

Sternentstehung - Star Formation

Sommersemester 2009

Henrik Beuther

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26.6 High-mass star formation, clusters and the IMF

3.7 Extragalactic star formation (Eva Schinnerer)

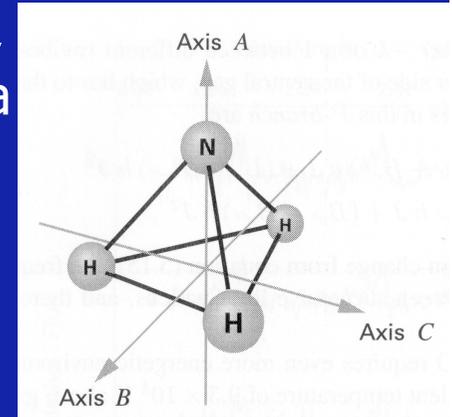
10.7 Summary

More Information and the current lecture files: http://www.mpia.de/homes/beuther/lecture_ss09.html
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Questions during last lecture

- Why does NH_3 inversions have transitions if both energies are the same?

→ Energies levels in both minima are actually the same, however, the quantum-mechanical tunneling causes a parity change (inversion of the spatial coordinates), and that causes a splitting of the levels and hence our observed transition.



- Why magnetic fields there at all?

→ Magnetic fields are produced in stars and disks via dynamos. Get ejected potentially to ISM. But also: We always have some degree of ionization and moved ions produce B-fields (and the other way round). Therefore, weak B-fields can always be produced in ISM.

- How does dust exactly gets produced?

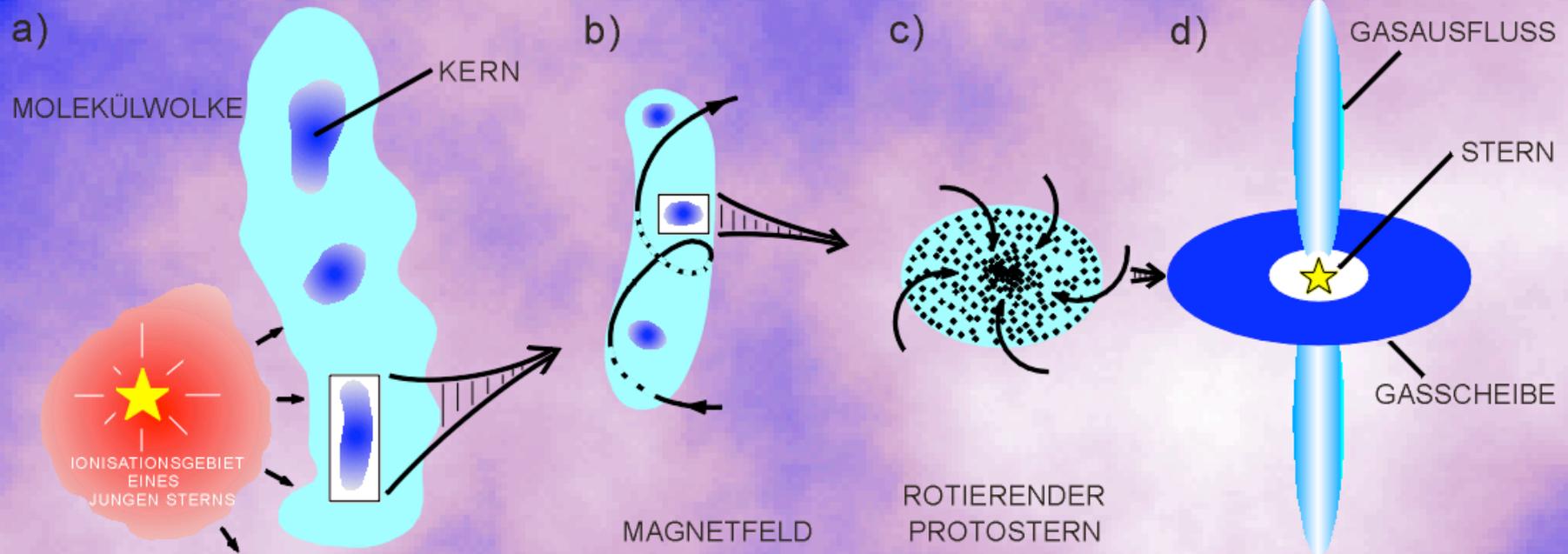
→ Dust produced mainly in atmospheres of cool giants. Temperature order 1000K and very high densities ideal environment. However, dust production time needs to be very short because due to the winds and expansion the material gets rarified very quickly. Dust formation time-scales need to be of order months to years! Growth rate proportional to density n and \sqrt{T} .

Topics today

- Isothermal sphere, hydrostatic equilibrium, grav. stability, Bonnor-Ebert spheres
- Jeans analysis
- Virial equilibrium
- Rotational support of molecular cloud stability
- Magnetic field support of molecular cloud stability

Star Formation Paradigm

DIE ENTWICKLUNGSTUFEN DER STERNENTSTEHUNG



Isothermal Sphere I

Three equations governing the equilibrium are:

Hydrostatic equilibrium

$$-\frac{1}{\rho}\nabla P - \nabla\Phi_g = 0 \quad (1)$$

Ideal isothermal gas

$$P = \rho a_t^2 \quad (2)$$

where the Φ_g obeys Poisson equation

$$\nabla^2\Phi_g = 4\pi G\rho \quad (3)$$

Substituting equation 2 in 1 and after integration

$$\ln\rho + \Phi_g/a^2 = \text{const.} \quad (4)$$

In the spherical case, this is

$$\rho(r) = \rho_c \exp(-\Phi_g/a^2) \quad (5)$$

Isothermal Sphere II

With ρ_c the density at the center and $\Phi_g(r=0) = 0$,
the Poisson eq. becomes

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Phi_g}{dr} \right) = 4\pi G \rho \quad (1)$$

$$= 4\pi G \rho_c \exp(-\Phi_g/a^2) \quad (2)$$

Often, this equations is used in dimensionless form
with the dimensionless potential:

$$\phi = \Phi_g/a^2$$

and the dimensionless length ξ

$$\xi = \sqrt{\frac{4\pi G \rho_c}{a^2}} r$$

Then the Poisson eq. turns into the Lane-Emden eq.

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left(\xi^2 \frac{d\phi}{d\xi} \right) = \exp(-\phi) \quad (3)$$

--> Numerical integration

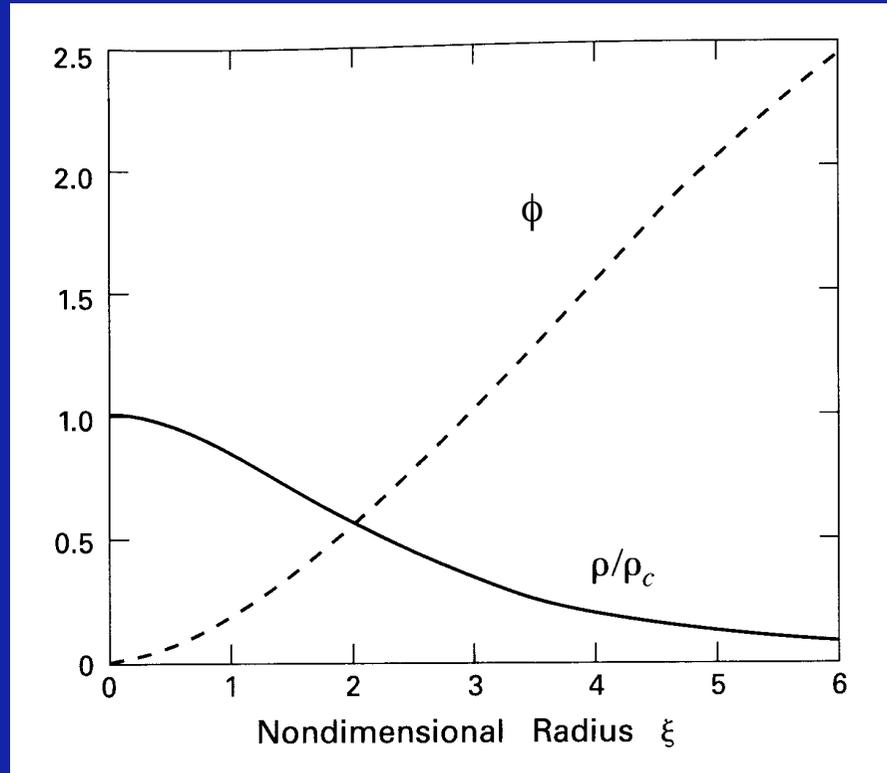
Boundary conditions:

$$\phi(0) = 0$$

$$\phi'(0) = 0$$

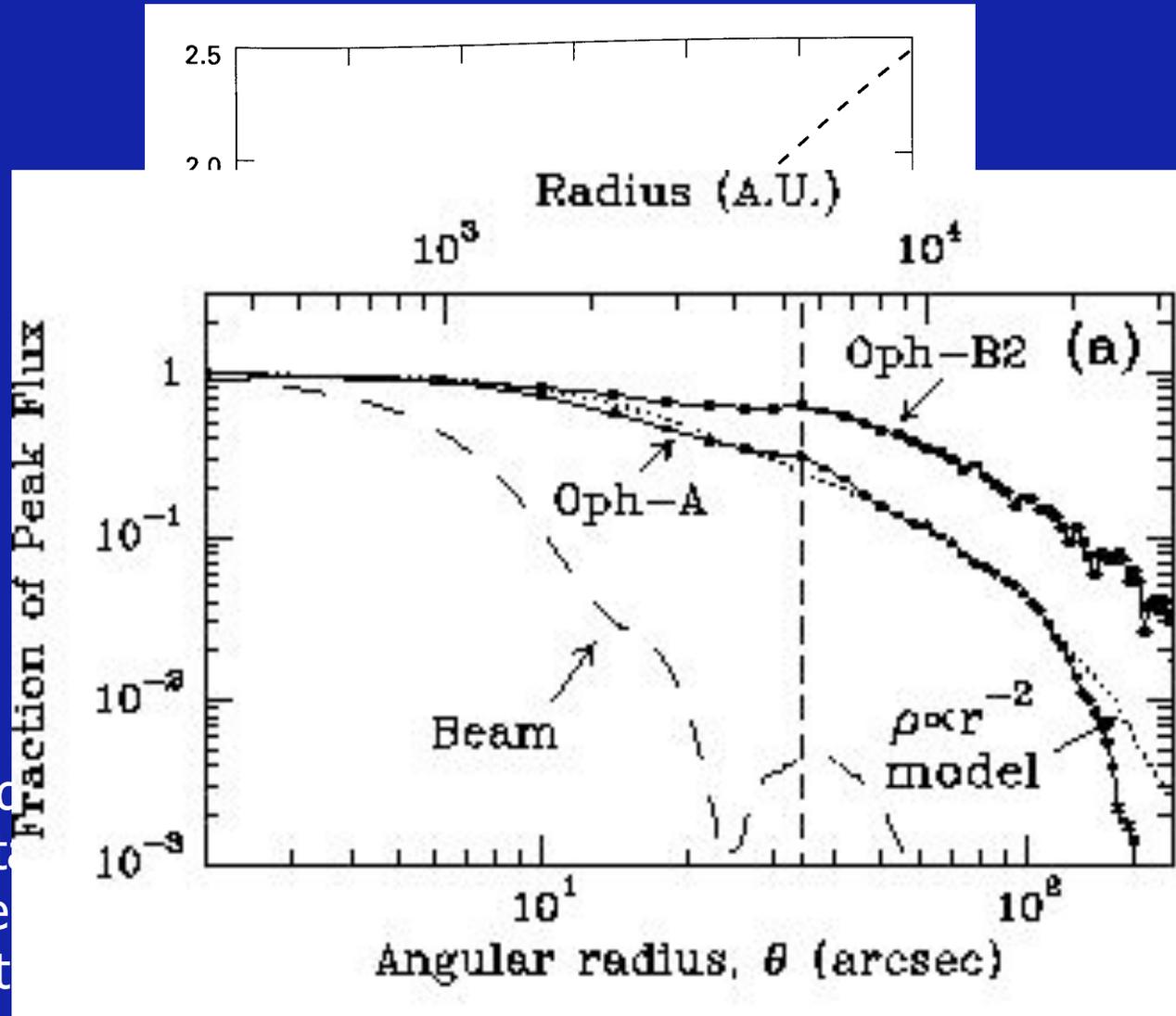
Gravitational potential and
force are 0 at the center.

Isothermal Sphere III



- Density and pressure ($P=\rho a^2$) drop monotonically away from the center.
 - > important to offset inward pull from gravity for grav. collapse.
- After numerical integration of the Lane-Emden equation, one finds that the density ρ/ρ_c approaches asymptotically $2/\xi^2$.
- Hence the dimensional density profile of the isothermal sphere is:
$$\rho(r) = a^2/(2\pi Gr^2).$$

Isothermal Sphere III



- Density and ...
- > import ...
- After nume ...
- one finds t ...
- Hence the dimensional density profile of the isothermal sphere is:

$$\rho(r) = a^2 / (2\pi G r^2).$$

center.

Isothermal Sphere IV

The dimensional mass is

$$M = 4\pi \int_0^{r_0} \rho r^2 dr \quad (1)$$

$$= 4\pi \rho_c \left(\frac{a_t^2}{4\pi G \rho_c} \right)^{3/2} \int_0^{\xi_0} e^{-\phi} \xi^2 d\xi \quad (2)$$

Using the Lane-Emden eq. and the boundary condition $\phi'(0) = 0$

$$\rightarrow M = 4\pi \rho_c \left(\frac{a_t^2}{4\pi G \rho_c} \right)^{3/2} \left(\xi^2 \frac{d\phi}{d\xi} \right)_{\xi_0} \quad (3)$$

Defining furthermore a dimensionless mass m

$$m = \frac{P_0^{1/2} G^{3/2} M}{a_t^4}, \quad \text{with } P_0 = \rho_0 a_t^2 \quad (4)$$

the dimensionless mass equals

$$m = \left(4\pi \frac{\rho_c}{\rho_0} \right)^{-1/2} \left(\xi^2 \frac{d\phi}{d\xi} \right)_{\xi_0} \quad (5)$$

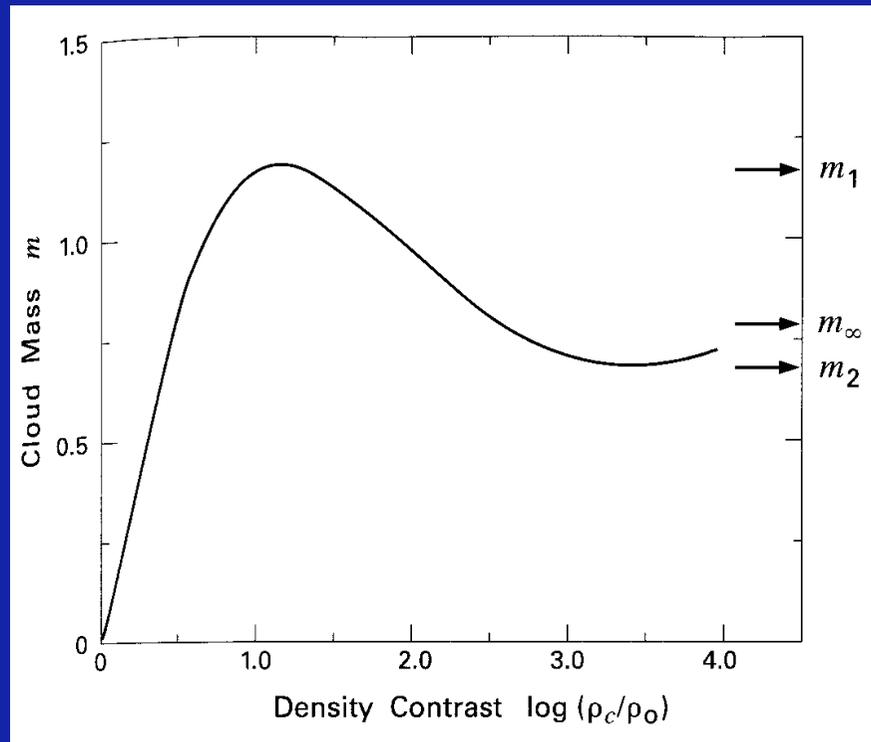
Since ξ_0 is known for each ρ_c/ρ_0 , and $\left(\xi^2 \frac{d\phi}{d\xi} \right)_{\xi_0}$ can be read from the previous figure, one can evaluate m .

With:

$$r = \sqrt{(a_t^2/(4\pi G \rho_c))} * \xi$$

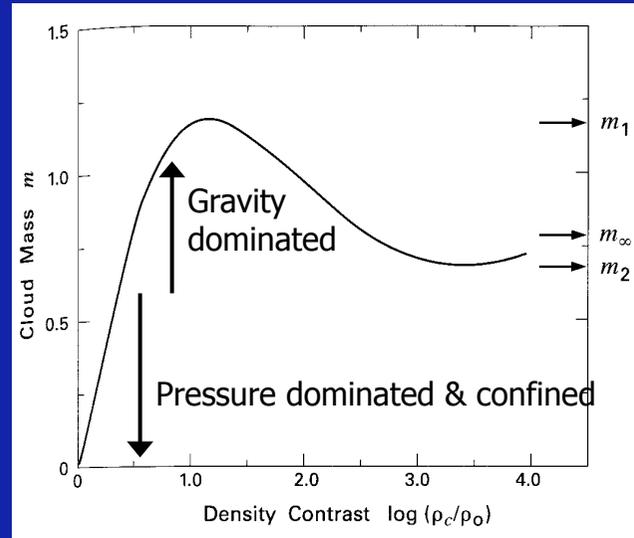
$$\rho = \rho_c \exp(\phi)$$

Isothermal Sphere V



The beginning is for a radius $\xi_0=0$, hence $\rho_c/\rho_0=1$ and $m=0$.
For increasing ρ_c/ρ_0 m then increases until $\rho_c/\rho_0=14.1$,
corresponding to the dimensionless radius $\xi_0=6.5$.

Gravitational stability



- Low density contrast cloud: Increasing outer pressure P_0 causes a rise of m and ρ_c/ρ_0 . With the internal pressure $P = \rho a_t^2$ and ρ decreasing outward, inner P rises more strongly than P_0 and the cloud remains stable.
- Since the physical radius r_0 is related to ξ_0 and ρ_c like

$$r_0 = \text{sqrt}(a_t^2 / (4\pi G \rho_c)) * \xi_0$$

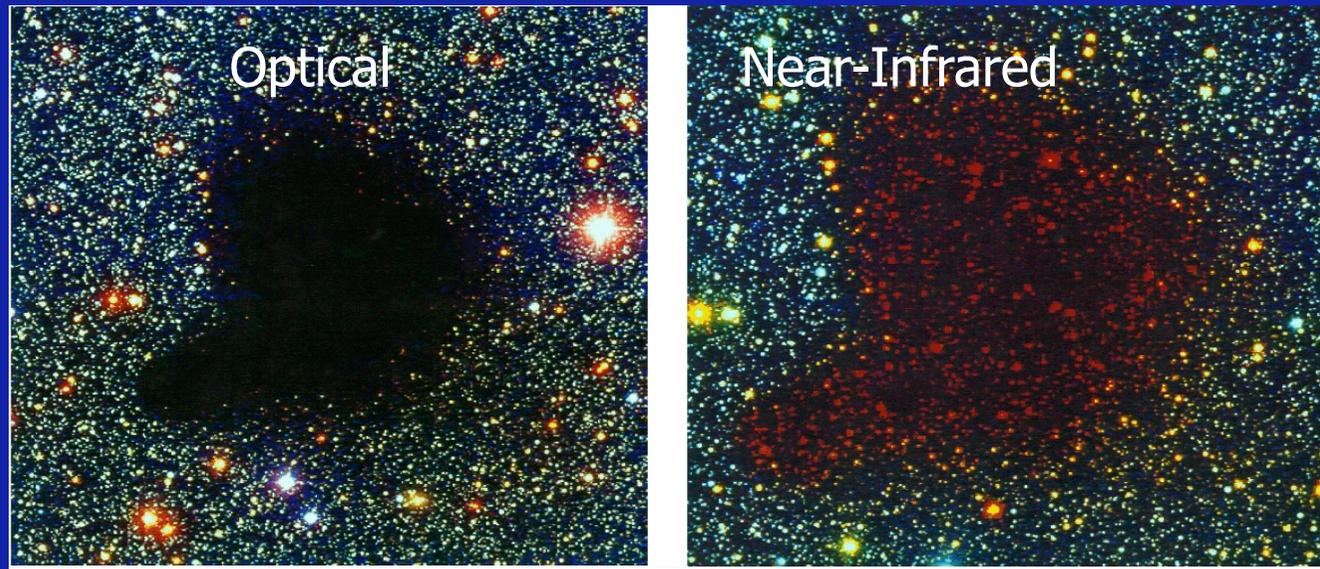
and ρ_c increases faster than ξ_0 , the core actually shrinks with increasing outer pressure P_0 . The same as Boyle-Mariotte law for ideal gas:

$$PV = \text{const.} \rightarrow P * 4/3\pi r^3 = \text{const.}$$

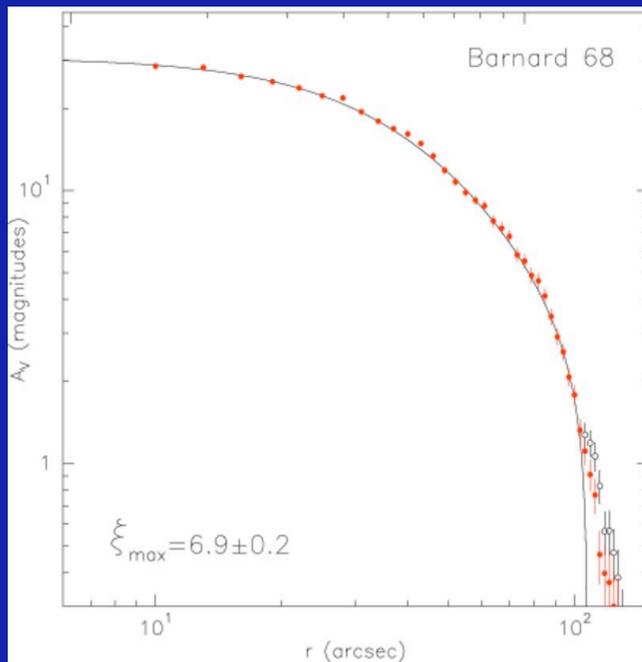
- All clouds with $\rho_c/\rho_0 > 14.1$ ($\xi_0 = 6.5$) are gravitationally unstable, and the critical mass is the Bonnor-Ebert mass (Ebert 1955, Bonnor 1956)

$$M_{BE} = (m_1 a_t^4) / (P_0^{1/2} G^{3/2})$$

Gravitational stability: The case of B68



$\xi_0 = 6.9$ is only marginally about the critical value 6.5
→ gravitational stable or at the verge of collapse



Topics today

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- Jeans analysis
- Virial equilibrium
- Rotational support of molecular cloud stability
- Magnetic field support of molecular cloud stability

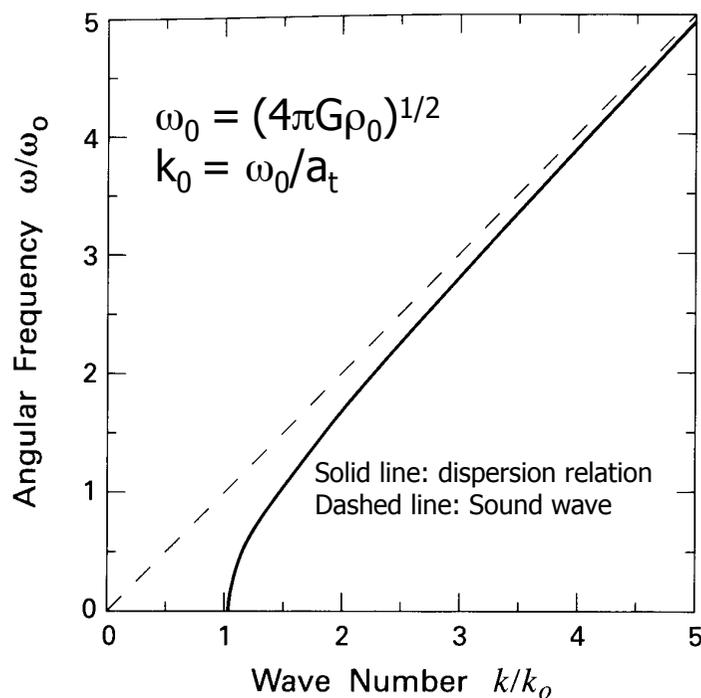
Jeans analysis I

- The previous analysis implies that clouds from certain size-scales upwards are prone to collapse --> Jeans analysis early 20th century
- A travelling wave in an isothermal gas can be described as:

$$\rho(x,t) = \rho_0 + \delta\rho \exp[i(kx - \omega t)] \quad \text{wave number } k=2\pi/\lambda \text{ and frequency } \omega$$

- Using this in all previous equations of the hydrostatic isothermal gas, one gets the *dispersion equation*

$$\omega^2 = k^2 a_t^2 - 4\pi G \rho_0$$



For large k high-frequency disturbances the wave behaves like sound wave $\omega = k a_t$
 → isothermal sound speed of background

However, for low k ($k \leq k_0$) $\omega^2 \leq 0$.
 The corresponding Jeans-length is

$$\lambda_j = 2\pi/k_0 = (\pi a_t^2 / G \rho_0)^{1/2}$$

Perturbations larger λ_j have exponentially growing amplitudes → instable

Using ρ_0 instead P_0 , Bonnor-Ebert mass M_{BE} is rather known as Jeans-Mass M_j

$$M_j = m_1 a_t^3 / (\rho_0^{1/2} G^{3/2})$$

Jeans analysis II

This corresponds in physical units to Jeans-lengths of

$$\lambda_j = (\pi a_t^2 / G \rho_0) = 0.19 \text{ pc } (T / (10 \text{ K}))^{1/2} (n_{\text{H}_2} / (10^4 \text{ cm}^{-3}))^{-1/2}$$

and Jeans-mass

$$M_j = m_1 a_t^3 / (\rho_0^{1/2} G^{3/2}) = 1.0 M_{\text{sun}} (T / (10 \text{ K}))^{3/2} (n_{\text{H}_2} / (10^4 \text{ cm}^{-3}))^{-1/2}$$

Clouds larger λ_j or more massive than M_j may be prone to collapse. Conversely, small or low-mass cloudlets could be stable if there is sufficient extrenal pressure. Otherwise only transient objects.

Example: a GMC with $T=10\text{K}$ and $n_{\text{H}_2}=10^3\text{cm}^{-3}$

$$\rightarrow M_j = 3.2 M_{\text{sun}}$$

Orders of magnitide too low.

→ Additional support necessary, e.g., magnetic field, turbulence ...

Topics today

- Isothermal sphere, hydrostatic equilibrium, grav. stability, Bonnor-Ebert spheres
- Jeans analysis
- **Virial equilibrium**
- Rotational support of molecular cloud stability
- Magnetic field support of molecular cloud stability

Virial Analysis

What is the force balance within any structure in hydrostatic equilibrium?

The generalized equation of hydrostatic equilibrium including magnetic fields \mathbf{B} acting on a current \mathbf{j} and the full convective fluid velocity \mathbf{v} is:

$$\rho \frac{D\mathbf{v}}{Dt} = -\text{grad}(P) - \rho \text{grad}(\Phi_g) + 1/c \mathbf{j} \times \mathbf{B}$$

$$\frac{D\mathbf{v}}{Dt} = \underbrace{\left(\frac{\partial \mathbf{v}}{\partial t}\right)_x}_{1/2(\partial^2 I / \partial t^2)} + \underbrace{(\mathbf{v} \text{ grad})\mathbf{v}}_{-2T} \quad \begin{matrix} \uparrow \\ 2U \end{matrix} \quad \begin{matrix} \uparrow \\ W \end{matrix} \quad \begin{matrix} \uparrow \\ M \end{matrix}$$

($D\mathbf{v}/Dt$ includes the rate of change at fixed spatial position x $(\partial \mathbf{v} / \partial t)_x$ and the change induced by transporting elements to new location with differing velocity.)

Employing the Poisson equation and requiring mass conservation, one gets after repeated integrations the **VIRIAL THEOREM**

$$1/2 (\delta^2 I / \delta t^2) = 2T + 2U + W + M$$

I: Moment of inertia, this decreases when a core is collapsing ($m \cdot r^2$)

T: Kinetic energy U: Thermal energy W: Gravitational energy M: Magnetic energy

All terms except W are positive. To keep the cloud stable, the other forces have to match W.

Application of the Virial Theorem I

If all forces are too weak to match the gravitational energy, we get

$$1/2 (\delta^2 I / \delta t^2) = W \sim -Gm^2/r$$

Approximating further $I = mr^2$, the free-fall time is approximately

$$t_{\text{ff}} \sim \text{sqrt}(r^3/Gm)$$

Since the density can be approximated by $\rho = m/r^3$, one can also write

$$t_{\text{ff}} \sim (G\rho)^{-1/2}$$

Or more exactly for a pressure-free 3D homogeneous sphere

$$t_{\text{ff}} = (3\pi/32G\rho)^{1/2}$$

For a giant molecular cloud, this would correspond to

$$t_{\text{ff}} \sim 7 \cdot 10^6 \text{ yr } (m/10^5 M_{\text{sun}})^{-1/2} (R/25 \text{ pc})^{3/2}$$

For a dense core with $\rho \sim 10^5 \text{ cm}^{-3}$ the t_{ff} is approximately 10^5 yr .

However, no globally collapsing GMCs observed --> add support!

Application of the Virial Theorem II

If the cloud complexes are in approximate force equilibrium, the moment of inertia actually does not change significantly and hence $1/2 (\delta^2 I / \delta t^2) = 0$

$$2T + 2U + W + M = 0$$

This state is called VIRIAL EQUILIBRIUM. What balances gravitation W best?

Thermal Energy: Approximating U by $U \sim 3/2 nk_B T \sim mRT/\mu$

$$U/|W| \sim mRT/\mu (Gm^2/R)^{-1}$$

$$= 3 \cdot 10^{-3} (m/10^5 M_{\text{sun}})^{-1} (R/25 \text{pc}) (T/15 \text{K})$$

--> Clouds cannot be supported by thermal pressure alone!

Magnetic energy: Approximating M by $M \sim B^2 r^3 / 6$ (cloud approximated as sphere)

$$M/|W| \sim B^2 r^3 / 6 (Gm^2/R)^{-1}$$

$$= 0.3 (B/20 \mu\text{G})^2 (R/25 \text{pc})^4 (m/10^5 M_{\text{sun}})^{-2}$$

--> Magnetic force is important for large-scale cloud stability!

Application of the Virial Theorem III

The last term to consider in $2T + 2U + W + M = 0$ is the kinetic energy T

$$\begin{aligned} T/|W| &\sim 1/2 m \Delta v^2 (Gm^2/R)^{-1} \\ &= 0.5 (\Delta v/4\text{km/s}) (M/10^5 M_{\text{sun}})^{-1} (R/25\text{pc}) \end{aligned}$$

Since the shortest form of the virial theorem is $2T = -W$, the above numbers imply that a typical cloud with linewidth of a few km/s is in approximate virial equilibrium.

The other way round, one can derive an approximate relation between the Observed line-width and the mass of the cloud:

$$\begin{aligned} 2T &= 2 * (1/2 m \Delta v^2) = -W = Gm^2/r \\ \rightarrow \text{virial velocity: } v_{\text{vir}} &= (Gm/r)^{1/2} \\ \rightarrow \text{or virial mass: } m_{\text{vir}} &= v^2 r / G \end{aligned}$$

Application of the Virial Theorem III

The last te

energy T

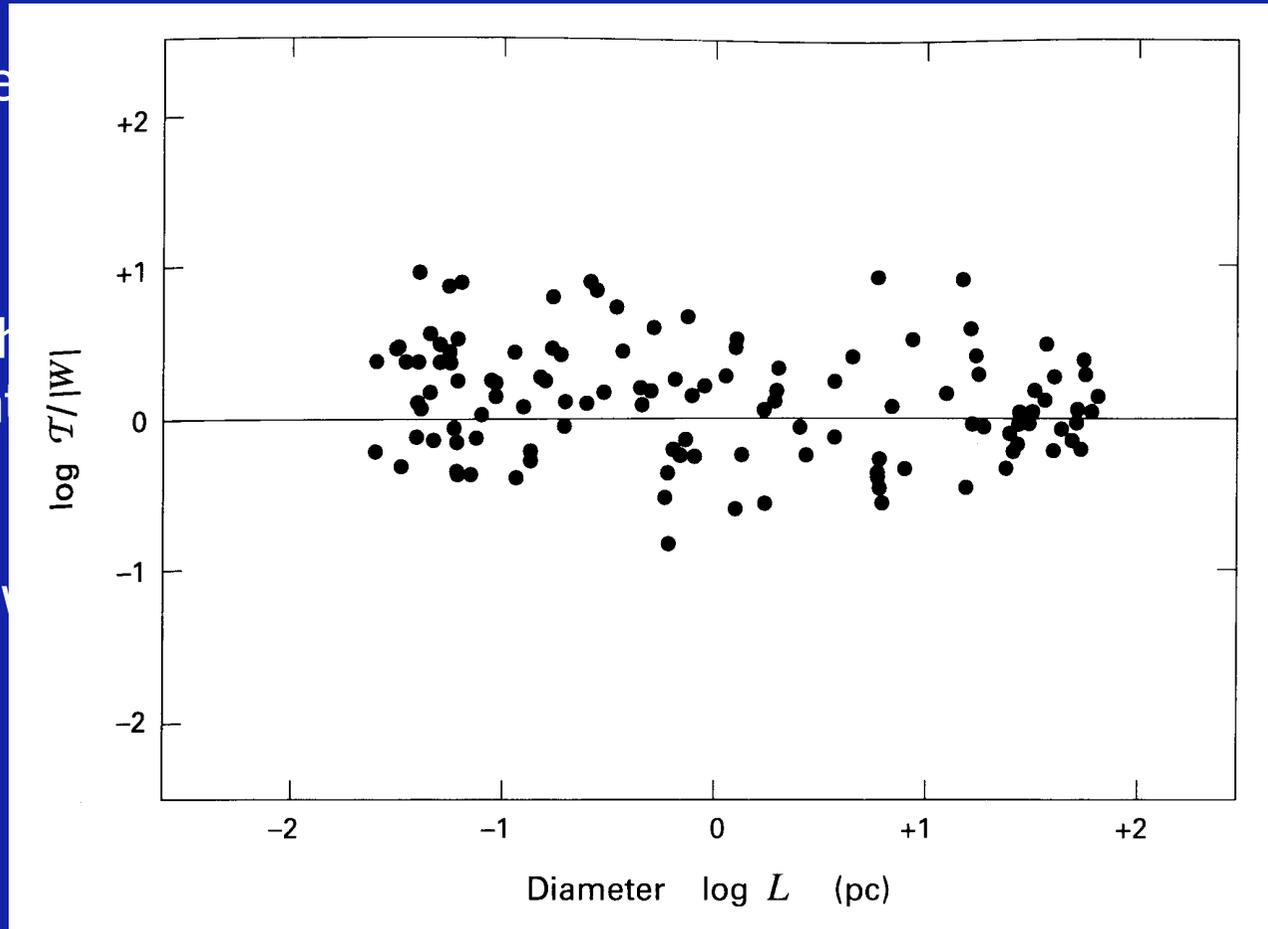
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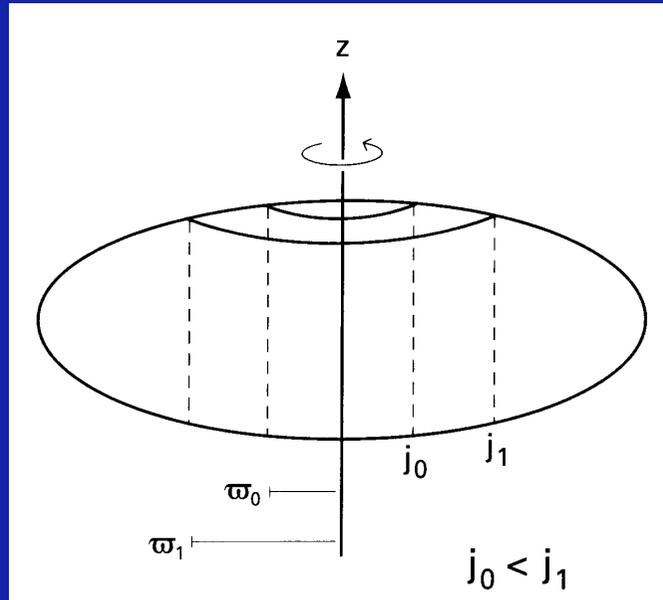
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- Isothermal sphere, hydrostatic equilibrium, grav. stability, Bonnor-Ebert spheres
- Jeans analysis
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- Rotational support of molecular cloud stability
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Basic rotational configurations I



Adding a centrifugal potential Φ_{cen} , the hydrodynamic equation reads

$$-1/\rho \text{ grad}(P) - \text{grad}(\Phi_g) - \text{grad}(\Phi_{\text{cen}}) = 0$$

With Φ_{cen} defined as

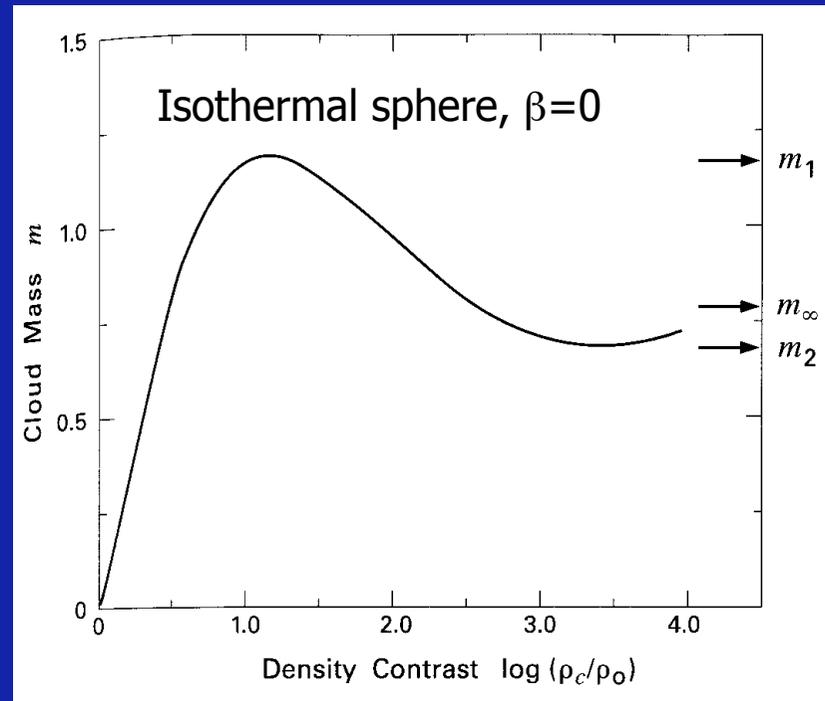
$$\Phi_{\text{cen}} = - \int (j^2/\omega^3) d\omega$$

j : angular momentum
 ω : cylindrical radius

and $j = \omega u$ with u the velocity around the rotation axis

Rotation flattens the cores and can be additional source of support against collapse.

Basic rotational configurations II



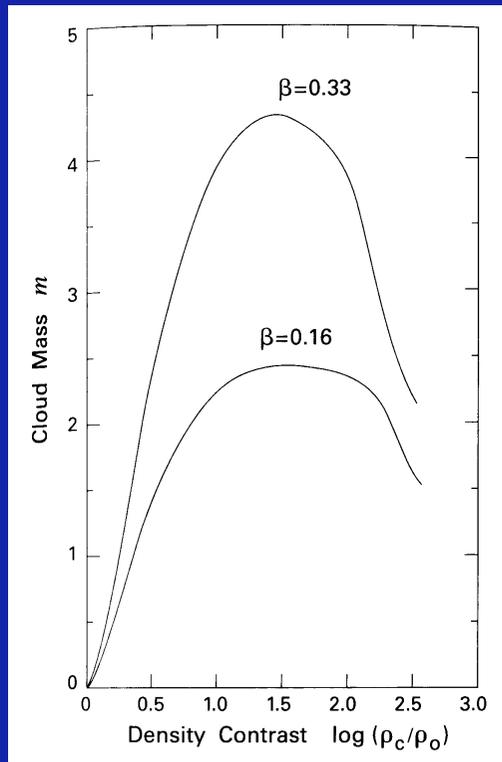
Compared to the previously discussed Bonnor-Ebert models, these rotational models now have in addition to the density contrast ρ_c/ρ_0 the other parameter β which quantifies the degree of rotation. β is defined as the ratio of rotational to gravitational energy

$$\beta = \Omega_0 R_0 / (3Gm)$$

with Ω_0 the angular velocity of the cloud and R_0 the initial cloud radius

$\beta > 1/3$ corresponds to breakup speed of the cloud. So $0 < \beta < 1/3$

Basic rotational configurations III



In realistic clouds, for flattening to appear, the rotational energy has to be at least 10% of the gravitational energy. T_{rot}/W equals approximately β (which was defined for the spherical case).

Examples:

Dense cores: aspect ratio ~ 0.6 . Estimated $T_{\text{rot}}/W \sim 10^{-3}$

GMCs: Velocity gradient of 0.05km/s representing solid body rotation, $200M_{\text{sun}}$ and 2pc size imply also $T_{\text{rot}}/W \sim 10^{-3}$

--> Cloud elongations do not arise from rotation, and centrifugal force NOT sufficient for cloud stability!

Other stability factors are necessary --> Magnetic fields

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Magnetic fields I

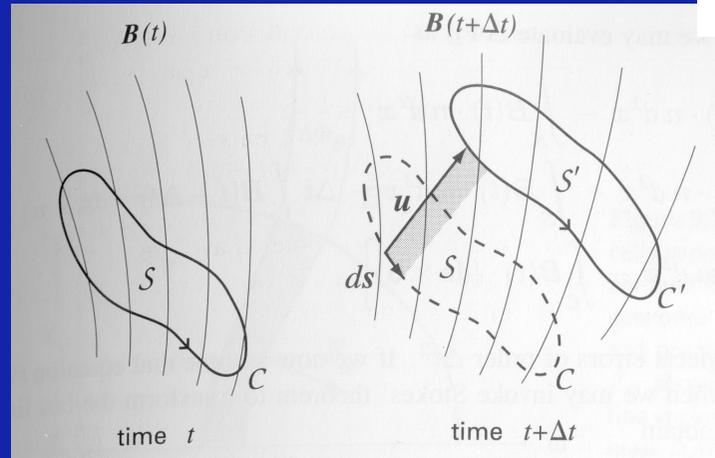
| Object | Type | Diagnostic | $ B_{ } $ [μG] |
|------------|---------------|------------|------------------------------|
| Ursa Major | Diffuse cloud | HI | 10 |
| NGC2024 | GMC clump | OH | 87 |
| S106 | HII region | OH | 200 |
| W75N | Maser | OH | 3000 |

Increasing magnetic field strength with increasing density indicate “field-freezing” between B-field and gas (B-field couples to ions and electrons, and these via collisions to neutral gas).

Magnetic fields II

This field freezing can be described by ideal MHD:

$$\frac{dB}{dt} = \nabla \times (\mathbf{u} \times \mathbf{B})$$



However, ideal MHD must break down at some point. Example:

Dense core: $1M_{\text{sun}}$, $R_0=0.07\text{pc}$, $B_0=30\mu\text{G}$

T Tauri star: $R_1=5R_{\text{sun}}$ If flux-freezing would hold, BR^2 should remain constant over time

--> $B_1=2 \times 10^7 \text{ G}$, which exceeds observed values by orders of magnitude

Ambipolar diffusion: neutral and ionized medium decouple, and neutral gas can sweep through during the gravitational collapse.

Magnetic fields III

The equation for magneto-hydrodynamic equilibrium now is:

$$-1/\rho \text{ grad}(P) - \text{grad}(\Phi_g) - 1/(\rho c) \mathbf{j} \times \mathbf{B} = 0$$

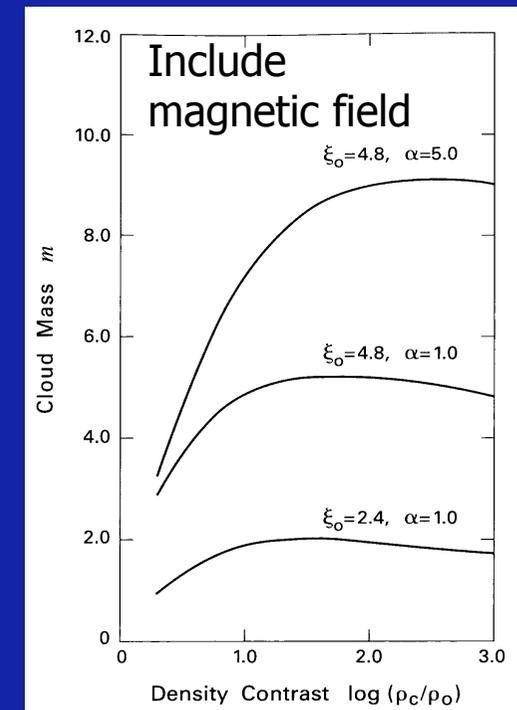
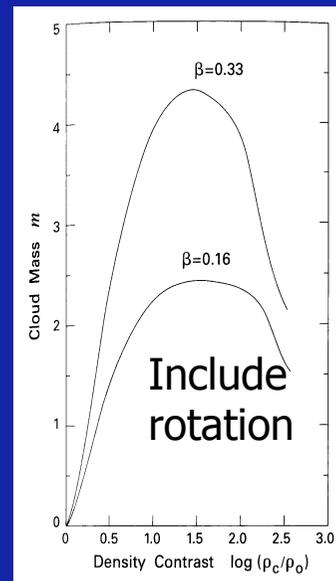
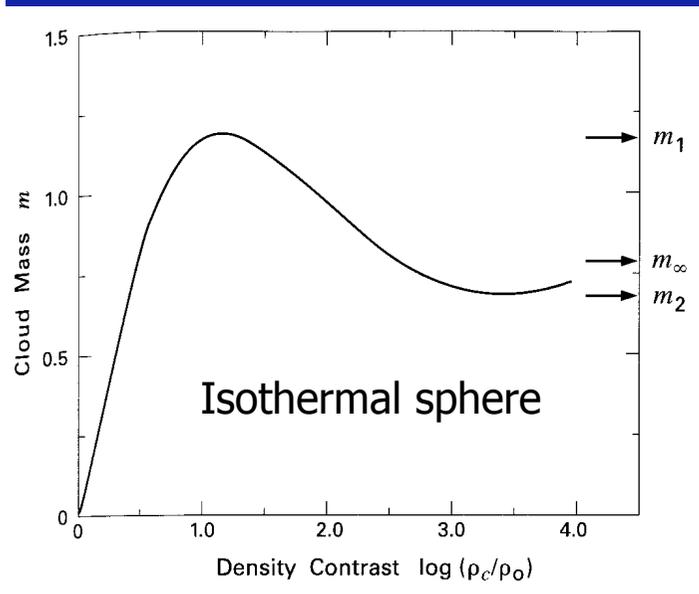
Solving the equations again numerically, one gets solutions with 3 free parameters: the density contrast ratio ρ_c/ρ_0 ,

the ratio α between magnetic to thermal pressure

$$\alpha = B_0^2/(8\pi P_0)$$

and the dimensionless radius of the initial sphere

$$\xi_0 = (4\pi G\rho_0/a_t^2)^{1/2} * R_0$$



A good fit to the numerical results is given by: $m_{\text{crit}} = 1.2 + 0.15 \alpha^{1/2} \xi_0^2$

Magnetic fields IV

Converting this to dimensional form (multiply by $a_t^4/(P_0^{1/2}G^{3/2})$), the first term equals the Bonnor-Ebert Mass ($M_{BE} = m_1 a_t^4/(P_0^{1/2}G^{3/2})$)

$$M_{crit} = M_{BE} + M_{magn}$$
$$\text{with } M_{magn} = 0.15 \alpha^{1/2} \xi_0^2 a_t^4/(P_0^{1/2}G^{3/2})$$
$$= 0.15 \frac{2}{\sqrt{2\pi}} (B_0 \pi R_0^2/G^{1/2}) \propto B_0$$

--> the magnetic mass M_{magn} is proportional to the B-field!

There is a qualitative difference between purely thermal clouds and magnetized clouds discussed here. If one increases the outer pressure P_0 around a low-mass core of mass M , the Bonnor-Ebert mass will decrease until $M_{BE} < M$, and then the cloud collapses. However, in the magnetic case, if $M < M_{magn}$ the cloud will always remain stable because M_{magn} is constant as long a flux-freezing applies.

Summary

- Hydrostatic equilibrium between thermal pressure and gravitational force.
→ Bonner Ebert mass for gravitationally stable cores.
- Jeans analysis to derive critical mass and length scales.
- Virial analysis and force (non-)equilibrium
- Rotational support of clouds/cores?
- Magnetic support of clouds/cores

Sternentstehung - Star Formation

Sommersemester 2009

Henrik Beuther

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29.5 *No lecture*

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