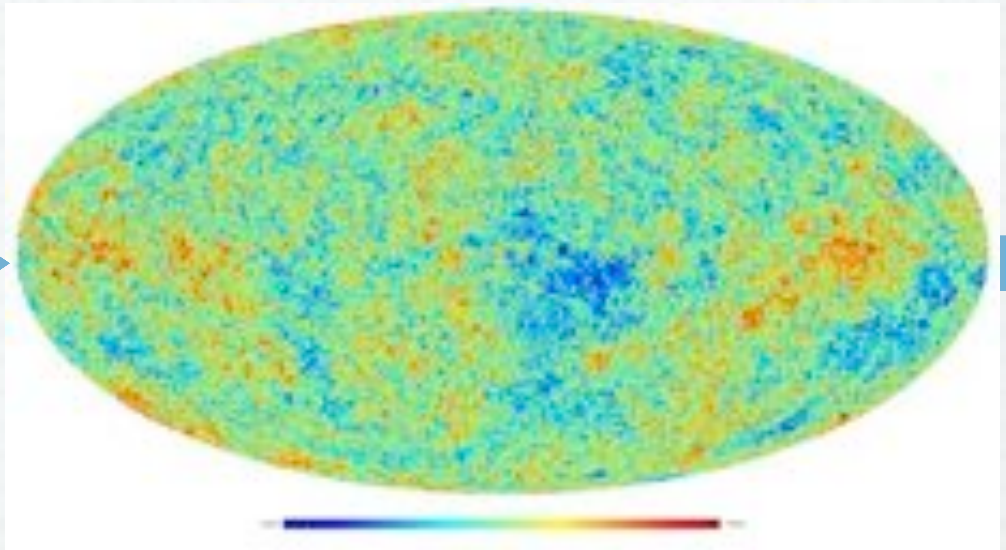
Plan

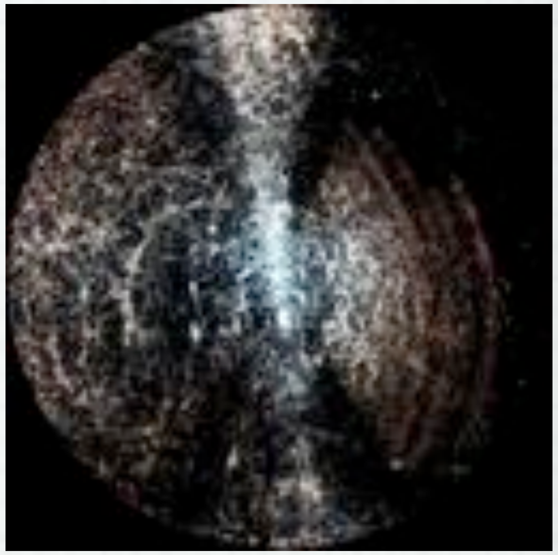
* Brief review of (resummed) perturbation theory (PT)
* improving resummed PT
* the limit of resummed PT

Understanding the LSS of the Universe

Inflation | **Decoupling** | **Today**







Linear, Gaussian

$$\left(\frac{\delta\rho}{\rho} \simeq 10^{-5}\right)$$

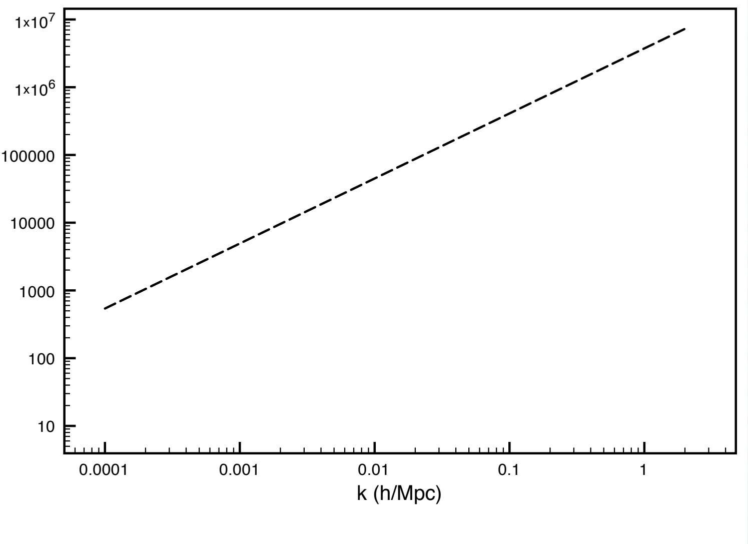
Linear, Gaussian

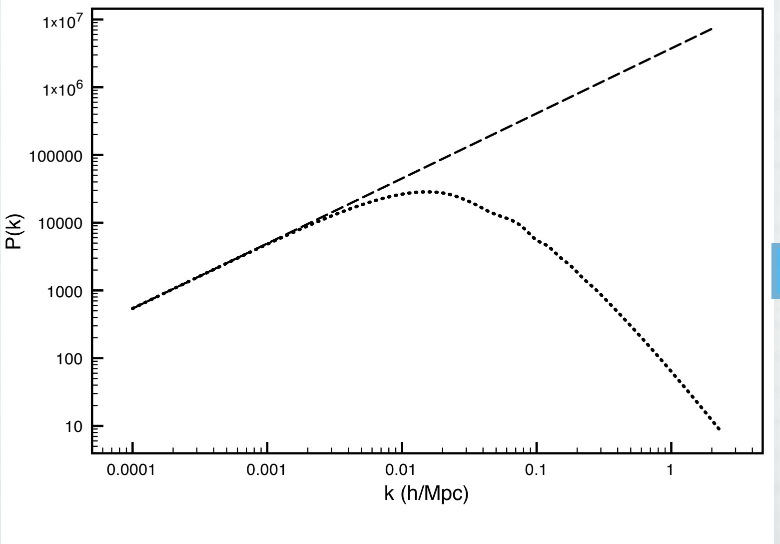
non-Linear, non-Gaussian

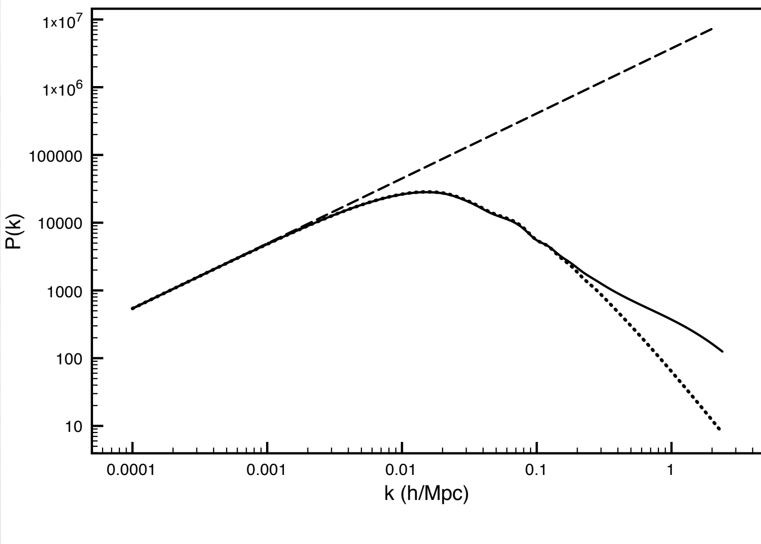
primordial density perturbations

photon-baryon-DM-neutrino....fluid

non-rel. matter







Why do we need to study the late (and non-linear) evolution?

* Dark Energy (Baryonic Acoustic Oscillations)
* Neutrino masses
* Non-Gaussianity
* Weak gravitational lensing
* ...

The future of precision cosmology: non-linear scales

matter density

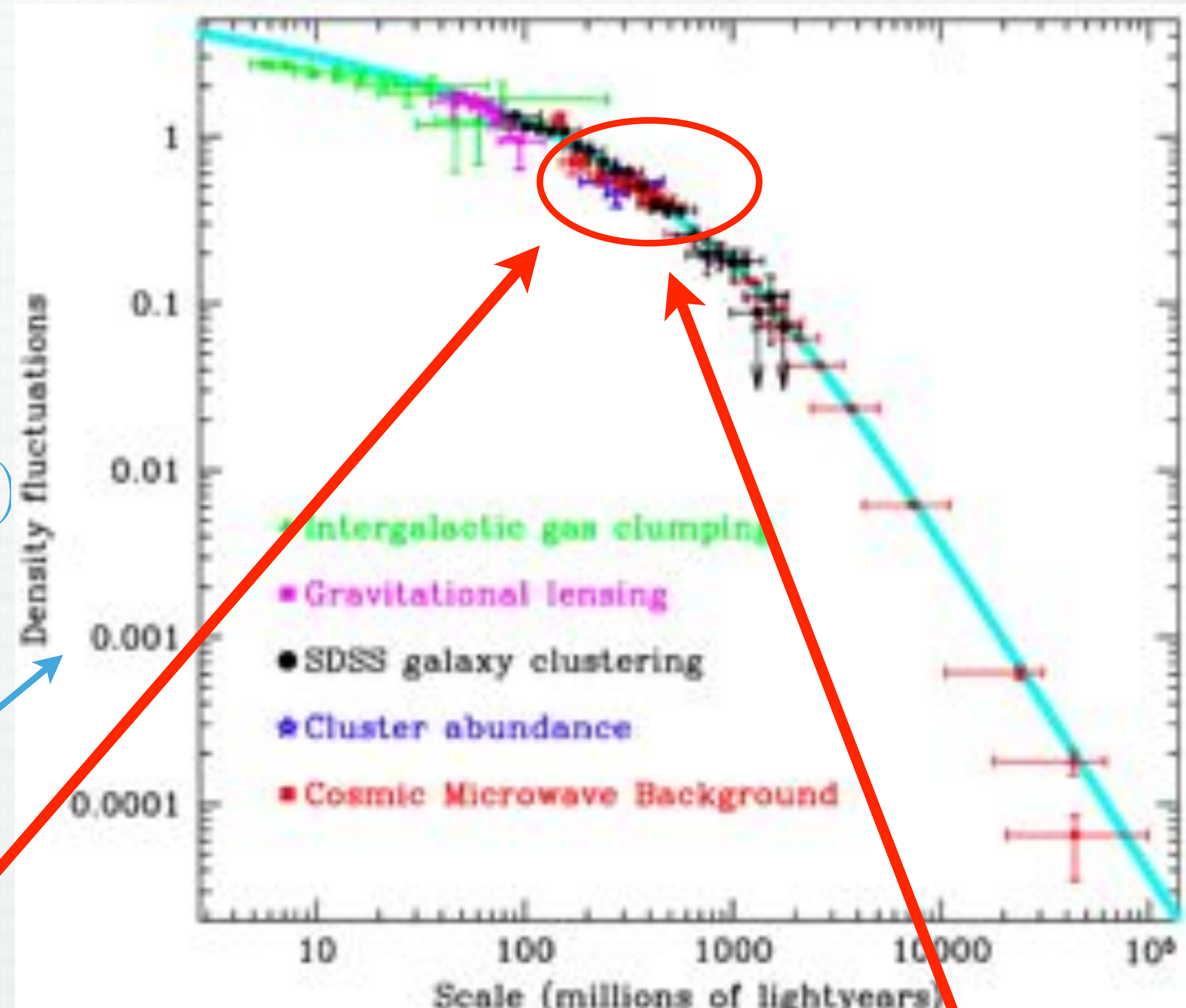
$$\rho(\mathbf{x},\,\tau) \equiv \bar{\rho}(\tau)[1+\delta(\mathbf{x},\,\tau)]$$

power spectrum

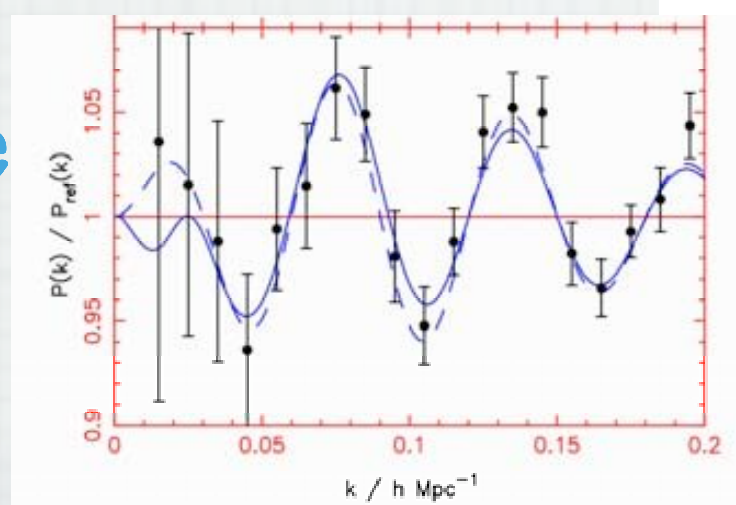
$$\langle\delta(\mathbf{k},\,\tau)\,\delta(\mathbf{k}',\,\tau)\rangle = P(k,\,\tau)\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

`size' of the fluctuations at different scales/epochs:

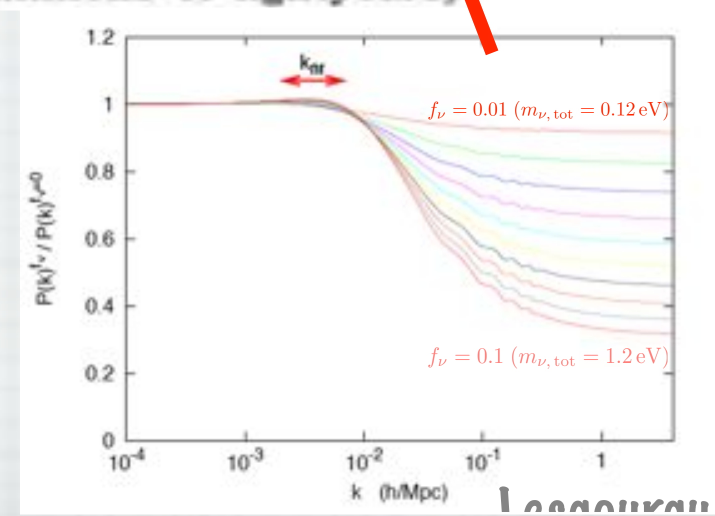
$$\Delta^2(k,\tau) = 4\pi k^3 P(k,\tau)$$

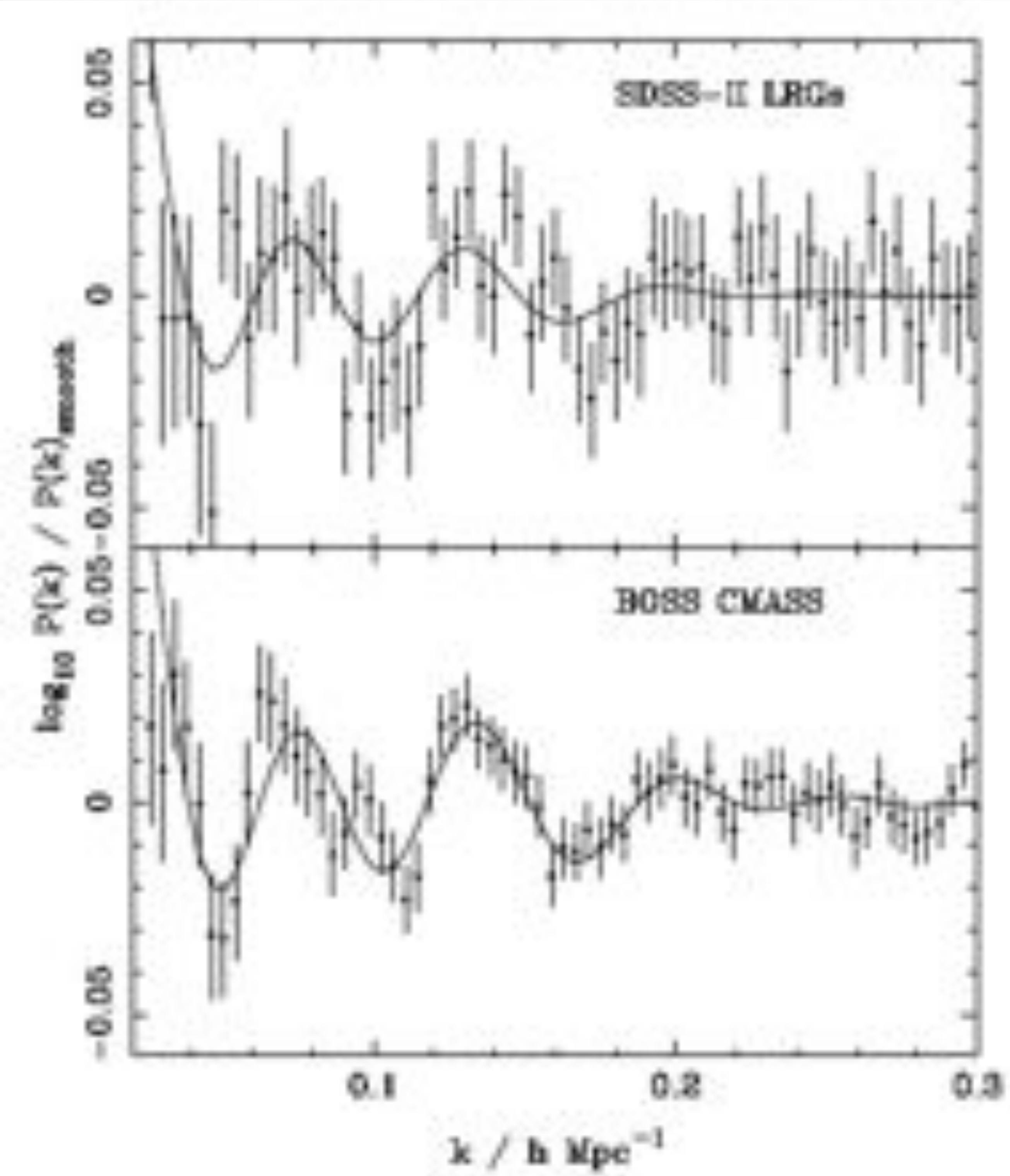


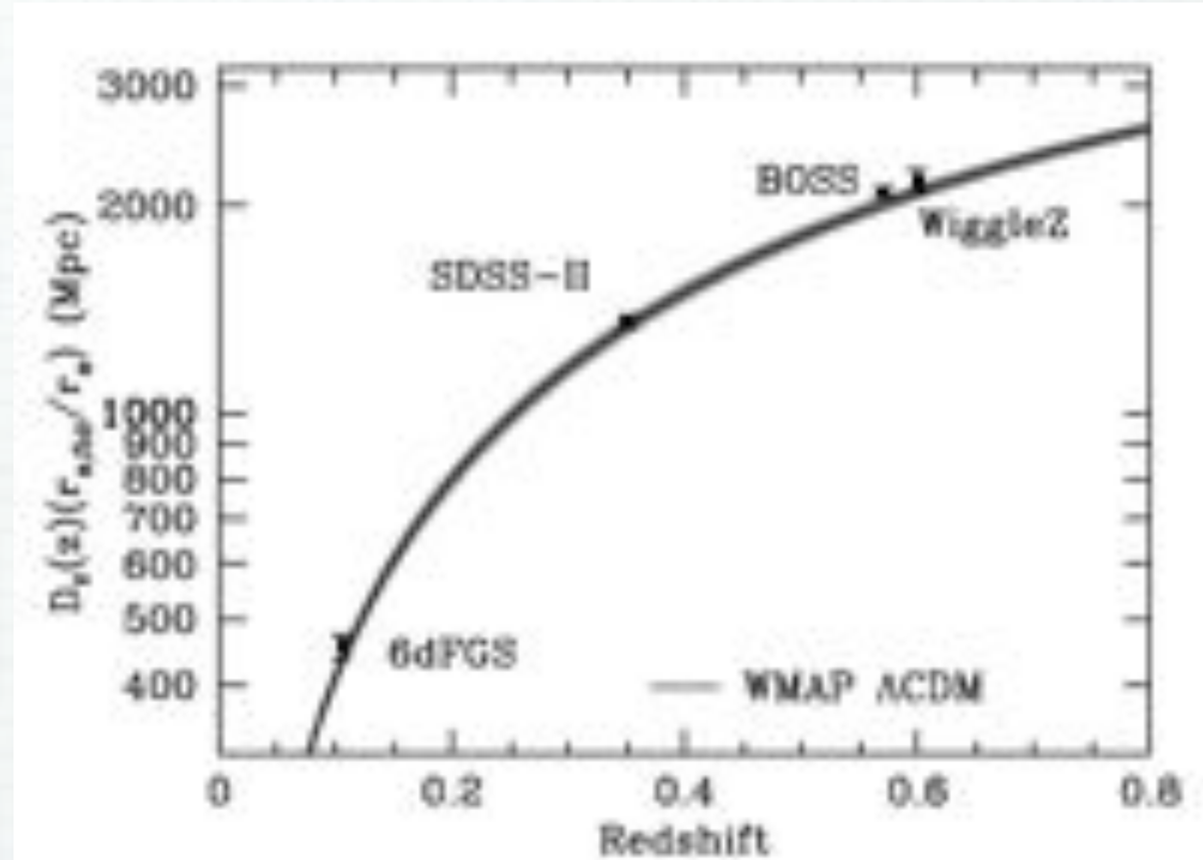
Baryonic Acoustic Oscillations (BAO)



Neutrino mass bounds







To be dramatically improved in the future: EUCLID, ...

WiggleZ 1105.2862

BOSS 1203.6594

The LSS mantra

%

... and fast

→ scan over different cosmologies

not trivial even for Nbody

* Initial conditions, large volumes, mass resolution, time-stepping (Heitmann et al 2010)
* non-LCDM models: (massive neutrinos, coupled quintessence, f(R), primordial NG, clustering DE,...)
* not fast!

The Eulerian way

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0\,,$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$

subhorizon scales, newtonian gravity
single stream approximation

Compact Perturbation Theory

Crocce, Scoccimarro '05

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

define $\left(\begin{array}{c} \varphi_1(\eta,\mathbf{k}) \\ \varphi_2(\eta,\mathbf{k}) \end{array}\right) \equiv e^{-\eta}\left(\begin{array}{c} \delta(\eta\,,\mathbf{k}) \\ -\theta(\eta\,,\mathbf{k})/\mathcal{H} \end{array}\right)$ with $\eta = \log\dfrac{D^+(\tau)}{D^+(\tau_i)}$

$$\mathbf{\Omega} = \left(\begin{array}{cc} 1 & -1 \\ -3/2 & 3/2 \end{array}\right)$$

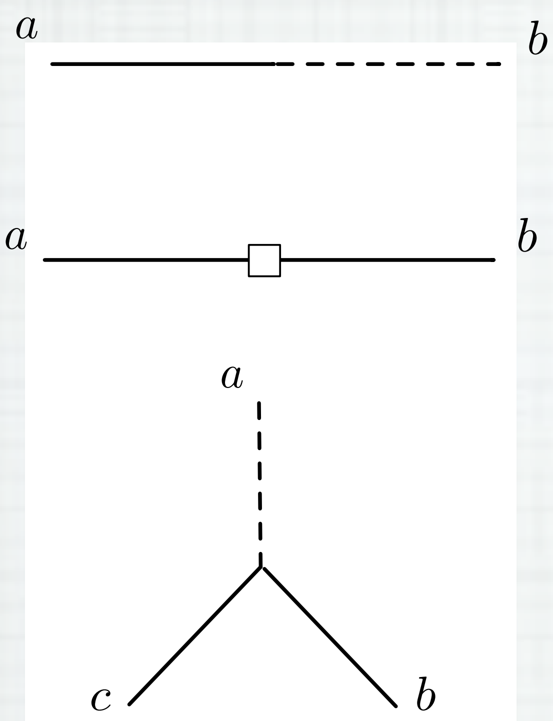
then we can write:

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta,\mathbf{k}) = e^{\eta}\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

linear

nonlinear

Perturbation Theory: Feynman Rules

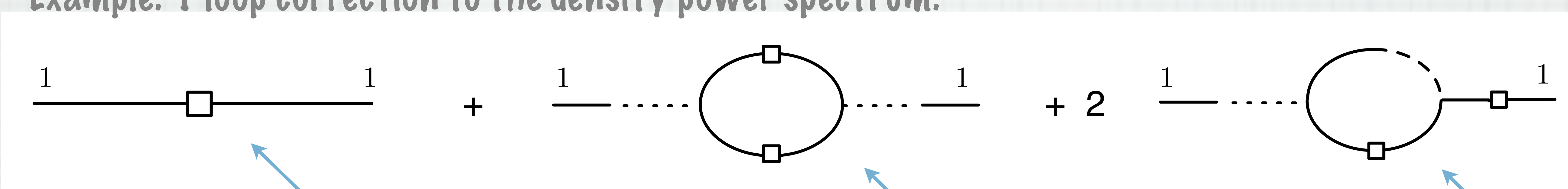


propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$

interaction vertex: (mode coupling) $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

Example: 1-loop correction to the density power spectrum:



Linear Power spectrum

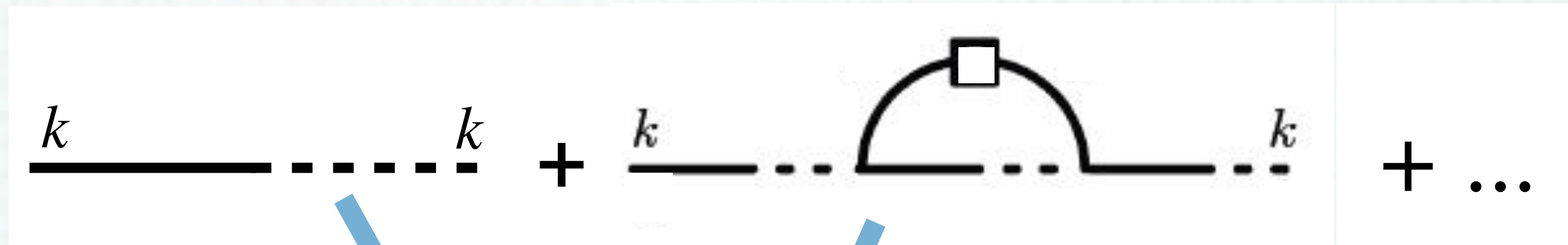
"P22"

"P13"

All known results in cosmological perturbation theory are expressible in terms of diagrams in which <u>only a trilinear fundamental interaction</u> appears

PT in the BAO range

1-loop propagator @ large k:



$$G_{ab}(k;\eta_a,\,\eta_b) = g_{ab}(\eta_a,\,\eta_b)\left[1 - k^2\sigma^2\,\frac{(e^{\eta_a} - e^{\eta_b})^2}{2}\right] + O(k^4\sigma^4)$$

$$\left(\sigma^2 \equiv \frac{1}{3}\int d^3q\,\frac{P^0(q)}{q^2}\right)$$

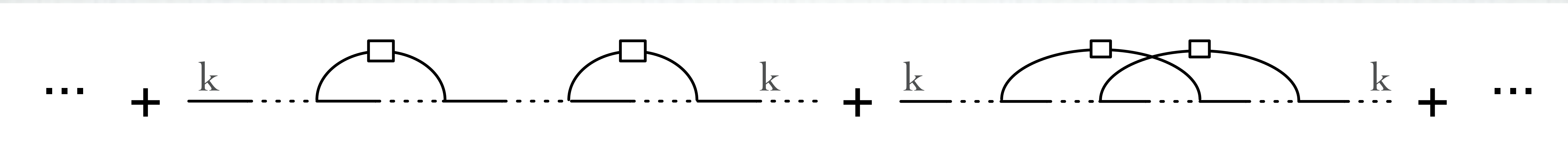
$(\sigma\, e^{\eta_a})^{-1} \simeq 0.15\,\mathrm{h\,Mpc^{-1}}$
in the BAO range!

2-loop

the PT series blows up in the BAO range

But it can be resummed!!

(Crocce-Scoccimarro '06)



$$G(k;\eta,\eta_{in}) = \frac{\langle \delta(k,\eta)\delta(k,\eta_{in})\rangle}{\langle \delta(k,\eta_{in})\delta(k,\eta_{in})\rangle} \sim e^{-\frac{k^2\sigma^2}{2}e^{2\eta}}$$

physically, it represents the effect of multiple interactions of the k-mode with the surrounding modes: memory loss

`coherence momentum' $k_{ch} = (\sigma\, e^{\eta})^{-1} \simeq 0.15\,\mathrm{h\,Mpc^{-1}}$

damping in the BAO range!

RPT: use G, and not g, as the linear propagator

More General Cosmologies

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,,$$

$$\frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}(1+A(\vec{x},\,\tau))\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

deviation from geodesic (e.g. DM-scalar field interaction)

$$\nabla^2\phi = 4\pi G\,(1+B(\vec{x},\,\tau))\,\rho\,a^2\,\delta$$

deviation from Poisson (e.g. scale-dep. growth factor)

$$(\delta_{ab}\partial_\eta + \Omega_{ab}(\eta,\,\mathbf{k}))\,\varphi_b(\eta,\mathbf{k}) = e^\eta\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

$$(\eta = \log a)$$

$$\mathbf{\Omega}_{ab} = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_M(1+B(\eta,\,\mathbf{k})) & 2+\frac{\mathcal{H}'}{\mathcal{H}}+A(\eta,\,\mathbf{k}) \end{pmatrix}$$

Chameleon, symmetron,... require new vertices!!

Partial (!) list of contributors to the field

- "traditional" P.T.: see Bernardeau et al, Phys. Rep. 367, 1, (2002), and refs. therein; Jeong-Komatsu; Saito et al; Sefusatti;...
- resummation methods: Valageas; Crocce-Scoccimarro; McDonald; Matarrese-M.P.; Matsubara; M.P.; Taruya-Hiratamatsu; Bernardeau-Valageas; Bernardeau-Crocce-Scoccimarro; Tassev-Zaldarriaga, Juergens-Bartelmann,...

Improving resummed PT methods

* PS not yet at % in the BAO range down to z=0
* rapidly degrading at smaller scales
* not fast enough: O(hrs) for a P(k,z)

Exact time-evolution equations I: the propagator

Anselmi, Matarrese, MP '11

a) start from the exact expression

$$G_{ab}(k;\eta,\eta') = g_{ab}(\eta,\eta') + \int ds\, ds' g_{ac}(\eta,s)\Sigma_{cd}(k;s,s')G_{cb}(k;s',\eta')$$

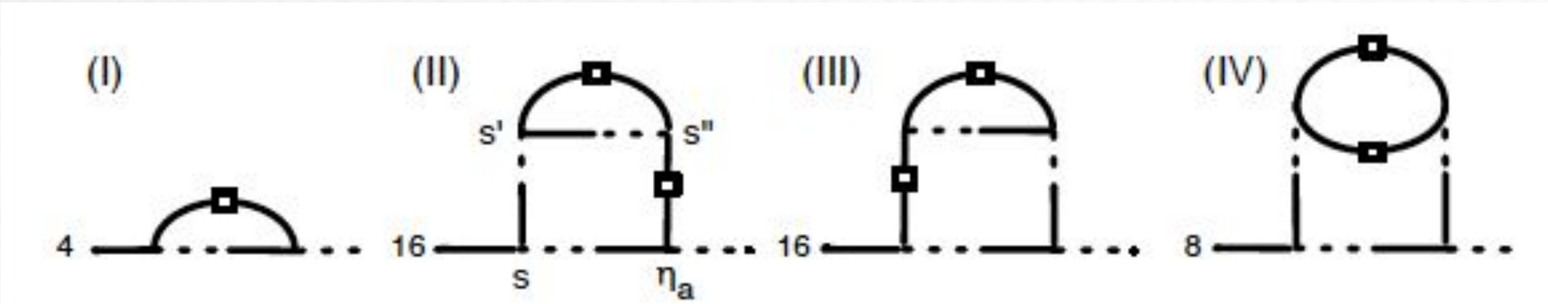
b) take the time derivative...

$$\partial_\eta G_{ab}(k;\eta,\eta') = \delta_{ab}\,\delta_D(\eta-\eta') - \Omega_{ac}\,G_{cb}(k;\eta,\eta') + \Delta G_{ab}(k;\eta,\eta')$$

$$\Delta G_{ab}(k;\eta,\eta') \equiv \int ds'\,\Sigma_{ad}(k;\eta,s')\,G_{db}(k;s',\eta')$$

c) compute Σ ...

$$\Sigma_{ab}(k; s, s')$$



+ higher loops...

small k: $\Sigma_{ab}(k; s, s') \simeq \Sigma_{ab}^{1-loop}(k; s, s') \longrightarrow G_{ab} \to G_{ab}^{1-loop}$

large k: $$\int ds' \Sigma_{ad}(k; \eta, s') G_{db}(k; s', \eta') \simeq -k^2 \sigma^2 e^{\eta} (e^{\eta} - e^{\eta'}) G_{ab}(k; \eta, \eta')$$

$$G_{ab} \to \exp\left(- \frac{k^2 \sigma^2 (e^{\eta} - e^{\eta'})^2}{2} \right) g_{ab}$$

1-loop evaluation of Σ gives the propagator at all loops!!

Exact time-evolution equations II: the power spectrum

Anselmi, MP '12

a) exact expression

$$P_{ab}(k;\eta,\eta') = G_{ac}(k;\eta,\eta_{in})G_{bd}(k;\eta',\eta_{in})P_{cd}(k;\eta_{in},\eta_{in}) + \int ds\,ds'\; G_{ac}(k;\eta,s)G_{bd}(k;\eta',s')\Phi_{cd}(k;s,s')$$

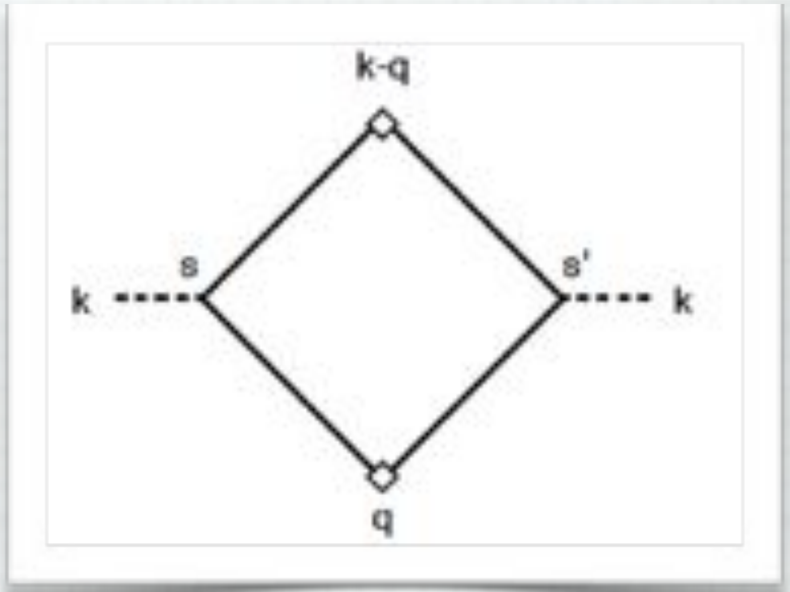
b) time derivative

$$\partial_\eta P_{ab}(k;\eta) = -\Omega_{ac}P_{cb}(k;\eta) - \Omega_{bc}P_{ac}(k;\eta) + H_{\mathrm{a}}(k;\,\eta,\eta_{in})\,P_{\mathrm{ab}}(k;\eta) + H_{\mathrm{b}}(k;\,\eta,\eta_{in})\,P_{\mathrm{ab}}(k;\eta) + \int ds\,[\tilde{\Phi}_{ad}(k;\eta,s)\bar{G}_{bd}(k;\,\eta,s) + \bar{G}_{ad}(k;\,\eta,s)\tilde{\Phi}_{db}(k;s,\eta)]$$

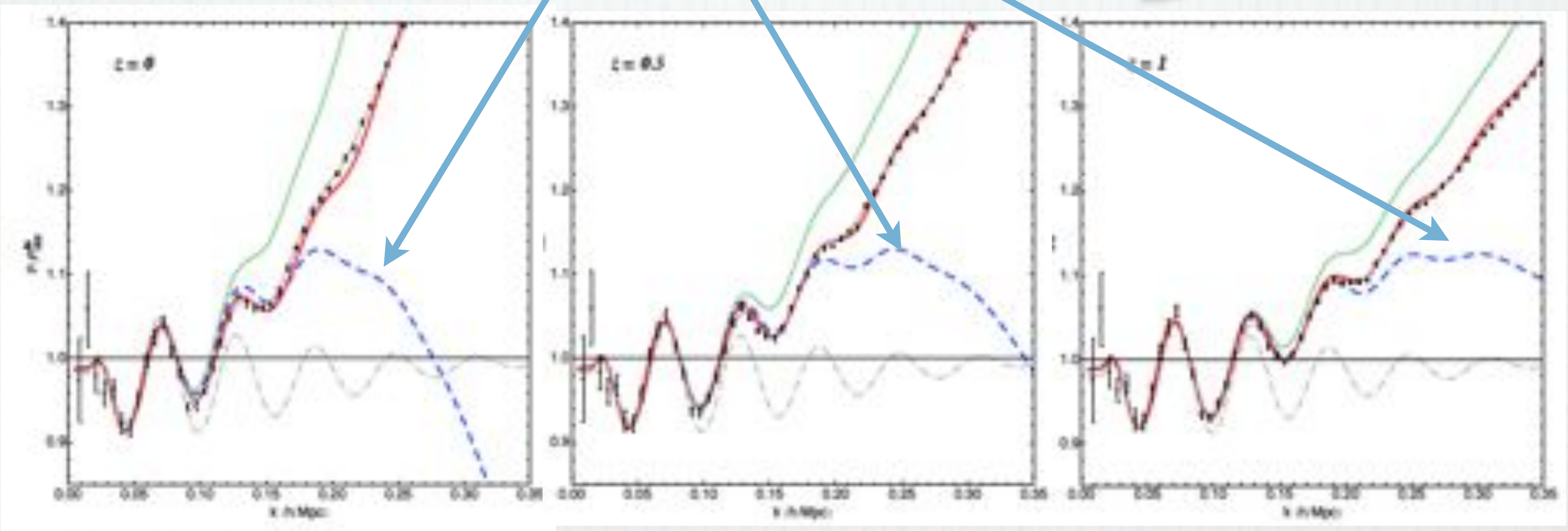
$$H_a(k;\,\eta,s) \equiv \int_s^\eta ds''\,\Sigma^{(1)}_{ae}(k;\,\eta\,,s'')\,u_e$$

Already computed for the propagator

$$\Phi_{ab}(k; s, s')$$



small k: $\Phi_{ab}(k; s, s') \simeq \Phi_{ab}^{1-loop}(k; s, s')$

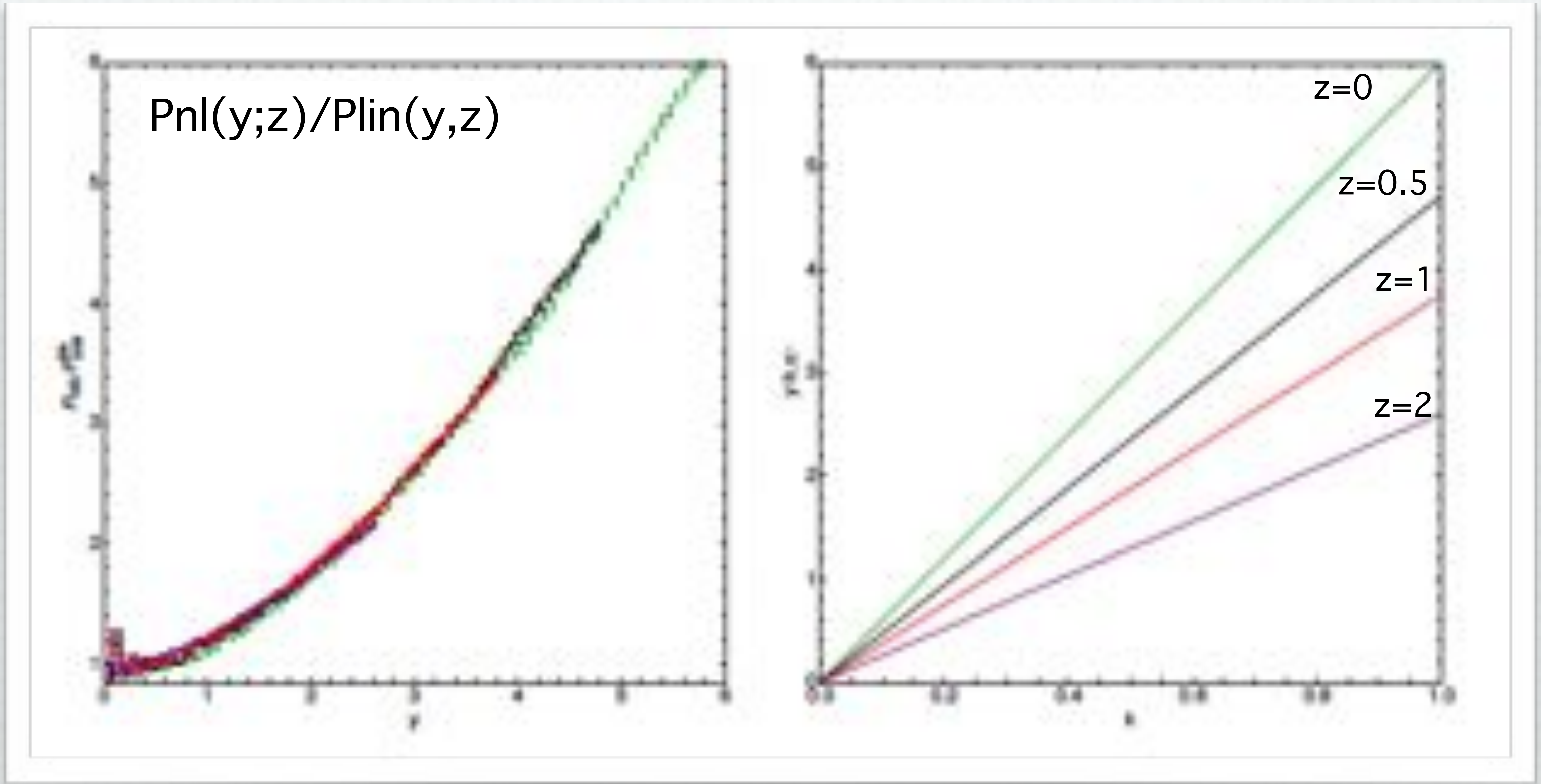


1-loop evaluation of Σ and Phi reproduces the BAO's at the % level at all z

The large k regime for the PS

for the nonlinear propagator, the relevant variable in the large k regime is : $y \equiv k\,\sigma(e^{\eta} - e^{\eta_i})$

$$G(k;\eta,\eta_i) \sim \exp\left(-\frac{y^2}{2}\right)$$

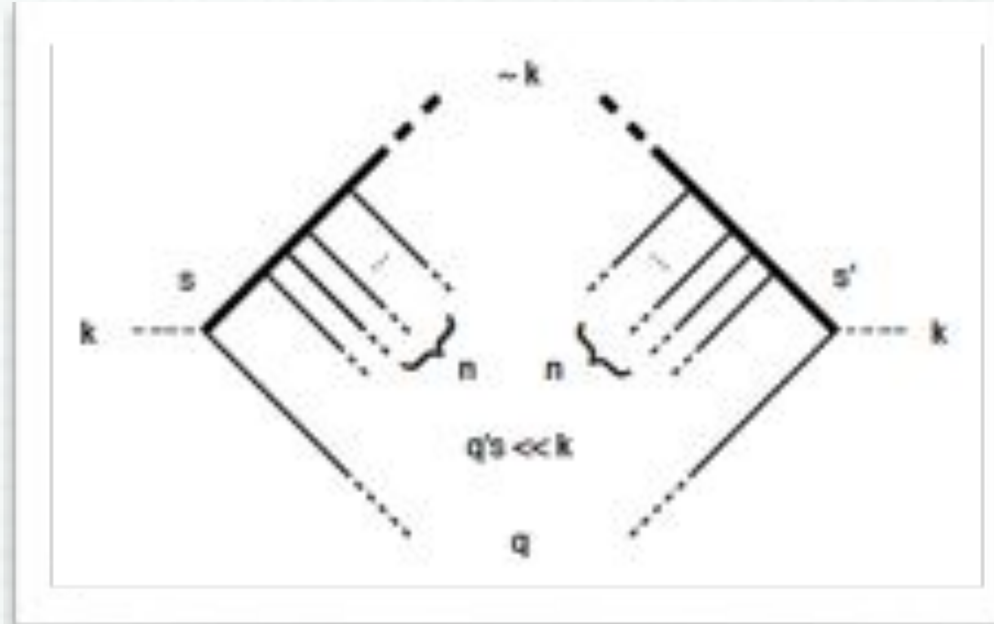


The non-linear PS is y-dependent too. resummation possible !!

$$\Phi_{ab}(k; s, s')$$

large k

leading contributions to Phi:

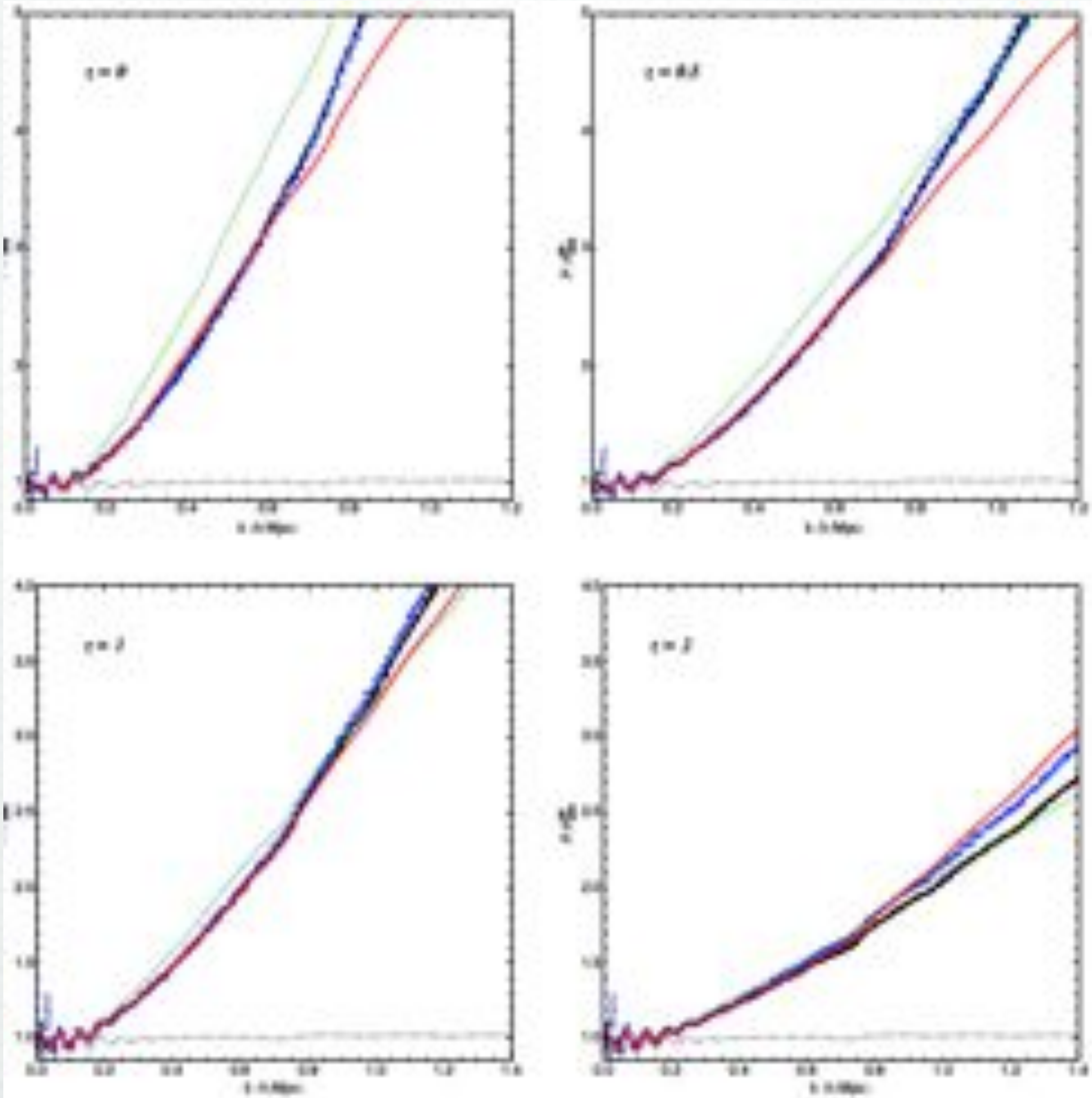


1 "hard" loop momentum, n-1 "soft" ones

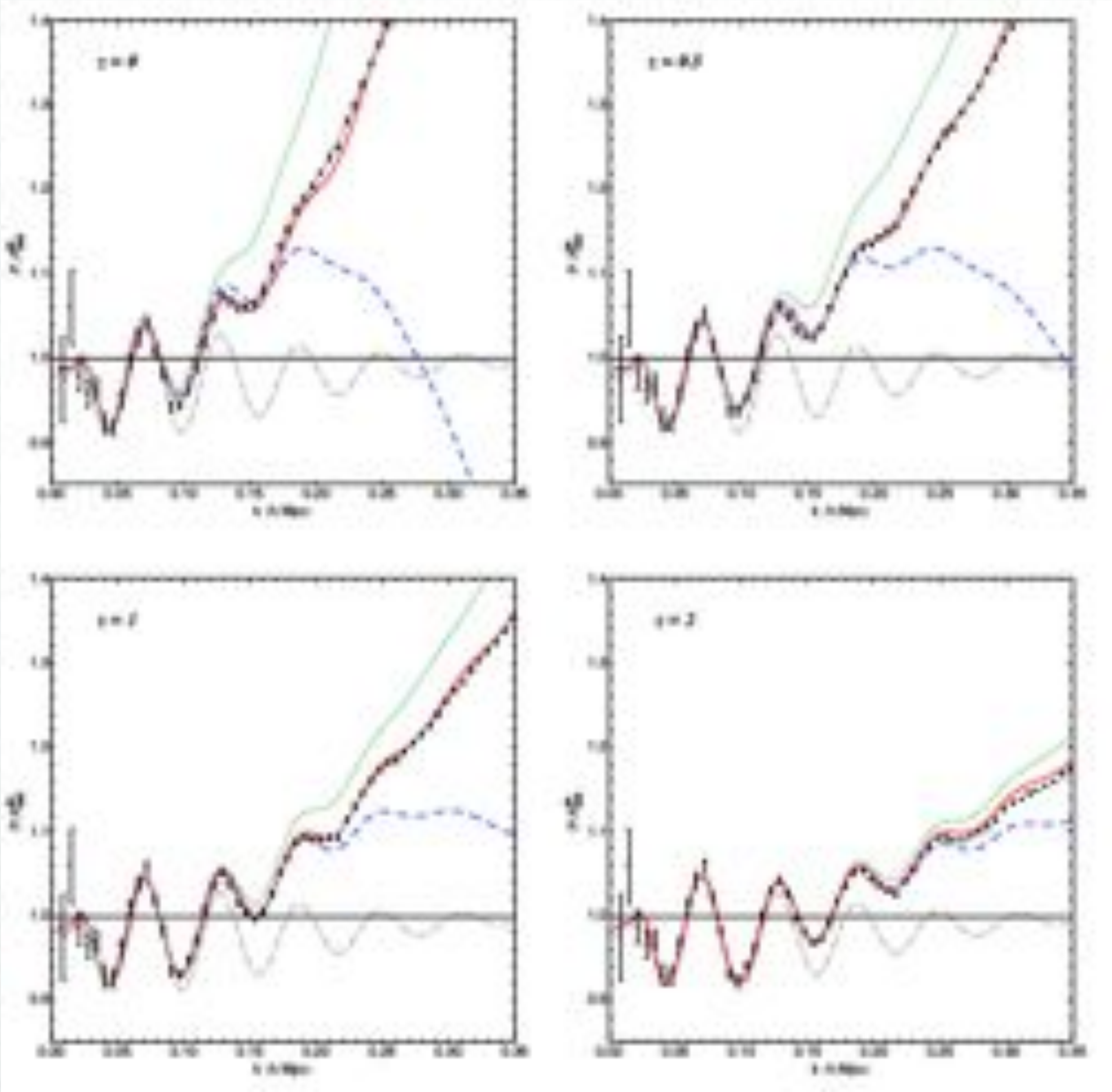
(neglect vertex renormalization)

$$\tilde{\Phi}_{ab}(k; s, s') \to e^{-\frac{k^2\sigma_v^2}{2}(e^s - e^{s'})^2} \left[\Phi_{ab}^{(1)}(k; s, s') + \left(k^2 \sigma_v^2 \, e^{s+s'} \right)^2 P(k) u_a u_b \right]$$

large k



BAO scales



Practical Implementation

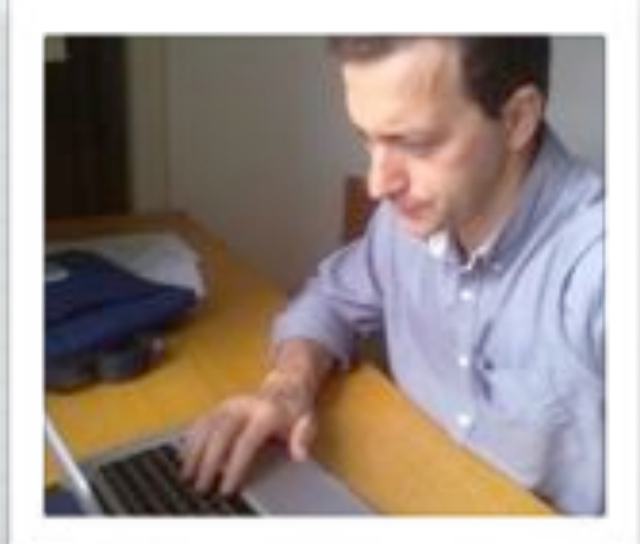
* linear PS at zin
* compute 5 momentum integrals:

$$H_1(k;\eta,-\infty) = -\,e^{2\eta}\frac{k^3\pi}{21}\int dr\left[19-24r^2+9r^4-\frac{9}{2r}(r^2-1)^3\log\left|\frac{1+r}{1-r}\right|\right]P^0(kr)$$

$$\Phi^{(1)}_{11}(k;\eta,\eta') = e^{\eta+\eta'}\frac{\pi}{4k}\int_0^\infty dq\int_{|k-q|}^{k+q} dp\frac{\left[k^2(p^2+q^2)-(p^2-q^2)^2\right]^2}{p^3q^3}P^0(q)P^0(p)$$

* integrate the evolution equation from zin to z: get Pdd, Pdt, Ptt
* public code available soon

O(1 min) for PS at all z



How far can (resummed) PT go on its own?

The DM particle distribution function, $f(\mathbf{x}, \mathbf{p}, \tau)$, obeys the Vlasov equation:

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{am} \cdot \nabla f - am \nabla\phi \cdot \nabla_{\mathbf{p}} f = 0$$

with $p = am \frac{d\mathbf{x}}{d\tau}$ and $\nabla^2 \phi = \frac{3}{2} \Omega_M \mathcal{H}^2 \delta$ sub-horizon scales, Newtonian gravity

Taking moments,

$$\int d^3\mathbf{p}\, f(\mathbf{x}, \mathbf{p}, \tau) \equiv \rho(\mathbf{x}, \tau) \equiv \bar{\rho}\, [1 + \delta(\mathbf{x}, \tau)]$$

$$\int d^3\mathbf{p}\, \frac{p_i}{am} f(\mathbf{x}, \mathbf{p}, \tau) \equiv \rho(\mathbf{x}, \tau) v_i(\mathbf{x}, \tau)$$

$$\int d^3\mathbf{p}\, \frac{p_i\, p_j}{a^2 m^2} f(\mathbf{x}, \mathbf{p}, \tau) \equiv \rho(\mathbf{x}, \tau)\, [v_i(\mathbf{x}, \tau) v_j(\mathbf{x}, \tau) + \sigma_{ij}(\mathbf{x}, \tau)]$$

$$\ldots$$

$$\frac{\partial n}{\partial \tau} + \frac{\partial}{\partial x^i}(n\, v^i) = 0$$

source term

$$\frac{\partial v^i}{\partial \tau} + \mathcal{H} v^i + v^k \frac{\partial}{\partial x^k} v^i + \frac{1}{n}\frac{\partial}{\partial x^k}(n\sigma^{ki}) = -\frac{\partial}{\partial x^i}\phi$$

$$\frac{\partial \sigma^{ij}}{\partial \tau} + 2\mathcal{H}\sigma^{ij} + v^k \frac{\partial}{\partial x^k}\sigma^{ij} + \sigma^{ik}\frac{\partial}{\partial x^k} v^j + \sigma^{jk}\frac{\partial}{\partial x^k} v^i + \frac{1}{n}\frac{\partial}{\partial x^k}(n\omega^{ijk}) = 0$$

$$\frac{\partial \omega^{ijk}}{\partial \tau} + \cdots = 0$$

$$\cdots$$

$$\nabla^2 \phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$

No sources for $\sigma^{ij},\ \omega^{ijk}, \ldots, \vec{\nabla} \times \vec{v}, \ldots$

$\sigma^{ij} = \omega^{ijk} = \cdots = \vec{\nabla} \times \vec{v} = 0$ is a fixed point

neglecting σ_{ij} and higher moments...

$$\frac{\partial n}{\partial \tau} + \frac{\partial}{\partial x^i}(n\, v^i) = 0$$ continuity

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$ Euler

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$ Poisson

$$[n = n_0(1+\delta)]$$

(RESUMMED) PT IS BASED ON THE "SINGLE STREAM APPROXIMATION"

$$\sigma_{ij} = 0 \leftrightarrow f(\vec{x},\vec{p},\tau) = g(\vec{x},\tau)\delta_D(\vec{p} - am\vec{v}(\vec{x},\tau))$$

self-consistent, but wrong!

Large scale impact of velocity dispersion

$$q_i(\mathbf{x},\tau) \equiv \frac{1}{\rho}\nabla_j(\rho\sigma_{ij}).$$

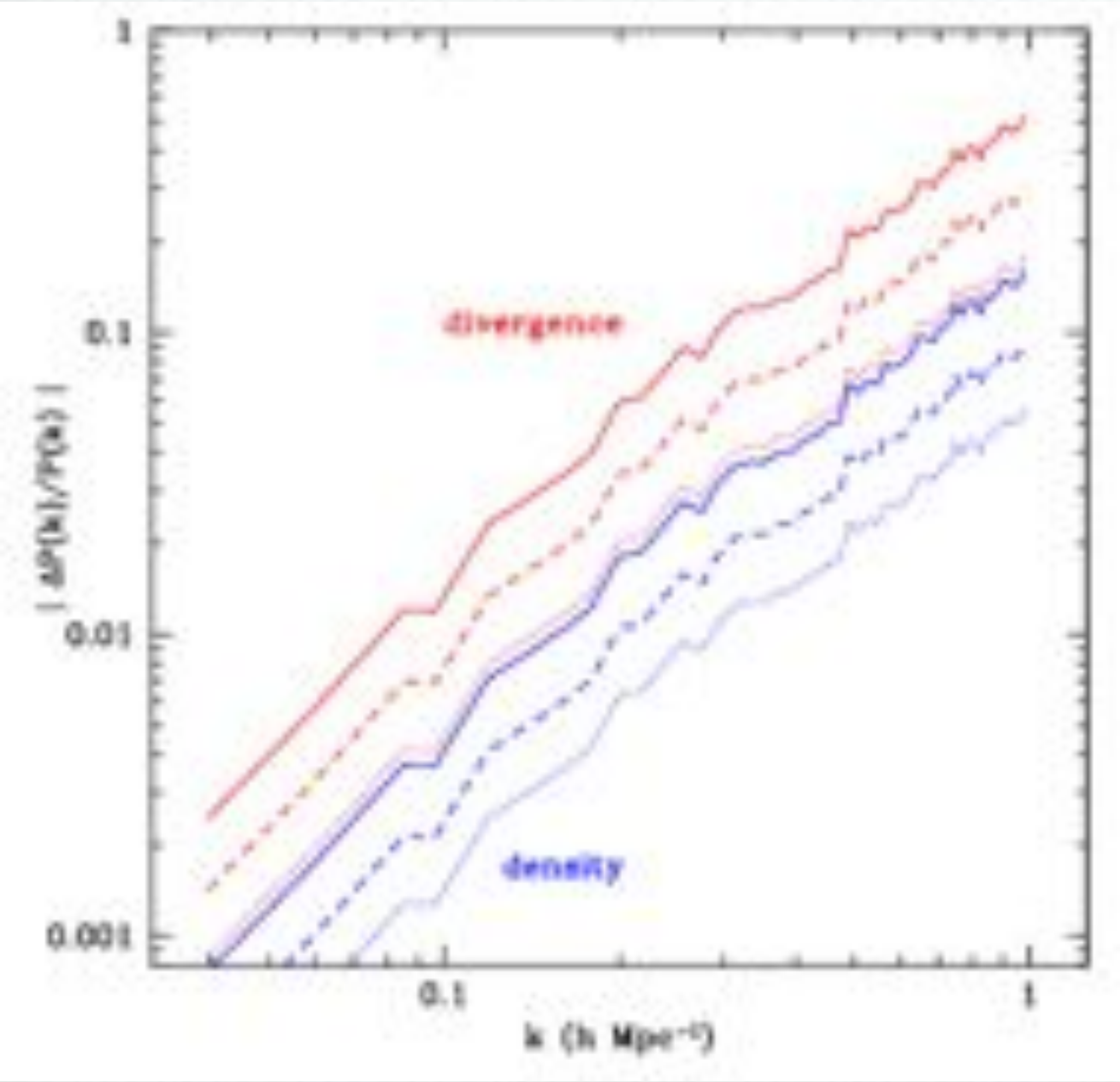
$$q_\theta \equiv \nabla \cdot \mathbf{q}, \qquad \mathbf{q}_w \equiv \nabla \times \mathbf{q},$$

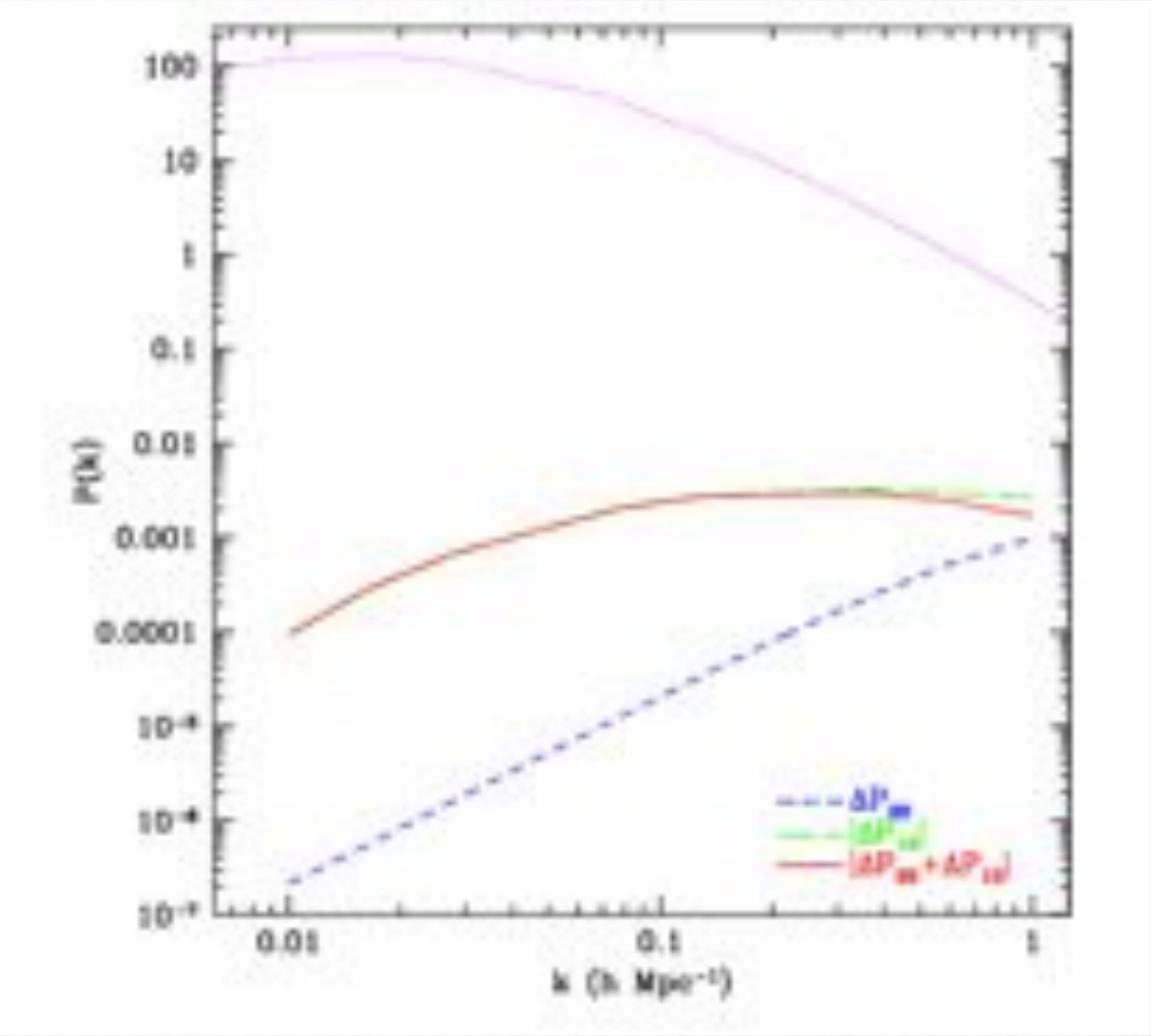
measure the q's from simulations

$$\begin{aligned} \partial_\eta \delta - \theta &= 0, \\ \partial_\eta \theta + \frac{\theta}{2} - \frac{3\delta}{2} &= \mathrm{q}_\theta, \\ \partial_\eta \mathbf{w} + \frac{\mathbf{w}}{2} &= \mathbf{q}_w. \end{aligned}$$

Pueblas Scoccimarro 2009

Effect on the power spectrum





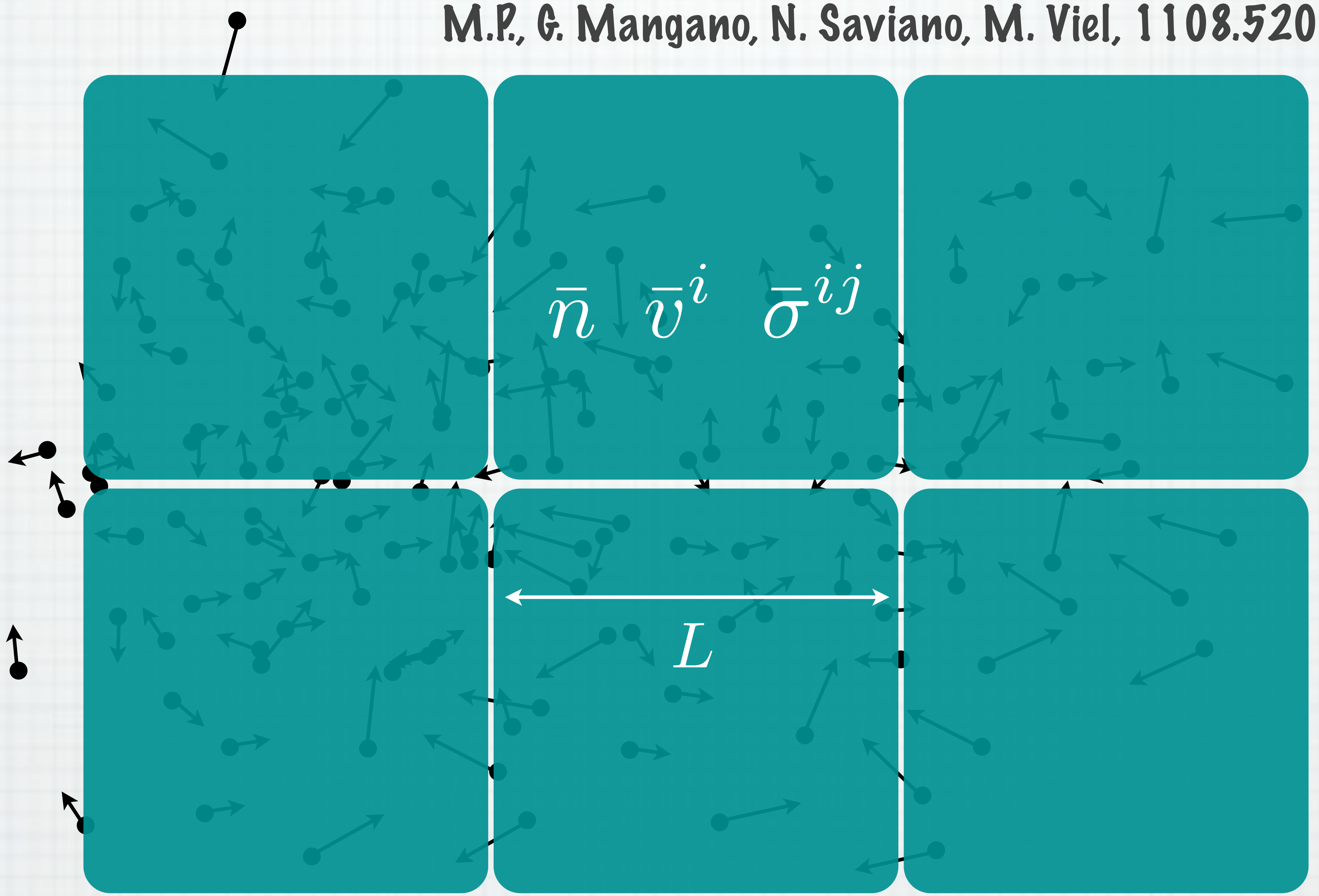
$q_\theta + q_w$ q_w only

around % at z=0 in the BAO range

Pueblas Scoccimarro 2009

Rederiving the fluid equations

Buchert, Dominguez, '05
M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203



$$f_{mic}(x,p,\tau) = \sum_n \delta_D(x - x_n(\tau))\,\delta_D(p - p_n(\tau))$$ Satisfies the "Vlasov eq."

$$\bar{f}(x,p,\tau) = \frac{1}{V}\int d^3y\, W(y/L)\, f_{mic}(x+y,p,\tau)$$

Coarse-Grained Vlasov eq.

large scales

$$\left[\frac{\partial}{\partial\tau} + \frac{p^i}{ma}\frac{\partial}{\partial x^i} - am\nabla^i_x\bar{\phi}(\mathbf{x},\tau)\frac{\partial}{\partial p^i}\right]\bar{f}(\mathbf{x},\mathbf{p},\tau)$$
$$= \frac{am}{V}\int d^3y\, \mathcal{W}\left(\left|\frac{\mathbf{y}}{L}\right|\right)\nabla^i_{x+y}\delta\phi(\mathbf{x}+\mathbf{y},\tau)\,\frac{\partial}{\partial p^i}\delta f(\mathbf{x}+\mathbf{y},\mathbf{p},\tau)$$

short scales

$$\delta\phi = \phi - \bar{\phi}$$
$$\delta f = f_{mic} - \bar{f}$$

Vlasov in the $L \to 0$ limit!

Short-distance sources

$q_\theta + q_w$

$$\frac{\partial}{\partial\tau}\bar{n}(\mathbf{x}) + \frac{\partial}{\partial x^i}\left(\bar{n}(\mathbf{x})\bar{v}^i(\mathbf{x})\right) = 0\,.$$

$$\frac{\partial}{\partial\tau}\bar{v}^i(\mathbf{x}) + \mathcal{H}\bar{v}^i(\mathbf{x}) + \bar{v}^k(\mathbf{x})\frac{\partial}{\partial x^k}\bar{v}^i(\mathbf{x}) + \frac{1}{\bar{n}(\mathbf{x})}\frac{\partial}{\partial x^k}(\bar{n}(\mathbf{x})\bar{\sigma}^{ki}(\mathbf{x}))$$
$$= -\nabla^i_x\bar{\phi}(\mathbf{x}) - \frac{1}{V}\int d^3y\,\mathcal{W}\left(\left|\frac{\mathbf{y}}{L}\right|\right)\frac{n(\mathbf{x}+\mathbf{y})}{\bar{n}(\mathbf{x})}\nabla^i_{x+y}\delta\phi(\mathbf{x}+\mathbf{y})\,,$$

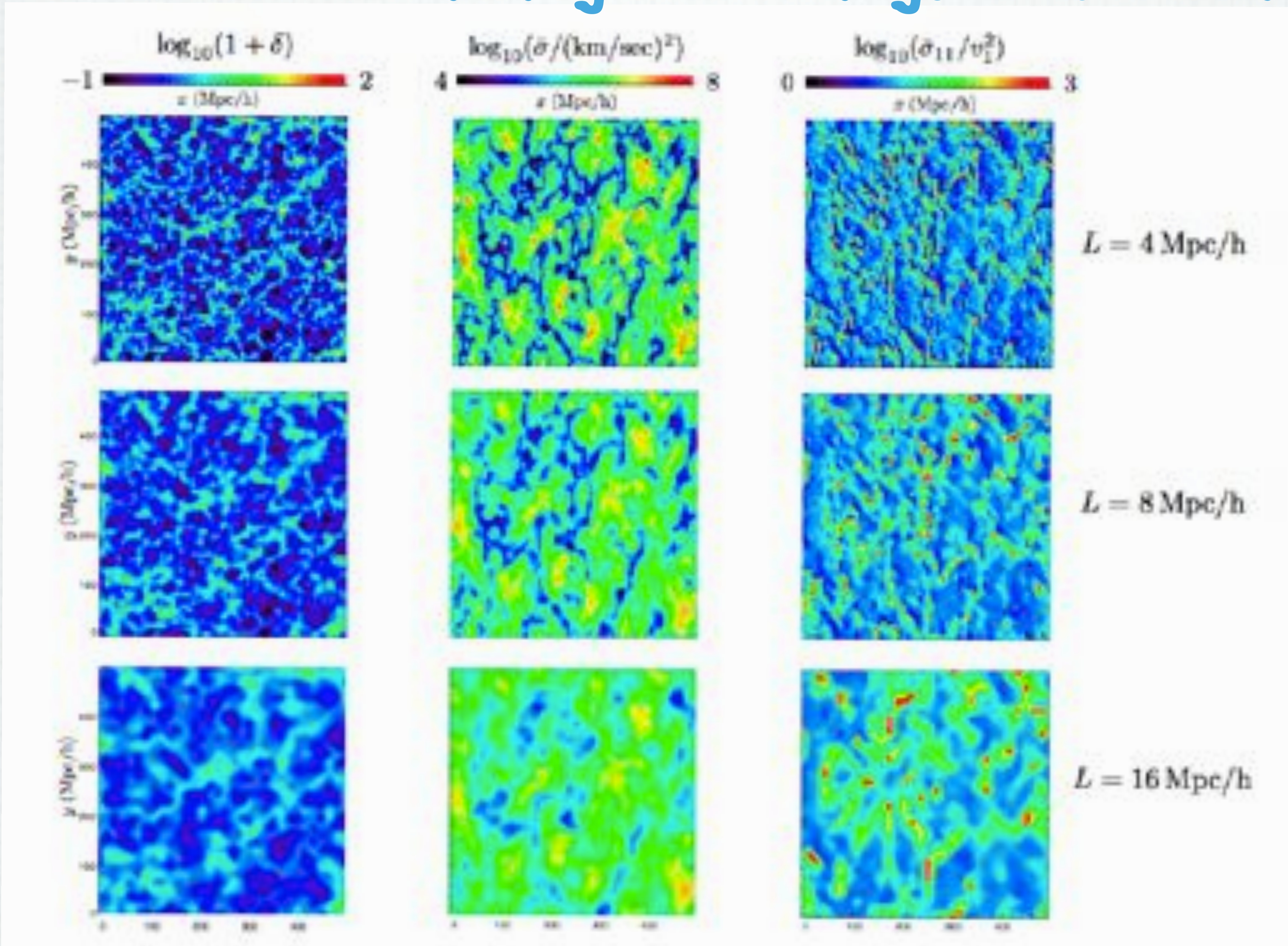
$$\frac{\partial}{\partial\tau}\bar{\sigma}^{ij} + 2\mathcal{H}\bar{\sigma}^{ij} + \bar{v}^k\frac{\partial}{\partial x^k}\bar{\sigma}^{ij} + \bar{\sigma}^{ik}\frac{\partial}{\partial x^k}\bar{v}^j + \bar{\sigma}^{jk}\frac{\partial}{\partial x^k}\bar{v}^i + \frac{1}{\bar{n}}\frac{\partial}{\partial x^k}\left(\bar{n}\,\bar{\omega}^{ijk}\right)$$
$$= -\frac{1}{V}\int d^3y\;\mathcal{W}\left(\left|\frac{\mathbf{y}}{L}\right|\right)\frac{n(\mathbf{x}+\mathbf{y})}{\bar{n}(\mathbf{x})}$$
$$\times\left[\delta v^j(\mathbf{x}+\mathbf{y})\nabla^i_{x+y} + \delta v^i(\mathbf{x}+\mathbf{y})\nabla^j_{x+y}\right]\delta\phi(\mathbf{x}+\mathbf{y})\,.$$

Short-distance sources

$\bar{\sigma}^{ij}$ and all higher-order moments are dynamically generated by coarse-graining!

Coarse-Graining vs. Single-Stream

PT gets better ⟶



SSA gets worse ⟶

Well-behaved PT calls for dropping the single stream approximation

Compact form

$$\bar{\varphi}_a(\mathbf{k},\eta) = e^{-\eta}\begin{pmatrix} \bar{\delta} \\ -\frac{\bar{\theta}}{\mathcal{H}f} \\ \frac{k^2}{\mathcal{H}^2 f^2}\bar{\sigma} \\ \frac{k^2}{\mathcal{H}^2 f^2}\bar{\Sigma} \end{pmatrix}$$

$$\bar{\sigma}(\mathbf{k}) = \bar{\sigma}^{ii}(\mathbf{k})\,, \quad \bar{\Sigma}(\mathbf{k}) = \frac{k^i k^j}{k^2}\bar{\sigma}^{ij}(\mathbf{k})$$

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\bar{\varphi}^b(\mathbf{k},\eta) =$$
$$e^{\eta}\int d^3q_1 d^3q_2 \delta_D(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)\gamma_{abc}(k,q_1,q_2)\bar{\varphi}_b(\mathbf{q}_1,\eta)\bar{\varphi}_c(\mathbf{q}_2,\eta) - h_a(\mathbf{k},\eta)$$

(resummed) PT expansion in γ_{abc}

$$0 \leq k \leq k_{(R)PT} \simeq \frac{2\pi}{L}$$

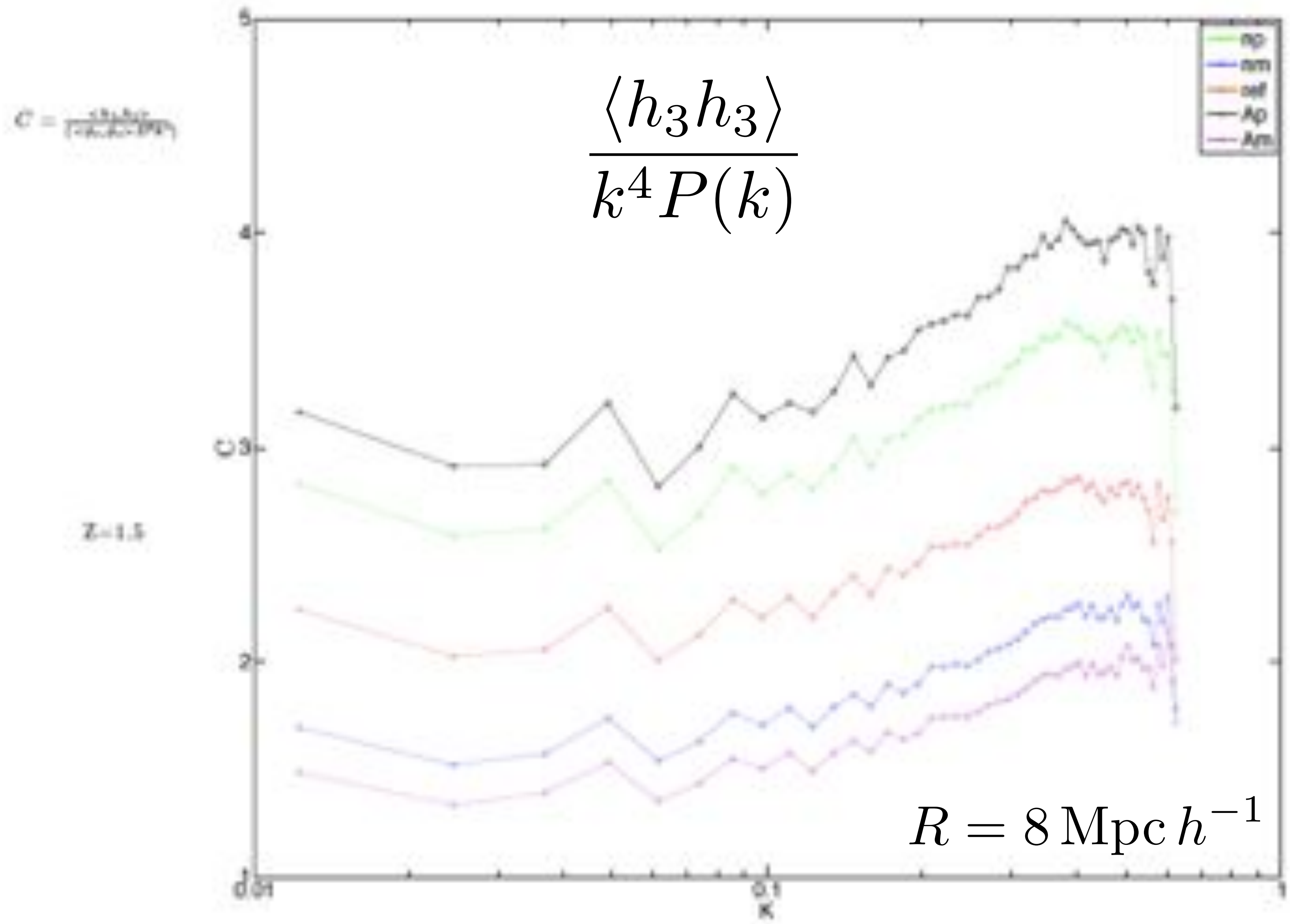
cosmology up to mildly non linear scales

short-distance sources: measure from simulations

$$k > \frac{2\pi}{L}$$

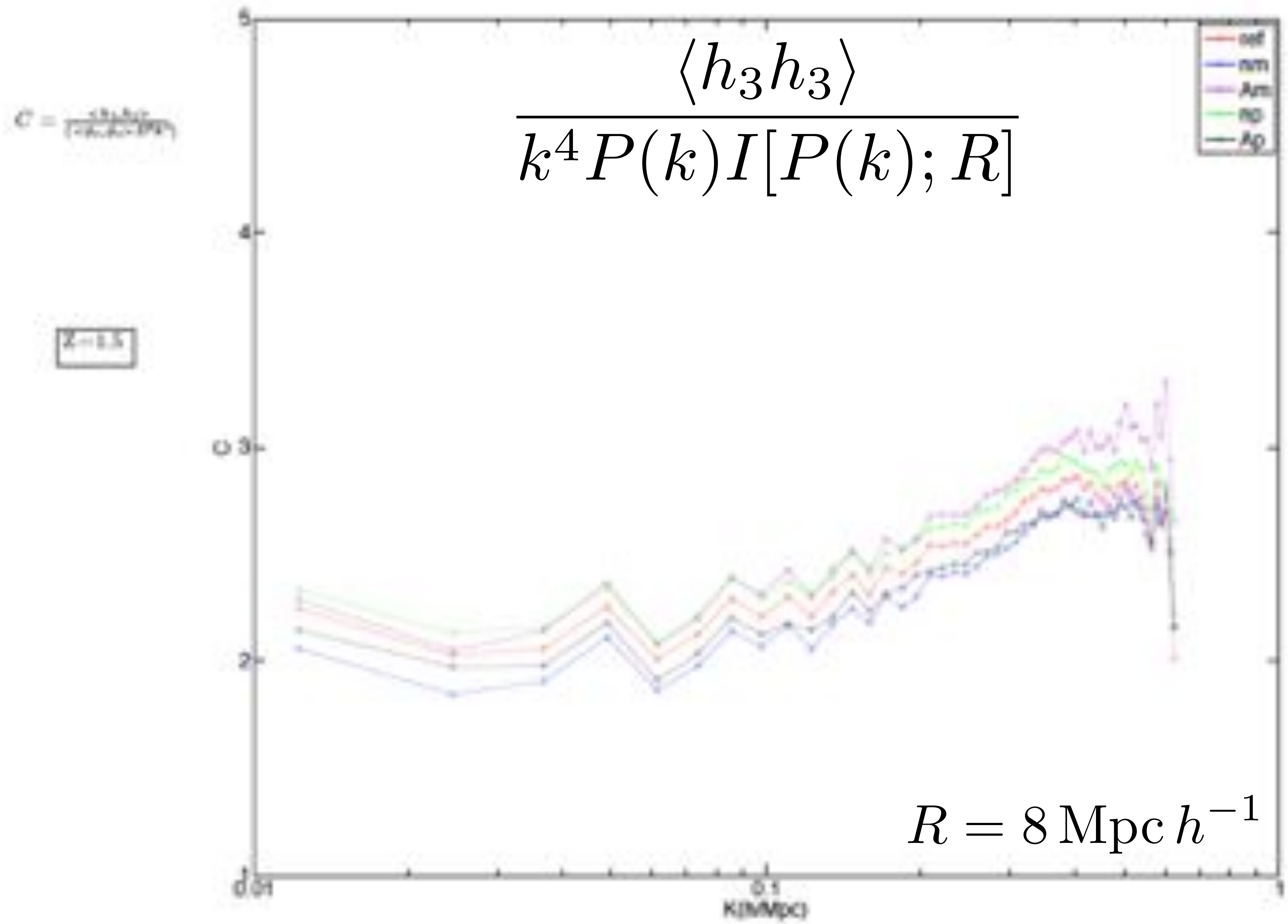
cosmology-independent?

Sources: cosmology dependence



$1.95 \cdot 10^{-9} < A_s < 3.0 \cdot 10^{-9}$ $\quad$ $0.932 < n_s < 1$

Sources: cosmology dependence



$1.95 \cdot 10^{-9} < A_s < 3.0 \cdot 10^{-9}$ $0.932 < n_s < 1$

Summary

* Improved PT methods OK for BAOs and beyond (up to O(1 h/Mpc))
* Good: speed, flexibility
* Extend to bispectrum, RSD, ...
* Common limit: the single stream approximation
* PT and N-body are complementary tools: let's exploit it! Coarse-grained PT

Thank you!!