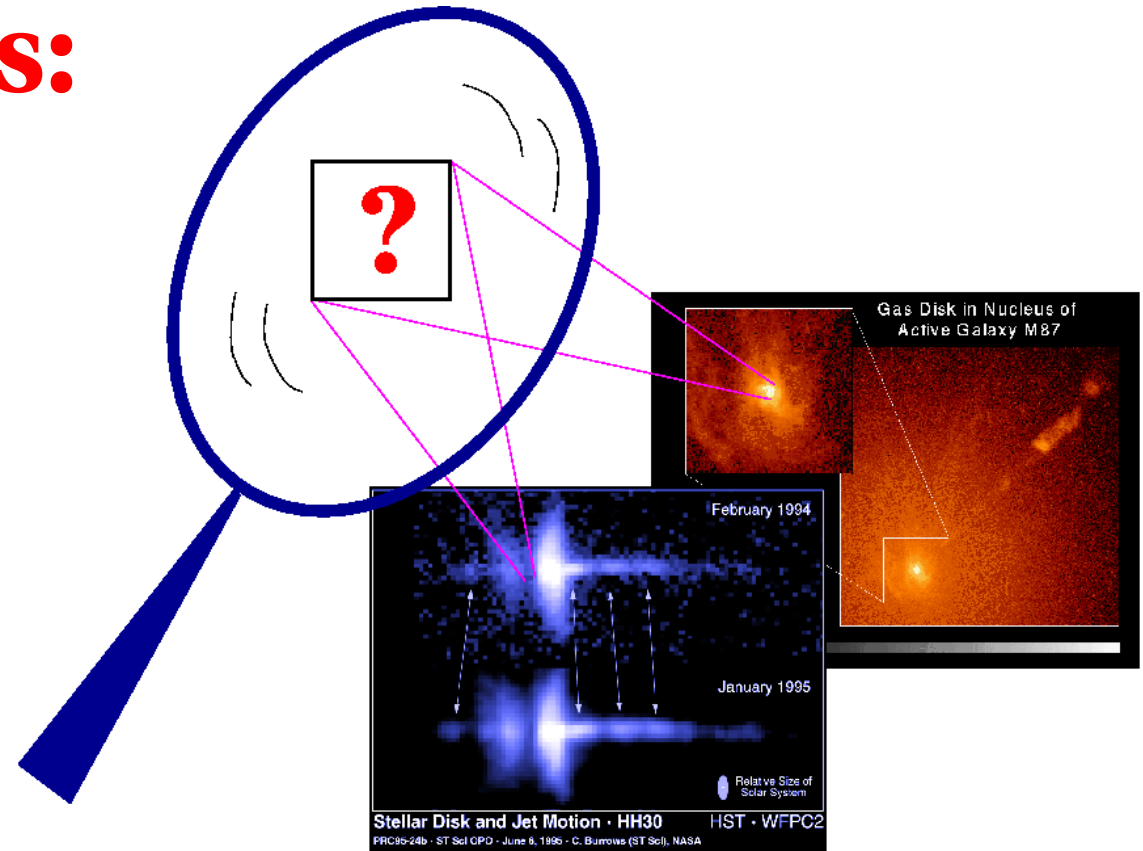


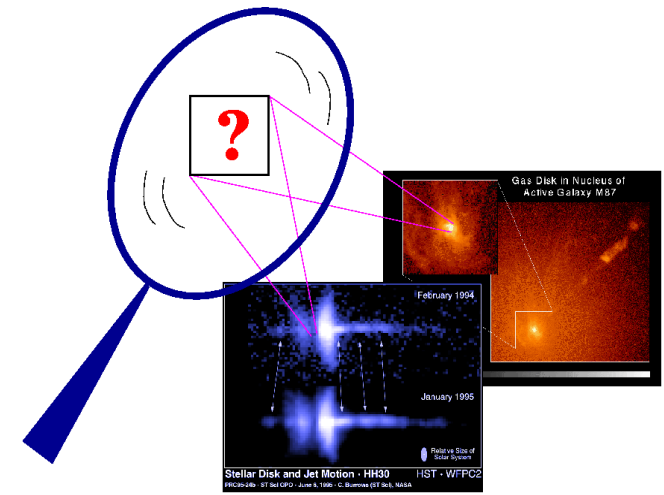
# Outflows & Jets: Theory & Observations



Lecture winter term 2008/2009

Henrik Beuther & Christian Fendt

# Outflows & Jets: Theory & Observations



## 10.10 Introduction & Overview ("H.B." & C.F.)

17.10 Definitions, parameters, basic observations (H.B.)

24.10 Basic theoretical concepts & models I (C.F.): Astrophysical models, MHD

31.10 Basic theoretical concepts & models II (C.F.): MHD, derivations, applications

07.11 Observational properties of accretion disks (H.B.)

14.11 Accretion, accretion disk theory and jet launching (C.F.)

21.11 Outflow-disk connection, outflow entrainment (H.B.)

28.11 Outflow-ISM interaction, outflow chemistry (H.B.)

**05.12 Theory of outflow interactions; Instabilities (C.F.)**

12.12 Outflows from massive star-forming regions (H.B.)

19.12 Radiation processes - 1 (C.F.)

*26.12 and 02.01 Christmas and New Year's break*

09.01 Radiation processes - 2 (H.B.)

16.01 Observations of AGN jets (C.F.)

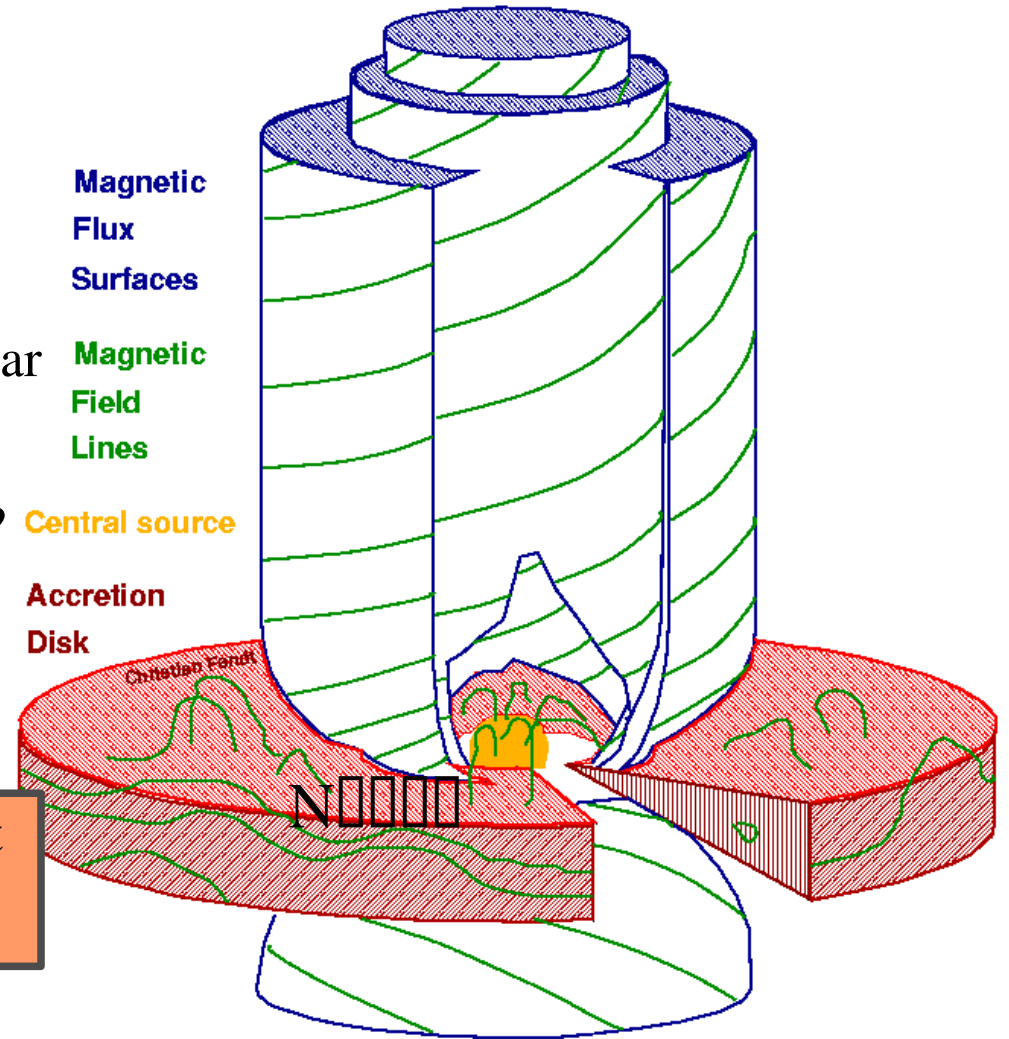
23.01 Some aspects of AGN jet theory (C.F.)

30.01 Summary, Outlook, Questions (H.B. & C.F.)

## Standard model of jet formation

### -> 5 basic questions of jet theory:

- collimation & acceleration of a disk/ stellar wind into a jet?
- ejection of disk/stellar material into wind?
- accretion disk structure?
- generation of magnetic field?
- jet propagation / interaction with ambient medium



### Topics today:

- molecular outflows
- jet instabilities
- shocks (HD, MHD)

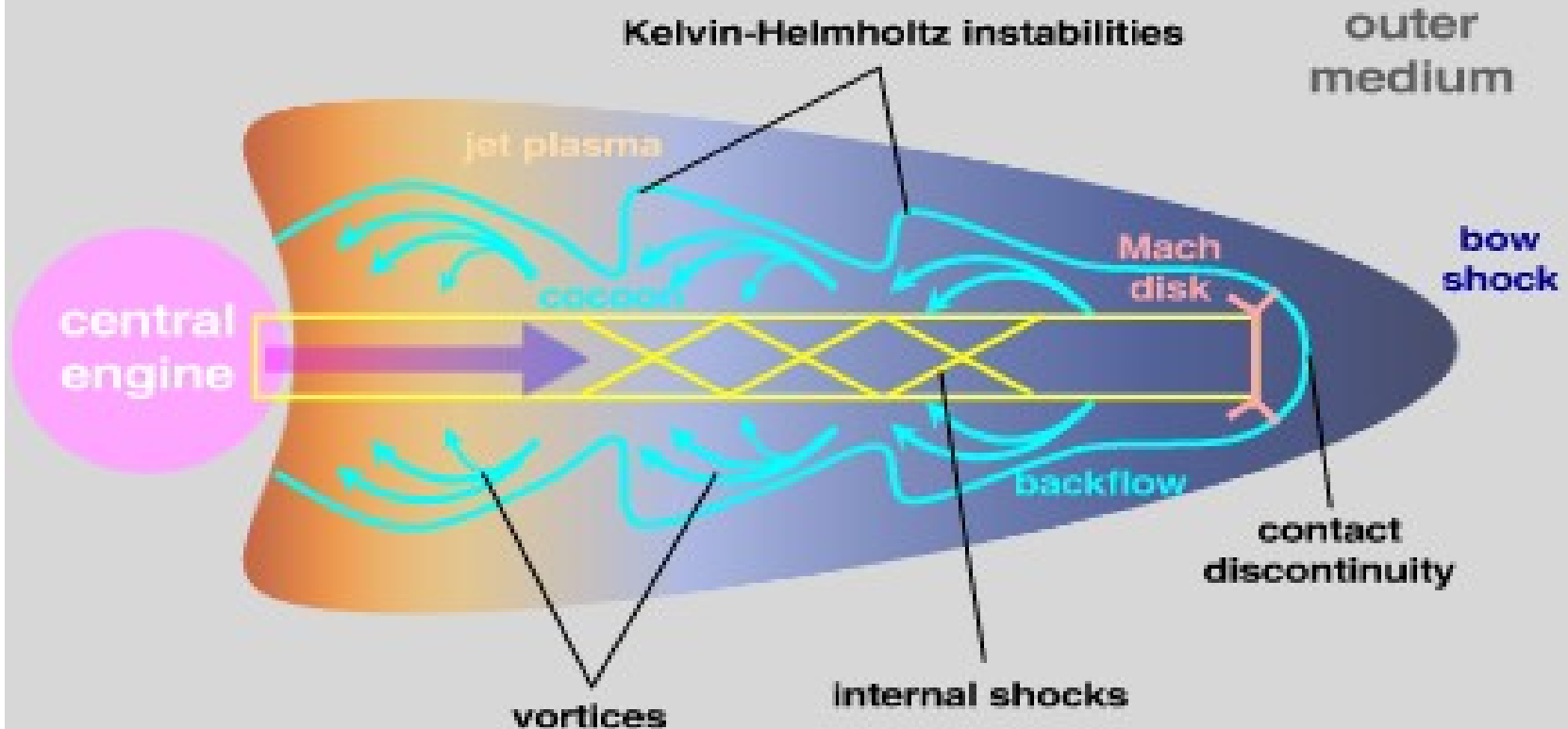
## Outflows & Jets: Theory & Observations

### Standard model of jet formation

- jet propagation / interaction with ambient medium

Topics today: molecular outflows, jet instabilities, shocks (HD, MHD)

## Jet propagation



## Driving of molecular outflows

### Molecular outflows are “momentum-driven” ...

- > momentum-driven: excess energy is radiated away
- > energy-driven: energy adiabatically converted into kinetic energy

-> observational constraints:

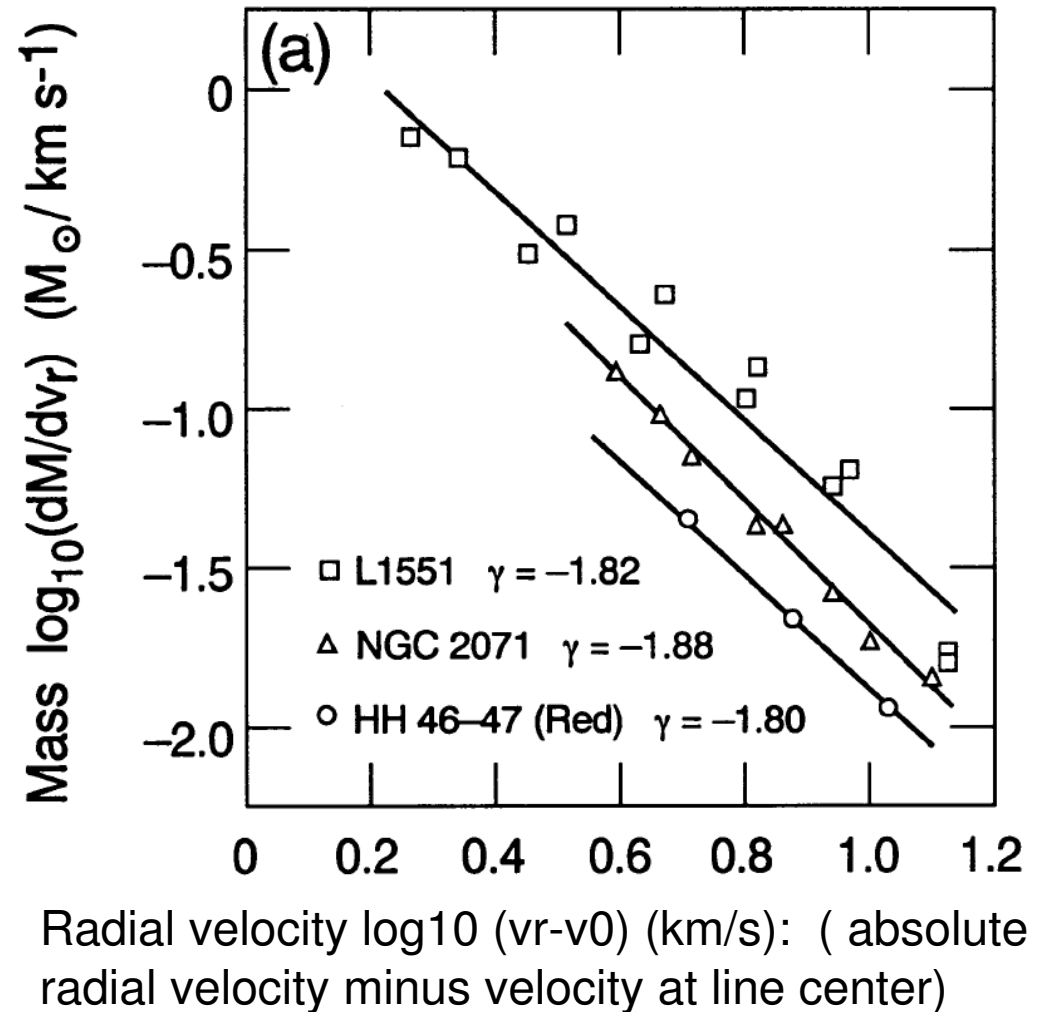
1) M-V relation ( $v_j$  fiducial jet speed):

$$\frac{dM(v)}{dv} = k \left( \frac{v}{v_j} \right)^{-\gamma}$$

2) increasing velocity ( $\sim$  linear) with distance, “acceleration”:  
 “Hubble law” (Lada & Fich 96)

-> defines dynamical time scale:

$$t_{dyn} = r / \bar{v}$$



## Outflows & Jets: Theory & Observations

### Driving of molecular outflows

Molecular outflows are “momentum-driven” ...

From observed M-V relation:

-> kinetic luminosity:

$$L_{kin} = \frac{1}{t_{dyn} u_o} \int du u^2 dM / du$$

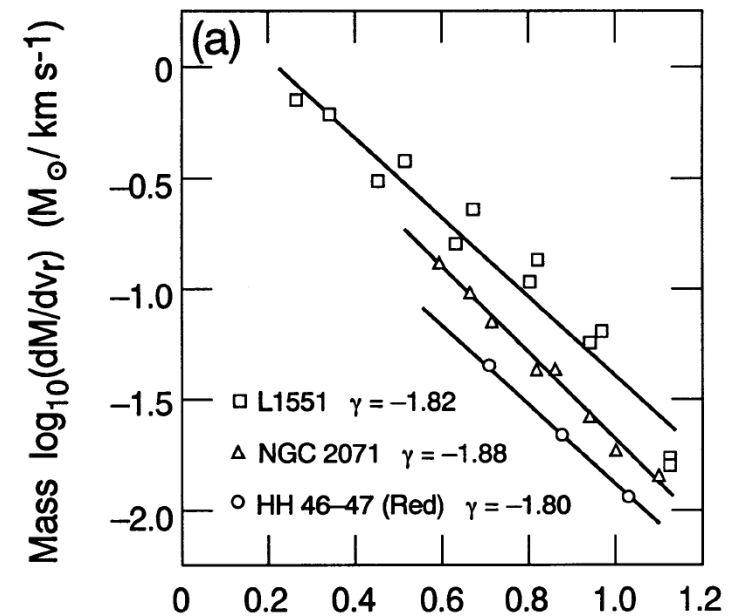
-> momentum flow (“force”):

$$F_{out} = \frac{1}{t_{dyn} u_o} \int du u dM / du$$

-> driver cannot be radiation of central star:

$$F_{out} = 10^{-3} M_o km/s/yr \gg L_{bol} / c$$

-> additional source for energy / momentum: **magnetic field** (via jet)



# Driving of molecular outflows

## Molecular outflows are “momentum-driven”

-> 4 model scenario of molecular flow acceleration (see Henrik's lecture):  
jet entrainment, bow shock, wide angle wind, circulation model

-> main **questions to be answered by theory** (see Downes & Ray 1999):

-> how much **momentum** is transferred to the ambient molecules?

-> is there a **power-law** relation between the mass in the molecular flow and velocity?

-> what are the **proper motions** of the molecular 'knots'?

-> how does the **knot emission** behave in time?

-> is the so-called **Hubble law** of molecular outflows reproduced?

-> is there extra entrainment of ambient gas along the jet due to velocity variations (i.e. jet pulses) ?

-> **answers are not yet known** .... preference for jet-driven models

## Driving of molecular outflows

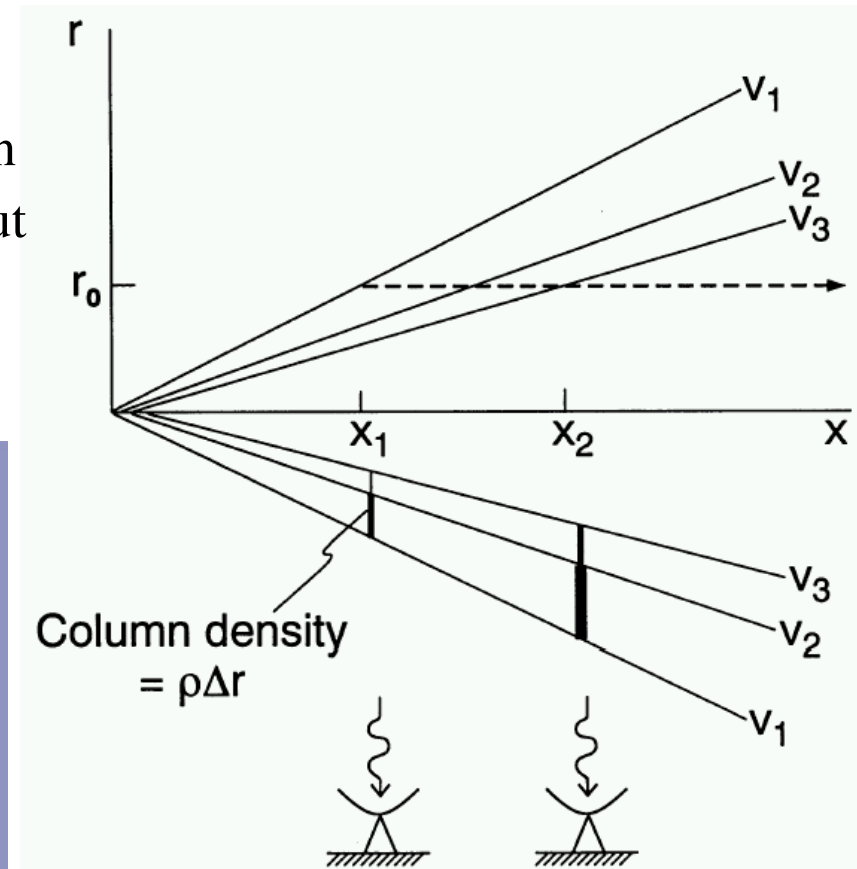
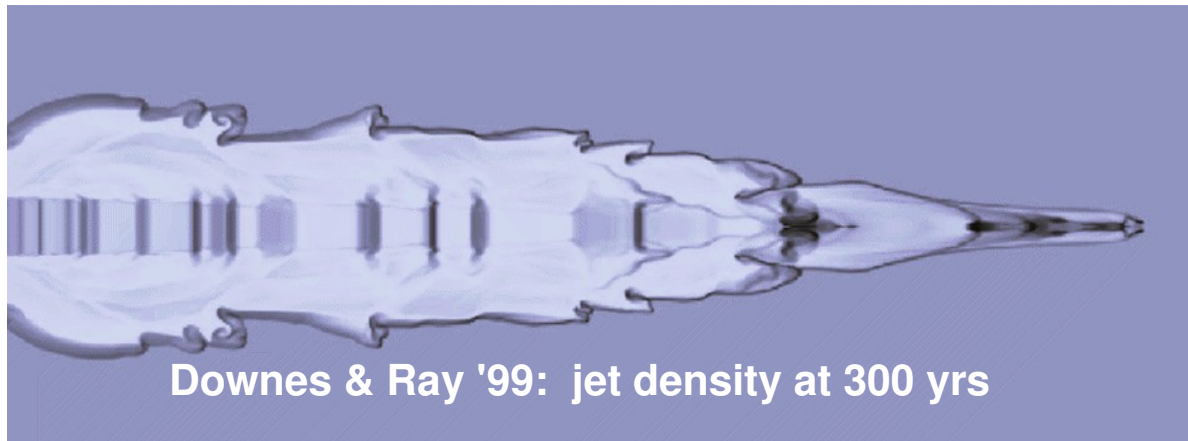
### Molecular outflows are “momentum-driven”

-> main questions to be answered by **theory** (see Downes & Ray 1999)

**-> answers are not yet known** .... preference for jet-driven models

-> indication that **Hubble law** is **apparent effect**:

- 1) l. o. s . **column density** increases (Stahler 94)  
along turbulent outflow;  $v_3 > v_2 > v_1$ ; opt. thin  
-> high velocities become “visible” further out
- 2) projected **bow shock** velocity  
distribution (Downes & Ray 99)



## Driving of molecular outflows

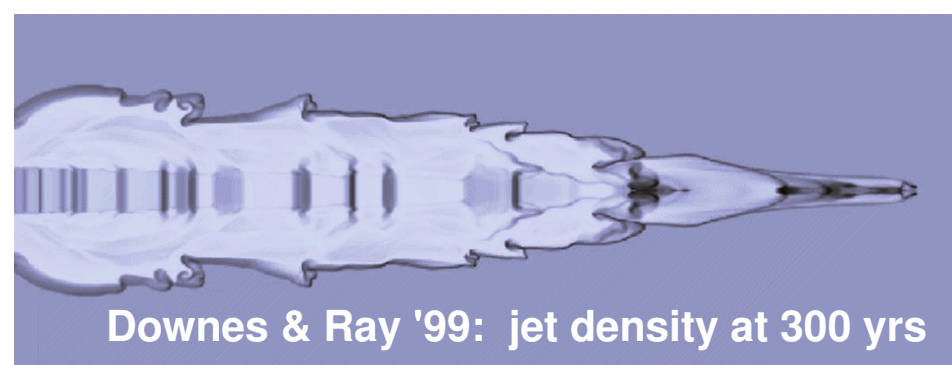
### Jet-driven molecular outflows

Numerical simulations (Downes & Ray 1999)

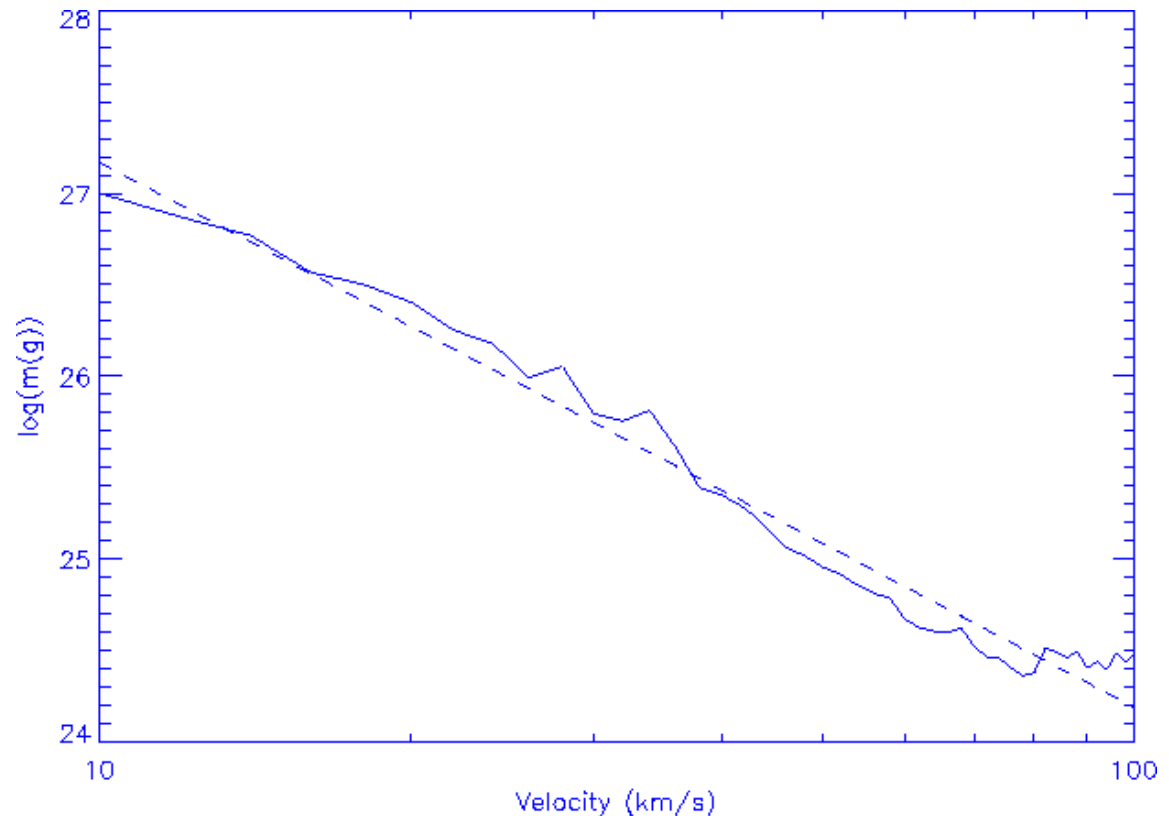
- > tricky: accelerate molecules without dissociating them -> proper cooling function ...
- > **measure momentum transfer:**
  - momentum in molecules / total momentum in box  $\sim 0.1 \dots 0.4$
  - **inefficient momentum transfer** for molecules, particularly for increasing density
  - jet cooling narrows the jet head thus the cross section thus reduces momentum transfer

- > **power-law mass-velocity**  
relation reproduced  $\gamma = 2 - 4$

**mass-velocity relation**  
**( 300 yrs,  $i = 60^\circ$  )**



Downes & Ray '99: jet density at 300 yrs

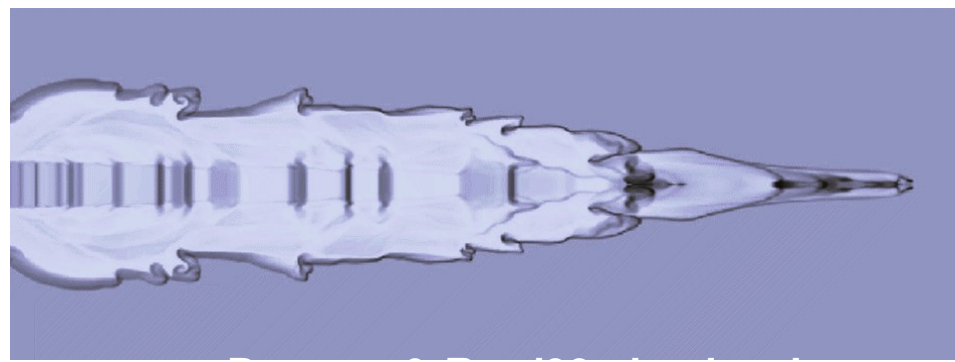


## Outflows & Jets: Theory & Observations

### Driving of molecular outflows

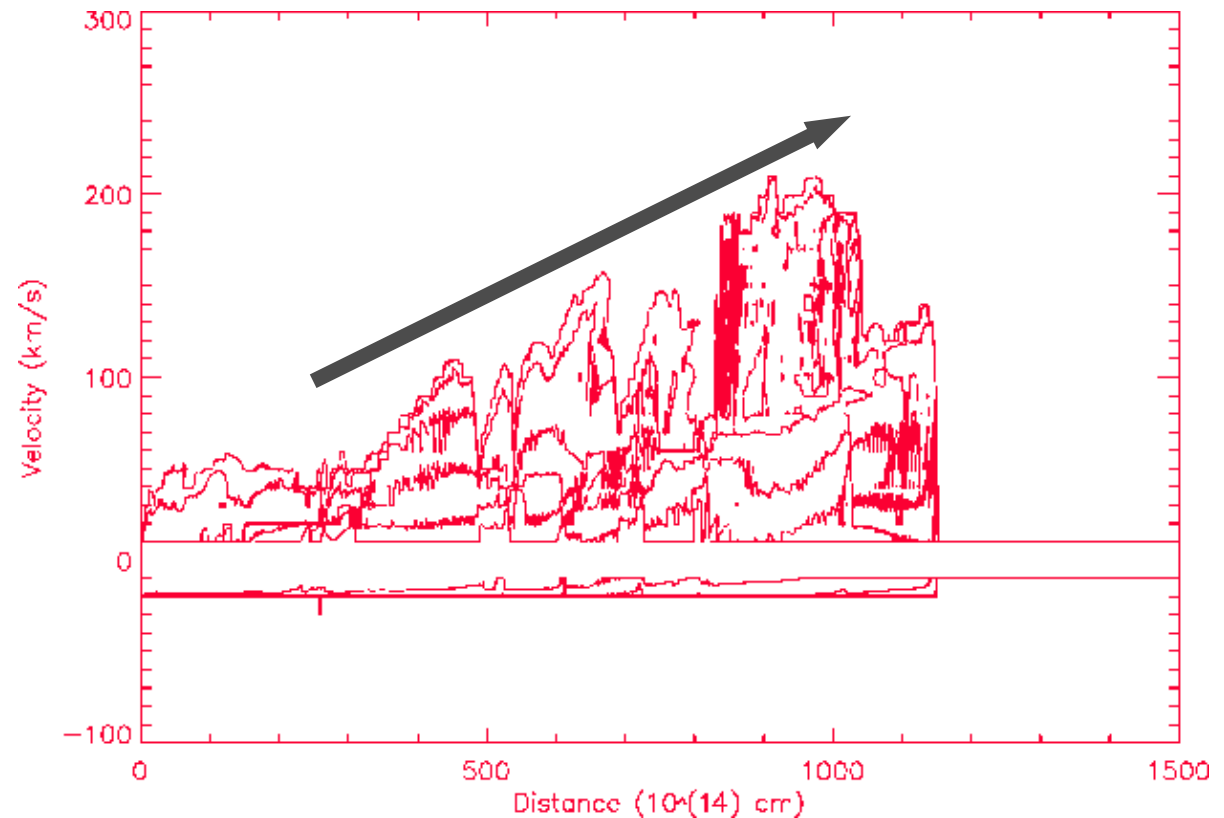
#### Jet-driven molecular outflows

Numerical simulations (e.g. Downes & Ray 1999)



- > tricky: accelerate molecules without dissociating them → proper cooling function ...
- > **measure momentum transfer:**
  - momentum in molecules / total momentum in box  $\sim 0.1 \dots 0.4$
  - **inefficient momentum transfer** for molecules, particularly for increasing density
  - jet cooling narrows the jet head thus the cross section thus reduces momentum transfer

-> **Hubble law reproduced**



**Hubble law**  
( 300 yrs,  $i = 60^\circ$  )

# Outflows & Jets: Theory & Observations

## Driving of molecular outflows

### Jet-driven molecular outflows

Analytical model for Hubble law  
(Downes & Ray 1999)

#### Idealized bow shock:

-> shape  $z = a - r^s$ ;  $s \geq 2$

('a' is apex position of bow shock)

-> velocity ratio along the bow shock (= streamline ...):

$$\mathfrak{R} \equiv -v_z / v_r = s(a - z)^{\frac{s-1}{s}}$$

-> postshock velocity (would be derived from jump conditions):  $v_1(z)$

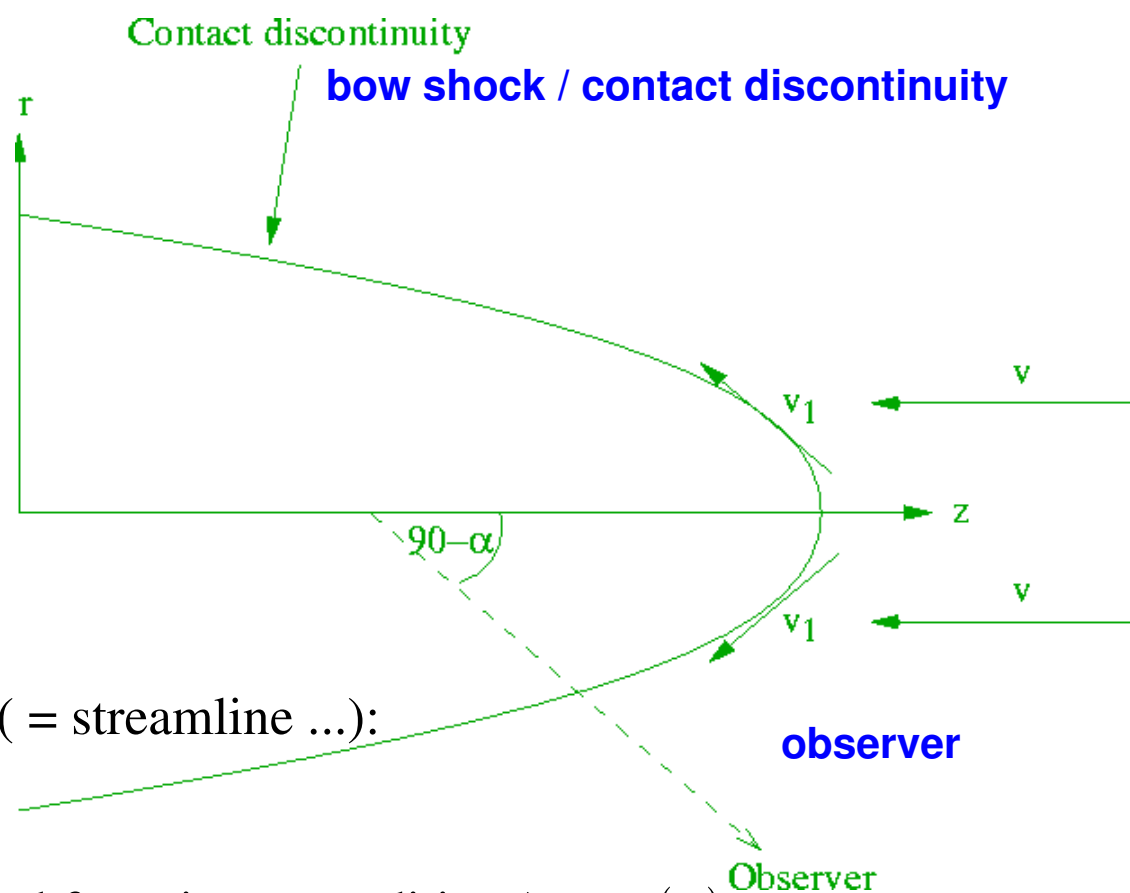
-> radial component:  $v_r(z) = \frac{v_1}{\sqrt{1 + \mathfrak{R}^2}}$

-> component along line of sight:  $v_{los}(z) = \frac{v_1}{\sqrt{1 + \mathfrak{R}^2}} (\cos \alpha + \mathfrak{R} \sin \alpha)$

-> for strong shock, compression ratio of 4:  $v_{los}(z) = v \cos(\arctan \mathfrak{R}) \sqrt{\frac{1}{16} + \mathfrak{R}^2}$

-> this implies Hubble-like position-velocity diagram

-> Hubble law is (partly) artifact of geometric



## Outflows & Jets: Theory & Observations

### Instabilities in jet flows

#### Primer on jet instabilities

-> “instability”: fluid system is unstable if small perturbations grow unbounded

-> instabilities usually investigated from **equilibrium state**

-> instabilities may lead to **disruption** of entire flow

-> **jets propagation** is affected by a number of instabilities

-> **Kelvin-Helmholtz-i.**, **current driven i.**, **sausage i.**, **kink i.** ...

-> magnetic field may stabilise some instability modes (e.g. KHI)

-> magnetic field may cause additional instabilities (“current driven”)

**-> in summary, real jets are surprisingly stable:**

**protostellar jets:** jet length (several pc)  $\sim$  100 jet radii ( $<$  100 AU)

**AGN jets :** jet length (several Mpc)  $\sim$  100 jet radii (on kpc scale)

-> **stability analysis:** dispersion relation between angular frequency of perturbation and wave vector by wave ansatz:

$$D(\omega, \mathbf{k}) = 0 \quad ; \quad V \rightarrow V + V' \quad ; \quad V' \simeq \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t))$$

-> stability is inferred from roots of dispersion relation; solution  $\mathbf{k}(\omega)$

-> roots with negative imaginary part of  $\mathbf{k}$  correspond to **spatially growing** perturbations in some direction

## Instabilities in jet flows

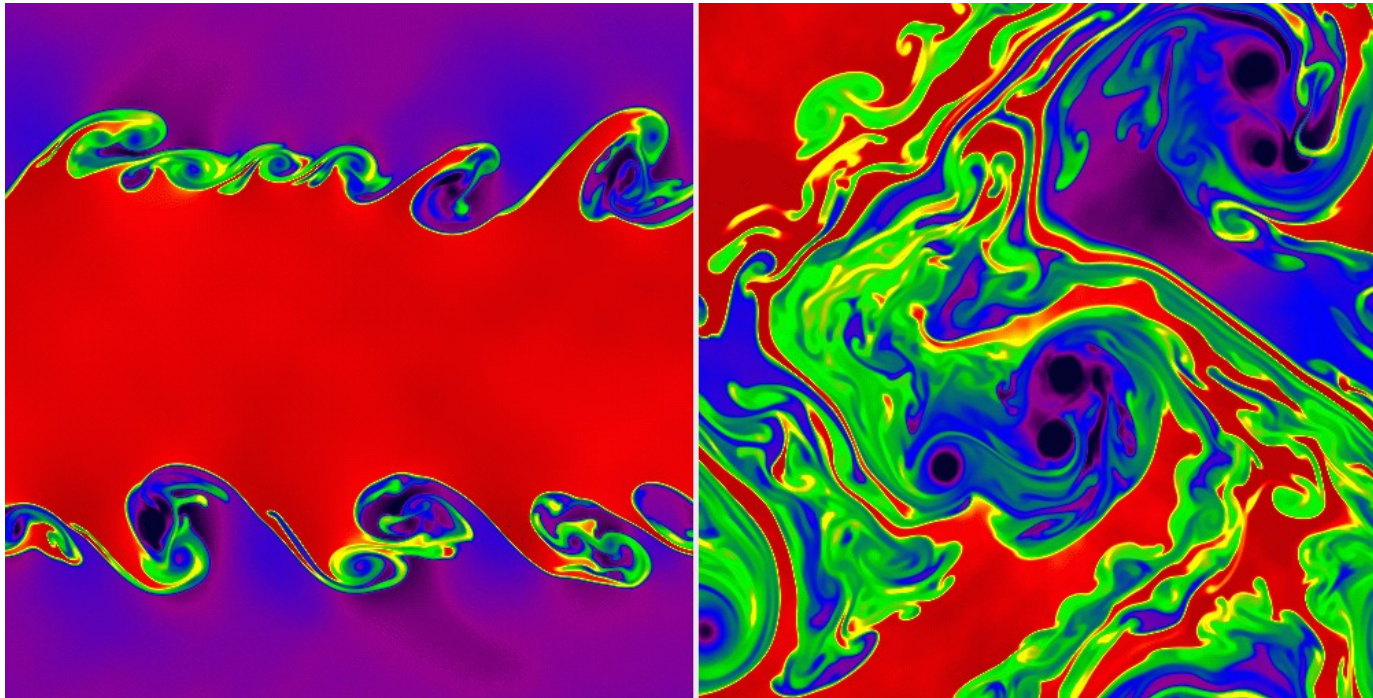
### Kelvin Helmholtz instability (KHI)

-> fluid layers, velocity shear -> growth of initial undulation  
growth mechanism: centrifugal force due to flow along curved interface (see Shu 1992)

-> mathematical approach: (M)HD equations, linear perturbation,

wave ansatz -> instability if relative Mach number of two streams  $M^2 \cos^2 \phi < 8$

-> numerical simulations to investigate nonlinear regime of instability



J. Stone et al; see  
[www.astro.princeton.edu/  
~jstone/tests/kh/kh.html](http://www.astro.princeton.edu/~jstone/tests/kh/kh.html)

Hydrodynamic simulation; time steps 1.0, 5.0; box of 512 x 512 cells; density 0.9 ...2.1

$[v_x]_1 = -0.5$  ;  $[v_x]_2 = 0.5$  ;  $\rho_1 = 1$  ;  $\rho_2 = 2$  ,  $P_1 = P_2$  ;  $M_1 = 0.38$  ,  $M_2 = 0.27$

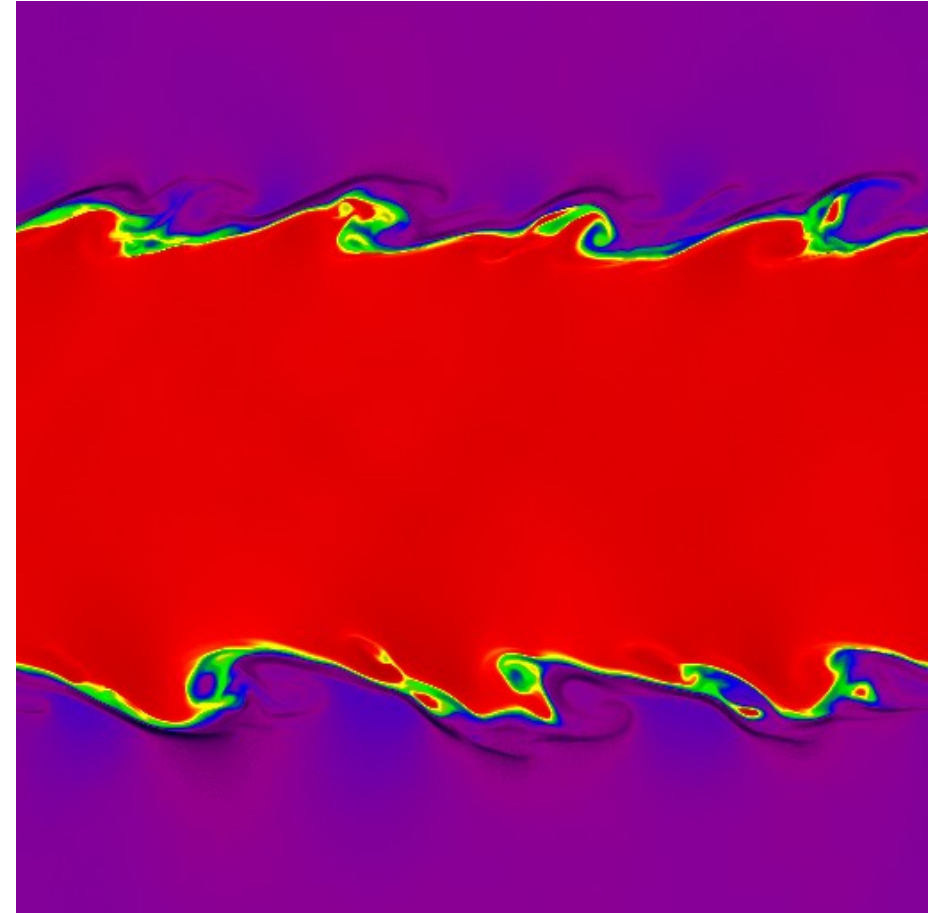
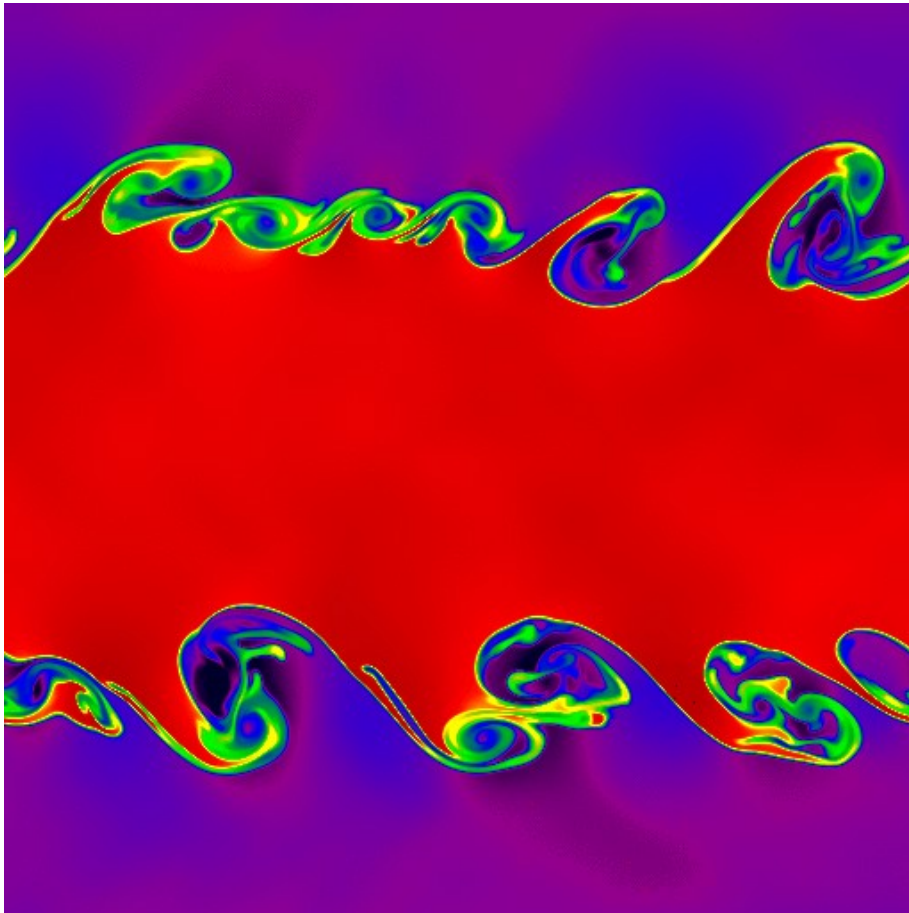
## Outflows & Jets: Theory & Observations

### Instabilities in jet flows

#### Kelvin Helmholtz instability

-> HD simulations of shearing fluid layers

MHD simulation: aligned magnetic field stabilises  
KHI <----> tension balances centrifugal force



Time step 5.0; box of 512 x 512 cells; density 0.9 ...2.1 ; seed disturbance  $\Delta v \sim 0.01$

$[v_x]_1 = -0.5$  ;  $[v_x]_2 = 0.5$  ;  $\rho_1 = 1$  ;  $\rho_2 = 2$  ,  $P_1 = P_2$  ;

$M_1 = 0.377$  ,  $M_2 = 0.267$  ,  $\mathbf{B} = B_x = \text{const} = 0.5 \sqrt{4\pi}$

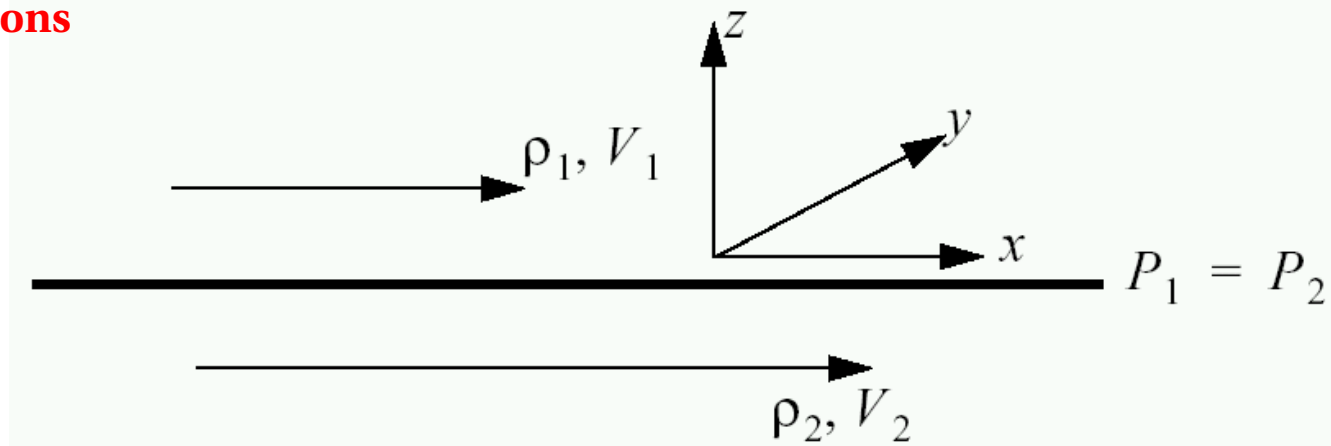
## Outflows & Jets: Theory & Observations

### Instabilities in jet flows

#### Kelvin Helmholtz instability

##### Derivations

(see lecture notes Bicknell)



-> mass & momentum conservation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho V_i) = 0$$

$$\rho \left[ \frac{\partial V_i}{\partial t} + V_j \frac{\partial V_i}{\partial x_j} \right] + \frac{\partial P}{\partial x_i} = 0$$

-> disturbance:

$$\rho = \rho_0 + \rho' \quad V_i = V_{0,i} + V_i'$$

-> implying that:

$$\rho V_i = \rho_0 V_{0,i} + \rho_0 V_i' + \rho' V_{0,i}$$

$$\frac{\partial V_i}{\partial x_j} = (V_{0,j} + V_j') \frac{\partial}{\partial x_j} V_i' = V_0 \frac{\partial}{\partial x} V_i'$$

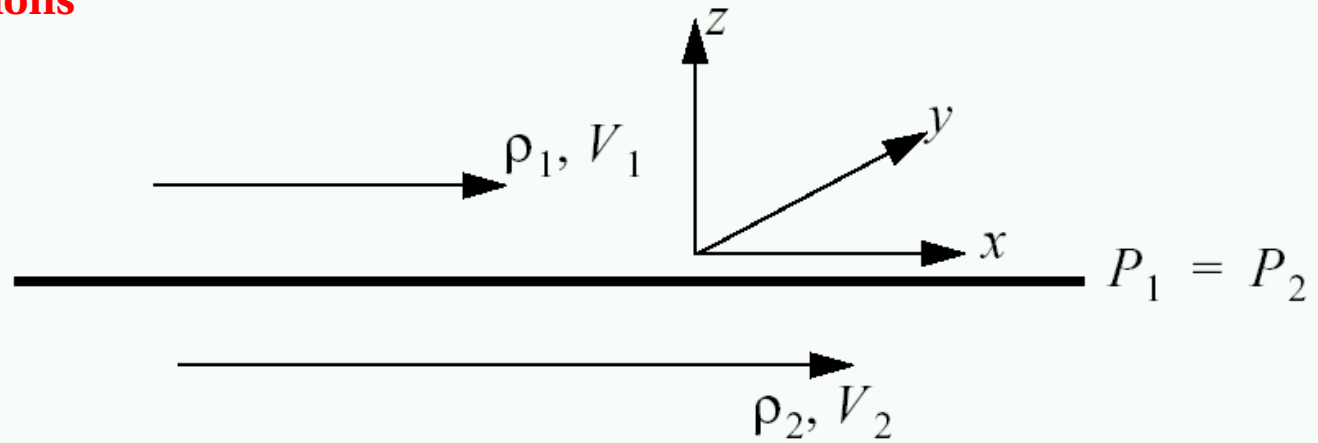
( used for perturbed mass & momentum conservation )

## Outflows & Jets: Theory & Observations

### Instabilities in jet flows

#### Kelvin Helmholtz instability

derivations ...



-> perturbed mass & momentum conservation:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial}{\partial x_j} V_j' + V_{0,j} \frac{\partial}{\partial x_j} \rho' = 0$$

$$\rho_0 \left[ \frac{\partial V_i}{\partial t} + V_0 \frac{\partial}{\partial x} V_i' \right] + \frac{\partial}{\partial x_i} P' = 0$$

-> substitute density by pressure using (note density might be discontinuous, pressure not)

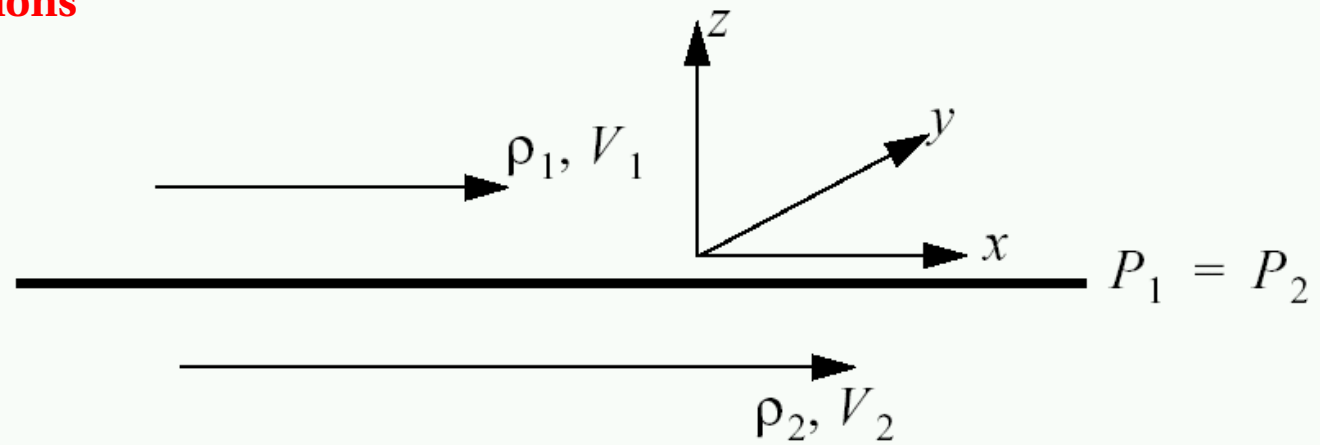
$$\left\{ \begin{array}{c} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x_i} \end{array} \right\} \rho = \frac{1}{c_s^2} \left\{ \begin{array}{c} \frac{\partial}{\partial t} \\ \frac{\partial}{\partial x_i} \end{array} \right\} P$$

## Outflows & Jets: Theory & Observations

### Instabilities in jet flows

#### Kelvin Helmholtz instability

derivations ....



-> summary of perturbed equations:

$$\left[ \frac{\partial}{\partial t} P' + V_0 \frac{\partial}{\partial x} P' \right] + \rho_0 c_0^2 \frac{\partial}{\partial x_j} V_j' = 0$$

$$\rho_0 \left[ \frac{\partial V_i}{\partial t} + V_0 \frac{\partial V_i'}{\partial x} \right] + \frac{\partial P'}{\partial x_i} = 0$$

-> apply wave ansatz for perturbed quantities:

$$P' = A \exp i[k_x x + k_y y + k_z z - \omega t]$$

$$V_i' = A_i \exp i[k_x x + k_y y + k_z z - \omega t]$$

-> with  $\rho c_0^2 = \gamma P_0$  we have  $-i(\omega - k_x V_0)A + \gamma P_0 (i k_j A_j) = 0$

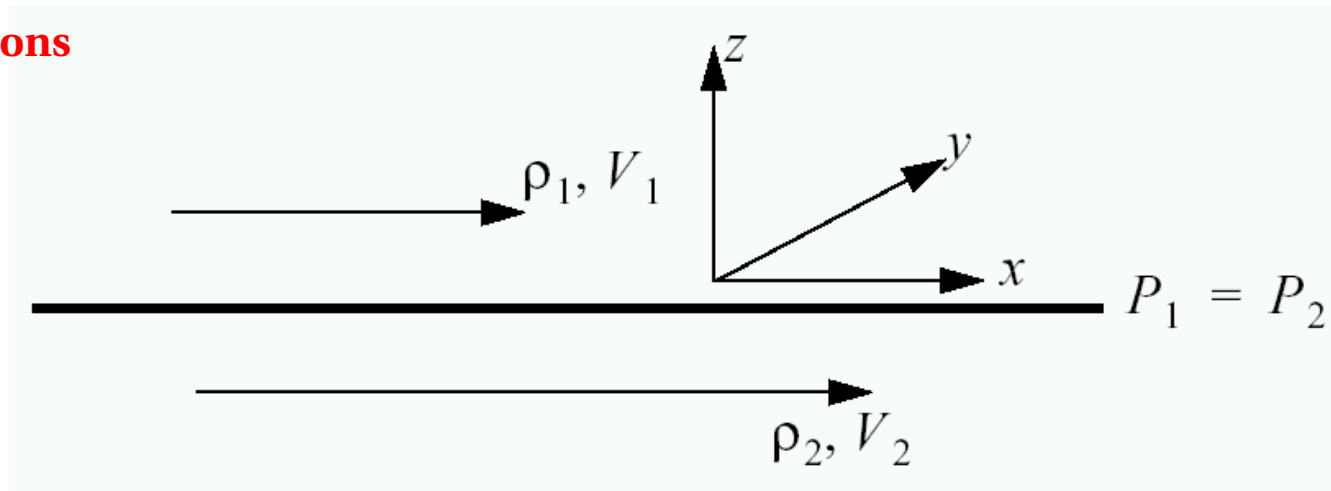
$$-i\rho_0(\omega - k_x V_0)A_i + i k_i A = 0$$

## Outflows & Jets: Theory & Observations

### Instabilities in jet flows

#### Kelvin Helmholtz instability

derivations ...



-> rewrite wave vector, parallel component parallel to interface

$$\mathbf{k} = (k_x, k_y, k_z) = (\mathbf{k}_{\parallel}, k_z)$$

$$k_x = k_{\parallel} \cos \phi \quad k_y = k_{\parallel} \sin \phi$$

$$k^2 = k_{\parallel}^2 + k_z^2 = k_i k_i$$

-> new perturbed equations (multiplied by  $k_{\parallel}$ ) -> **dispersion relation:**

$$(\omega - k_{\parallel} V_0 \cos \phi)^2 = \frac{\gamma P_0}{\rho_0} k^2 = c_0^2 k^2 = c_0^2 (k_{\parallel}^2 + k_z^2)$$

-> this is the dispersion relation of sound waves!

-> for the two sides of the interface:

$$(\omega - k_{\parallel} V_1 \cos \phi)^2 = c_1^2 k^2$$

$$(\omega - k_{\parallel} V_2 \cos \phi)^2 = c_2^2 k^2$$

# Outflows & Jets: Theory & Observations

## Instabilities in jet flows

### Kelvin Helmholtz instability

derivations ...

-> now check for displacement of interface ....

$$\frac{\partial \zeta}{\partial t} + V_0 \frac{\partial \zeta}{\partial x} = V_z' = A_z \exp i[k_x x + k_y y + k_z z - \omega t]$$

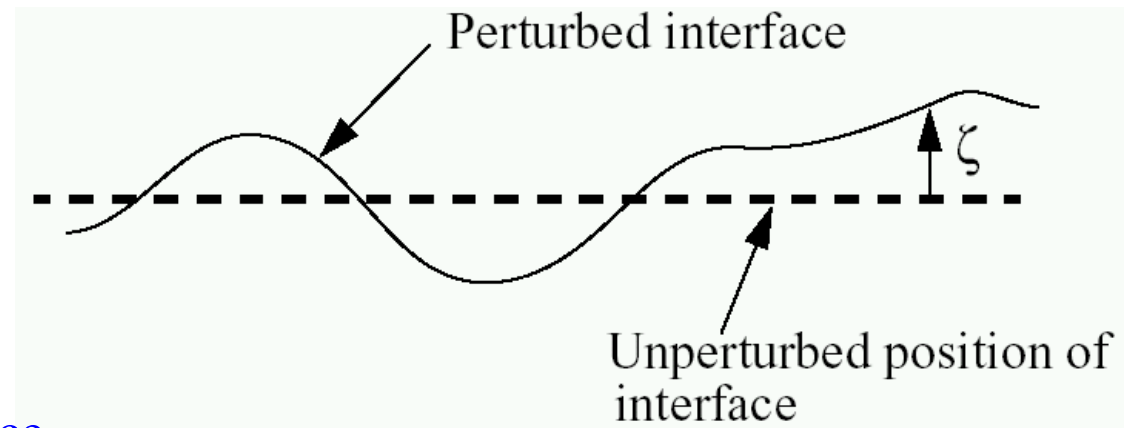
-> wave ansatz for z-displacement:  $\zeta = B_z \exp i[k_x x + k_y y + k_z z - \omega t]$

-> consider that 1)  $\mathbf{P}$ ,  $\zeta$  are continuous, 2)  $\rho$  is not, and 3) boundary conditions

-> equation for K-H instability:

$$\frac{\frac{V_{\text{ph}}'^2}{c_1^2} - 1}{\frac{\gamma_1^2 V_{\text{ph}}'^4}{c_1^4}} = \frac{\frac{(V_{\text{ph}}' - \Delta V \cos \phi)^2}{c_2^2} - 1}{\frac{\gamma_2^2 (V_{\text{ph}}' - \Delta V \cos \phi)^4}{c_2^4}}$$

with phase velocity  $V_{\text{ph}}' = \frac{\omega'}{k_{\parallel}}$  and velocity difference  $\Delta V = V_2 - V_1$

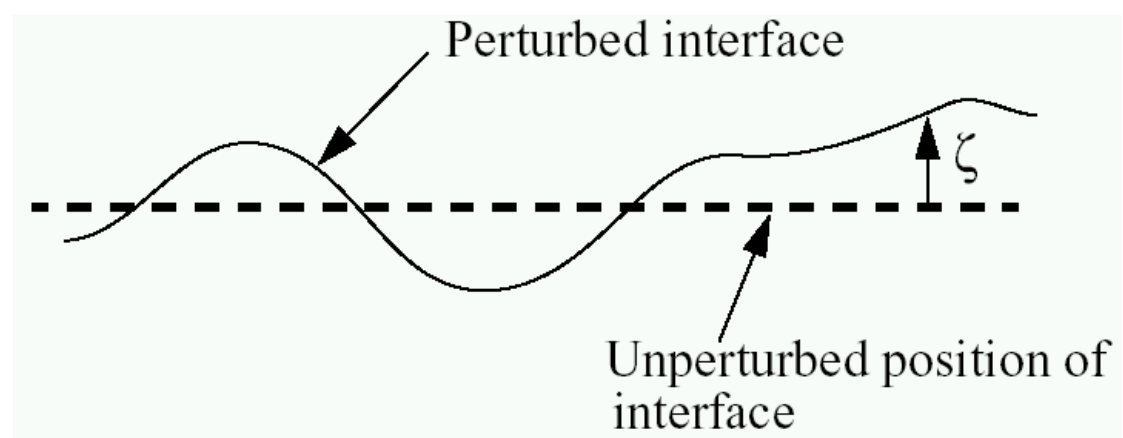


## Outflows & Jets: Theory & Observations

### Instabilities in jet flows

#### Kelvin Helmholtz instability

derivations ...



-> define **phase velocity** of perturbation relative to sound speed in frame of lower stream and **relative Mach number** of two streams in direction of perturbation ( ->  $\phi$  ):

$$x = \frac{V_{\text{ph}}'}{c_1} \quad m = \frac{\Delta V \cos \phi}{c_1}$$

-> basic dispersion relation for compressible KHI:

implicit 6<sup>th</sup> order polynomial equation:

$$\frac{x^2 - 1}{x^4} = \frac{\gamma_1^2 c_2^2}{\gamma_2^2 c_1^2} \left[ \frac{(x - m)^2 - c_2^2 / c_1^2}{(x - m)^4} \right]$$

-> example solution for instability for case  $\gamma_1 = \gamma_2$

-> eq. (\*\*) factorizes (quartic & quadratic part): roots of quadratic are stable solutions

-> roots of quartic correspond to **instability** if  $m^2 < 8$   
resp. correspond to **stability** if  $m = \frac{\Delta V \cos \phi}{c_1} > \sqrt{8}$   
( critical angle for instability ... )

-> growth rate for magnetized KHI  $\sim$  Alfvén crossing time

### Instabilities in jet flows

#### Current-driven (CD) instabilities

-> poloidal **electric current** equivalent to toroidal magnetic field:

$$\mathbf{rot} \mathbf{B} \sim \mathbf{j} \quad \text{integration} \rightarrow \mathbf{R} \mathbf{B}_\phi = \mathbf{I} \quad (\text{“identity”})$$

-> current carrying jets:

-> **KH** modes **slightly stabilised** compared to longitudinal field-only case (at same Mach number)

-> liable to additional pure **MHD instabilities** :

-> driven by **electric current** along the magnetic field (internal instability)

-> **thermal pressure** gradient in the jet (local instability -> turbulence)

-> CD instability growth on time scales < Alfvén crossing time scale

-> depending on: ratio **pitch length** / jet radius  $\sim r B_z / B_\phi / r_{\text{jet}}$

-> small pitch angle -> strongly unstable CD modes

-> **sausage & kink modes**: most dominant modes of CD instability

## Instabilities in jet flows

### Current-driven (CD) instabilities

-> current carrying jets:

-> non-linear evolution of CDIs  
by MHD simulations (e.g. Baty & Keppens '02):

“UNI”:  $B_\phi = 0$ ,  $B_z = 0.25$

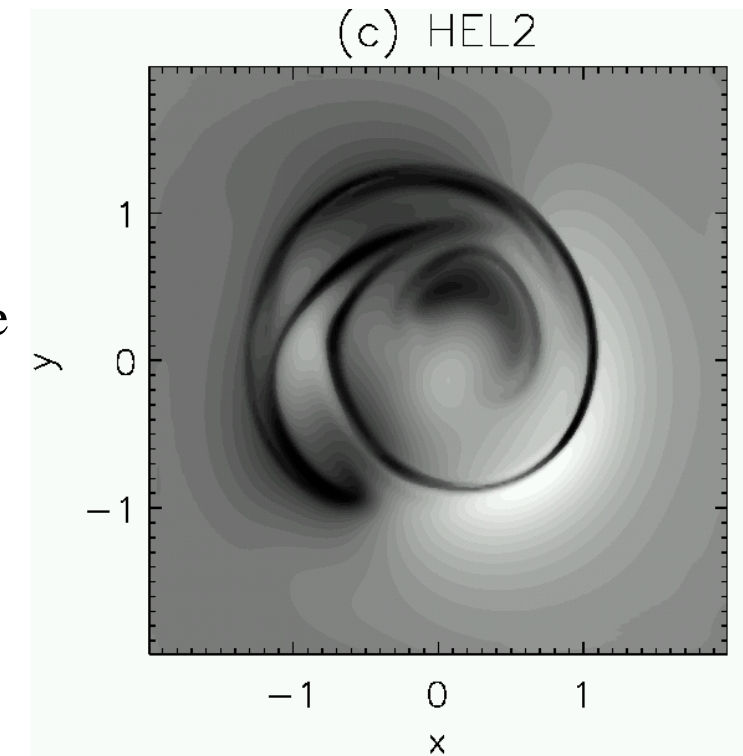
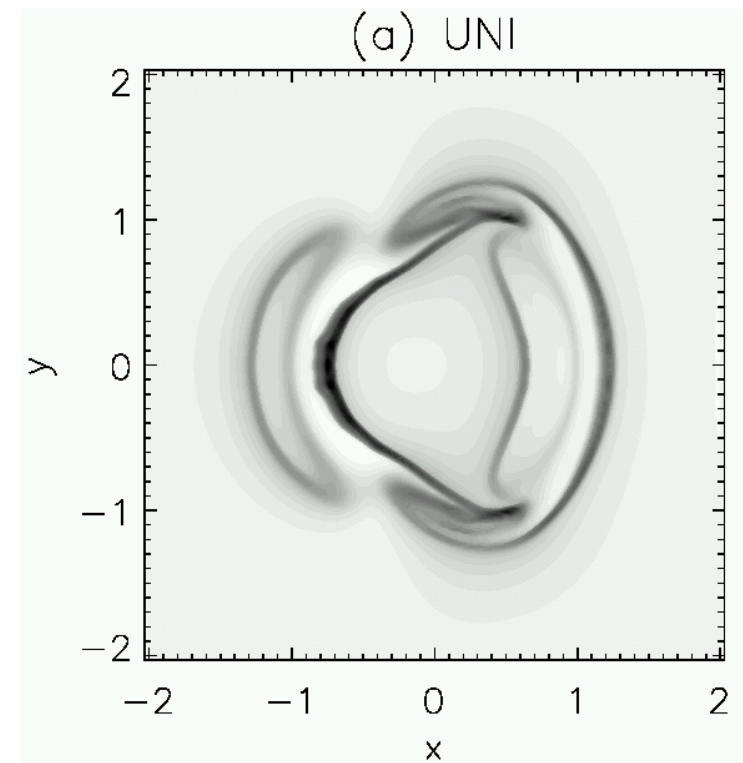
“HEL1”:  $B_\phi < 0.4$ ,  $B_z = 0.25$

Setup: 3D box: 200x200x100;  
 $M=1.26$ ,  $MA=6.52$ ,  $MF=1.24$

-> density distribution across the  
jet (linear scale 0.5 ...1.3)

-> mode coupling of CD with KH modes.

-> CD modes may saturate KH surface vortices,  
help to avoid jet disruption



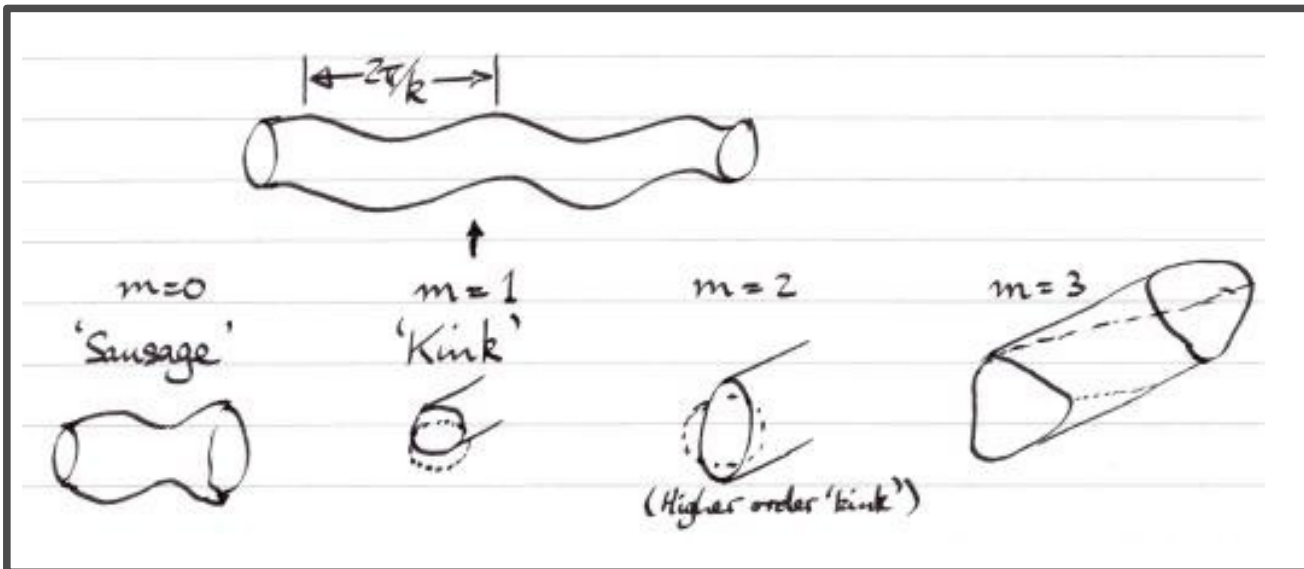
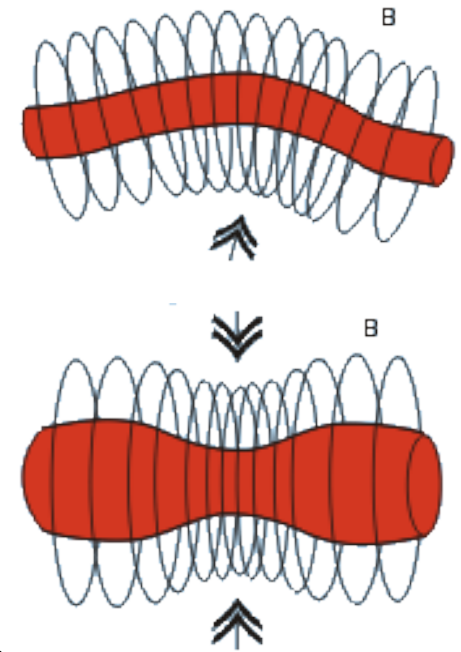
# Instabilities in jet flows

## Remarks on mode classification:

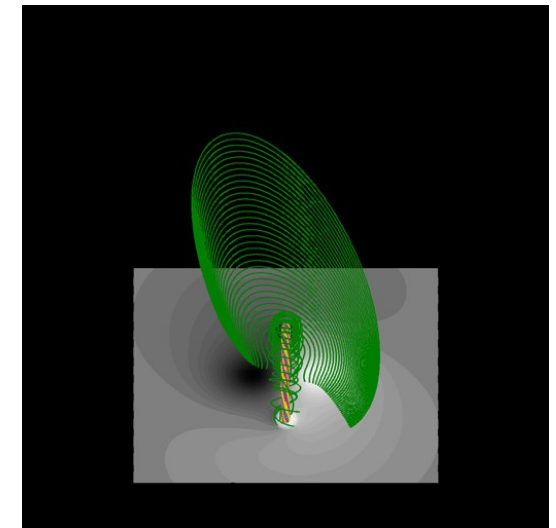
-> wave ansatz for different coordinate directions,  
e.g. cylindrical jet, cylindrical coordinate system:

$$f' = f'(r) \exp(i(kz + m\phi - \omega t))$$

- >  $m=0$  mode is **sausage** (pinch) mode;  $m=1$  is **kink** (helical) mode
- $m=2$  is higher order kink,  $m=3$  involves torsional kink
- > follows also from **Fourier expansion** of e.g. unstable flow



from Hutchinson lecture



# Shocks in jets and outflows

### Basic principles:

- > **compressible fluid**: disturbance travels within surrounding medium: **acoustic wave**
  - > **infinitesimal** disturbance: wave form is conserved, linear wave (linearized equations)
  - > **finite** disturbance: non-linear equations, non-linear acoustic wave
    - > **wave form steepens** -> **shock wave**
- > steepening of non-linear waves: sinusoidal wavelet steepens into triangular shock wave:
  - high density part of wave has higher sound speed, travels faster than average
    - > wave top catches up with bottom -> wave profile steepens
- > extreme example: compressible high speed fluid (jet) rams into low speed fluid (ISM)
  - > shock wave traveling on front of jet
- > **structure of thin shock** layer defined by **viscous** processes
  - > deceleration = momentum exchange, heating, compression defined by viscosity
  - > **shock thickness  $\Delta x \sim L$  mean free path**:
- > astrophysical plasmas thin -> mean free path long, **“collisionless”** shocks
  - > momentum exchange by **magnetic field** compression
- > Draine (1980): **J-shocks** (strong, thin, viscous), **C-shocks** (weak, wide, magnetic)

## Shocks in jets and outflows

### Hydrodynamic (viscous) shocks (J-shocks):

-> consider hydrodynamic equations in one direction:

$$\frac{d}{dx}(\rho u) = 0 \quad \frac{d}{dx} \left( \rho u^2 + P - \frac{4}{3} \mu \frac{du}{dx} \right) = 0 \quad \frac{d}{dx} \left[ \rho \left( \frac{1}{2} u^2 + \epsilon \right) u + \left( P - \frac{4}{3} \mu \frac{du}{dx} \right) u - \kappa \frac{dT}{dx} \right] = 0$$

-> frictional momentum flux:  $\pi_{xx} = \frac{4}{3} \mu \frac{du}{dx}$

-> integration -> conservation laws:  $\rho u = \text{const}$

$$\rho u^2 + P - \frac{4}{3} \mu \frac{du}{dx} = \text{const.} \quad \rho u \left( \frac{1}{2} u^2 + \epsilon + \frac{P}{\rho} \right) - \frac{4}{3} \mu u \frac{du}{dx} - \kappa \frac{dT}{dx} = \text{const.}$$

-> shock thickness:

in shock transition layer: momentum flux of same order as other terms

$$-\frac{4}{3} \mu \frac{du}{dx} \simeq \rho v \frac{\Delta u}{\Delta x} \simeq \rho u^2 \quad \rightarrow \quad \Delta x \simeq v \frac{\Delta x}{u^2}$$

for strong shocks:  $\Delta u \sim u$  and  $u \sim v_T$  since  $u$  becomes subsonic

since for kinematic viscosity  $v \sim L v_T \Rightarrow \Delta x \sim L$

## Shocks in jets and outflows

### Rankine-Hugoniot jump conditions:

-> conservation laws upstream and downstream of shock layer (derivatives neglected):

--> Rankine-Hugoniot jump conditions

$$\rho_1 u_1 = \rho_2 u_2 \quad \rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2 \quad u_1 \left( \frac{1}{2} \rho_1 u_1^2 + \rho_1 \epsilon + P_1 \right) = u_2 \left( \frac{1}{2} \rho_2 u_2^2 + \rho_2 \epsilon + P_2 \right)$$

-> solutions for R-H conditions: apply polytropic gas law  $P \sim \rho^\gamma$ ,  
define upstream Mach number  $M_1 \equiv u_1 / c_s$  with sound speed  $c_s \equiv \sqrt{\gamma P / \rho}$

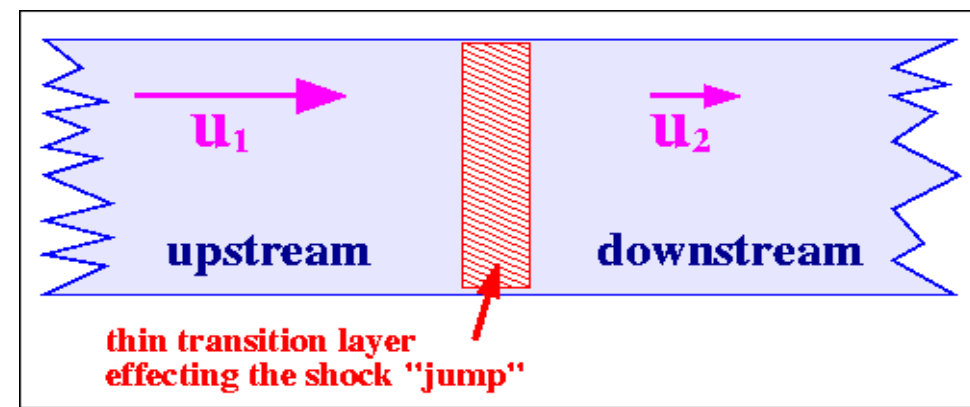
$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_1^2}{(\gamma + 1) + (\gamma - 1)(M_1^2 - 1)} = \frac{u_1}{u_2} \quad \frac{P_2}{P_1} = \frac{(\gamma + 1) + 2\gamma(M_1^2 - 1)}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{[(\gamma + 1) + 2\gamma(M_1^2 - 1)][(\gamma + 1) + (\gamma - 1)(M_1^2 - 1)]}{(\gamma + 1)^2 M_1^2}$$

Note that:  $P_2 \geq P_1$ ;  $\rho_2 \geq \rho_1$ ;  $u_2 \leq u_1$ ;  $T_2 \geq T_1$ ; for  $M_1 \geq 1$ , equality for  $M_1 = 1$

for  $M_1 \rightarrow \infty$ :  $\frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1} = 4$  for  $\gamma = 5/3$ ; but  $\frac{P_2}{P_1}$  is unlimited

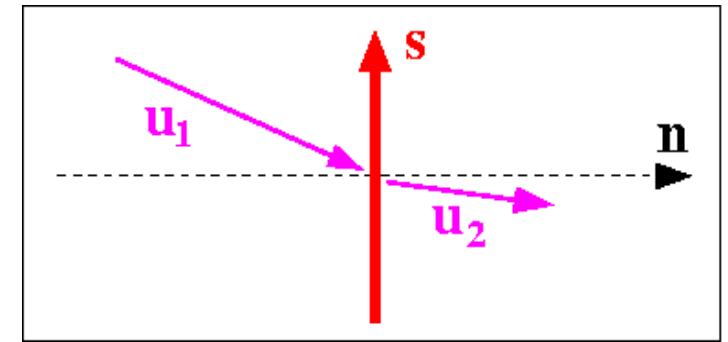
for  $M_1 > 1 \rightarrow M_2 < 1$  [compressive shocks]



## Shocks in jets and outflows

### Hydromagnetic shocks:

- > plane for local dynamics defined by inflow velocity  $\mathbf{u}$  and magnetic field  $\mathbf{B}$  (in shock frame)
- > decomposition of vectors:  $s$ -coordinate  $\parallel$  to shock,  $n$ -coordinate  $\perp$  to shock
- > Note: in addition to **viscous stress** tensor, now **Maxwell stresses** in order to exchange momentum, pressure etc
- > project **MHD conservation laws**  $\parallel$  and  $\perp$  to shock



$$\text{- mass conservation: } \frac{\delta \rho}{\delta T} + \frac{\delta}{\delta x_k} (\rho u_k) = 0 \quad \rightarrow \quad \frac{\delta}{\delta n} (\rho u_n) = \dots$$

$$\text{- momentum conservation } \frac{\delta}{\delta t} (\rho u_i) + \frac{\delta}{\delta x_k} (\rho u_i u_k + P \delta_{ik} - T_{ik}) = -\rho \frac{\delta \Phi}{\delta x_i} = 0$$

$$\rightarrow \frac{\delta}{\delta n} \left[ \rho u_n u_n + P - \frac{1}{8\pi} (B_n^2 - B_s^2) \right] = \dots \quad \frac{\delta}{\delta n} \left[ \rho u_s u_n - \frac{1}{4\pi} B_s B_n \right] = \dots$$

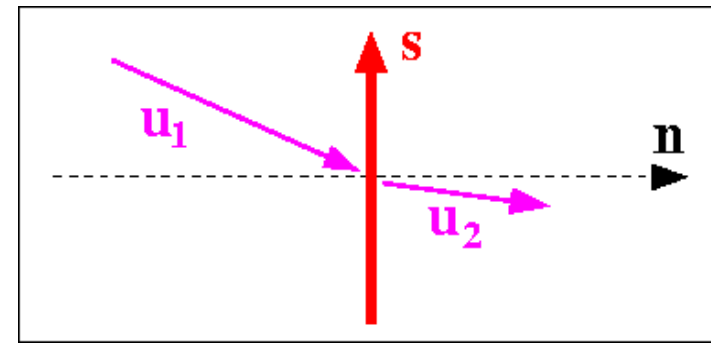
$$\text{- induction equation } \frac{\delta \vec{B}}{\delta t} + \vec{\nabla} \times (\vec{B} \times \vec{u}) = 0 \quad \rightarrow \quad \frac{\delta}{\delta n} (B_n u_s - B_s u_n) = \dots$$

$$\text{- no monopoles: } \vec{\nabla} \cdot \vec{B} = 0 \quad \rightarrow \quad \frac{\delta B_n}{\delta n} = \dots$$

## Shocks in jets and outflows

### Hydromagnetic shocks – jump conditions

jump conditions from integration of conservation laws:



$$\left[ \rho u_n \right]_2 = \left[ \rho u_n \right]_1$$

$$\left[ B_n u_s - B_s u_n \right]_2 = \left[ B_n u_s - B_s u_n \right]_1 \quad \left[ B_n \right]_2 = \left[ B_n \right]_1$$

$$\left[ \rho u_n u_s + \frac{B_s B_n}{4\pi} \right]_2 = \left[ \rho u_n u_s + \frac{B_s B_n}{4\pi} \right]_1$$

$$\left[ \rho u_n^2 + P + \frac{B_s^2}{8\pi} \right]_2 = \left[ \rho u_n^2 + P + \frac{B_s^2}{8\pi} \right]_1$$

$$\left[ \rho u_n \left( \frac{\gamma}{\gamma-1} \frac{P}{\rho} + \frac{1}{2} u^2 \right) - \frac{1}{4\pi} (B_n u_s - B_s u_n) B_s \right]_2 =$$

$$\left[ \rho u_n \left( \frac{\gamma}{\gamma-1} \frac{P}{\rho} + \frac{1}{2} u^2 \right) - \frac{1}{4\pi} (B_n u_s - B_s u_n) B_s \right]_1$$

## Shocks in jets and outflows

### Hydromagnetic shocks – jump conditions

jump conditions → behaviour of variables:

→  $(\rho \mathbf{u}_n)$  and  $\mathbf{B}_n$  conserved across the shock (mass flux, magnetic flux conservation)

→ for parallel velocity  $u_s$ :

$$[u_s]_2 - [u_s]_1 = \frac{B_n}{4\pi\rho u_n} \left( [B_s]_2 - [B_s]_1 \right)$$

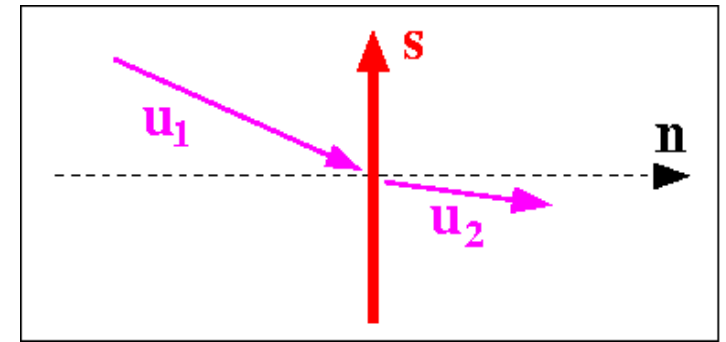
→ distinguishing features of MHD shocks:

→  $u_s$  is discontinuous unlike in the non-magnetic case

→ sudden deflections of tangential velocity possible

→ current sheet along shock:

$$j_s = \frac{c}{4\pi} \left( [B_s]_2 - [B_s]_1 \right)$$

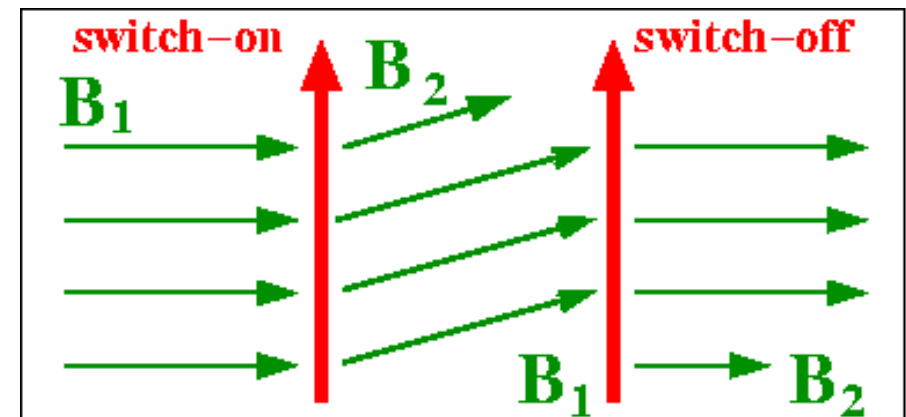
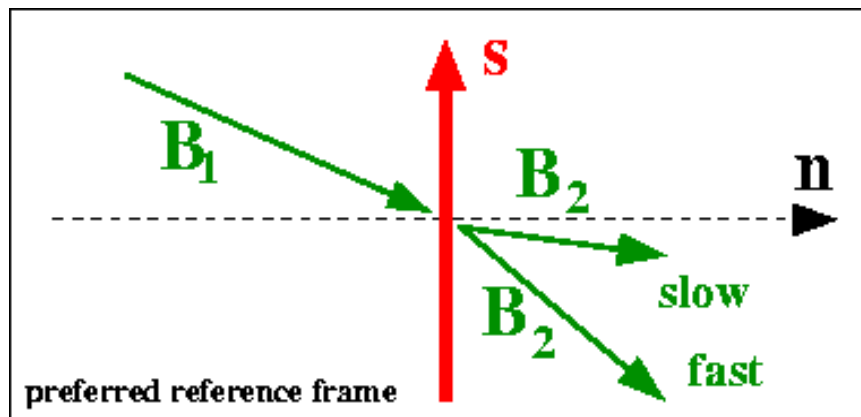
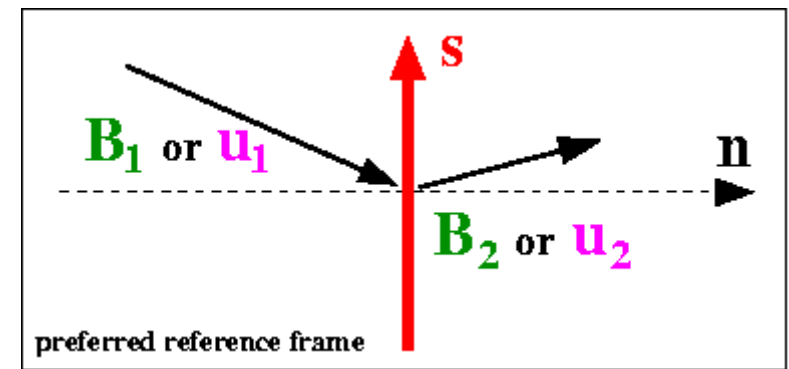


## Shocks in jets and outflows

### Hydromagnetic shocks – jump conditions

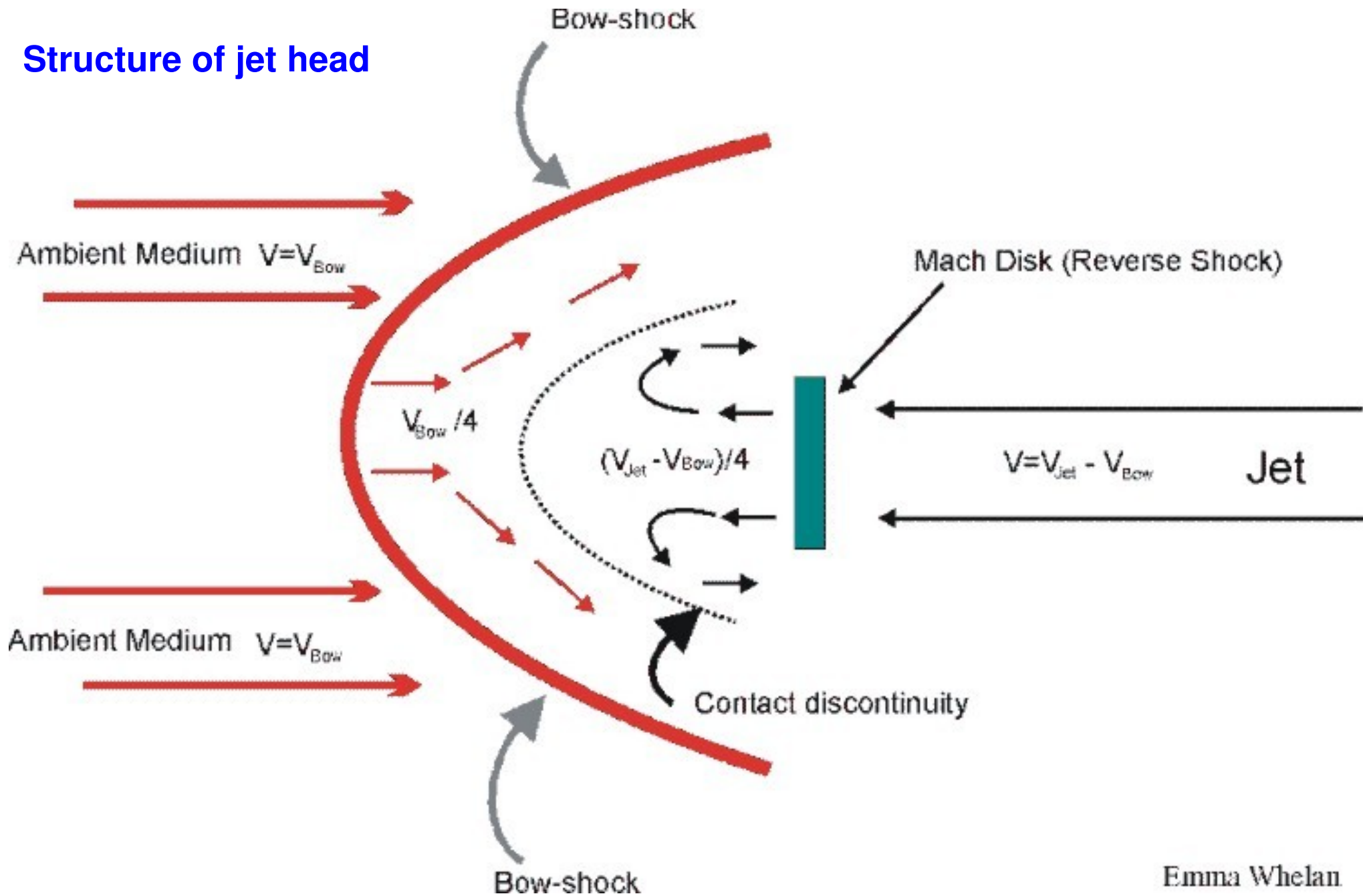
jump conditions --> shock classification:

- > for perpendicular velocity -> quartic relation to solve for  $U_{n,2}$  (requires some math ...)
- > solutions if  $U_{n,1} >$  MHD wave speeds  
(transform (slide tangentially) in reference frame where  $u \parallel B$  upstream & downstream)
- > **fast / slow shock**: tangential magnetic field increases / decreases across the shock  
(in reference frame where  $u \parallel B$  upstream & downstream)
- > switch off / on shock:  $B_s = 0$  behind / ahead of slow shock
- > **contact discontinuity**:  $u_s = 0$ ,  $B_n > 0$  (between jet & bow shock, shocked ambient gas)  
tangential discontinuity:  $u_s = 0$ ,  $B_n = 0$



# Shocks in jets and outflows

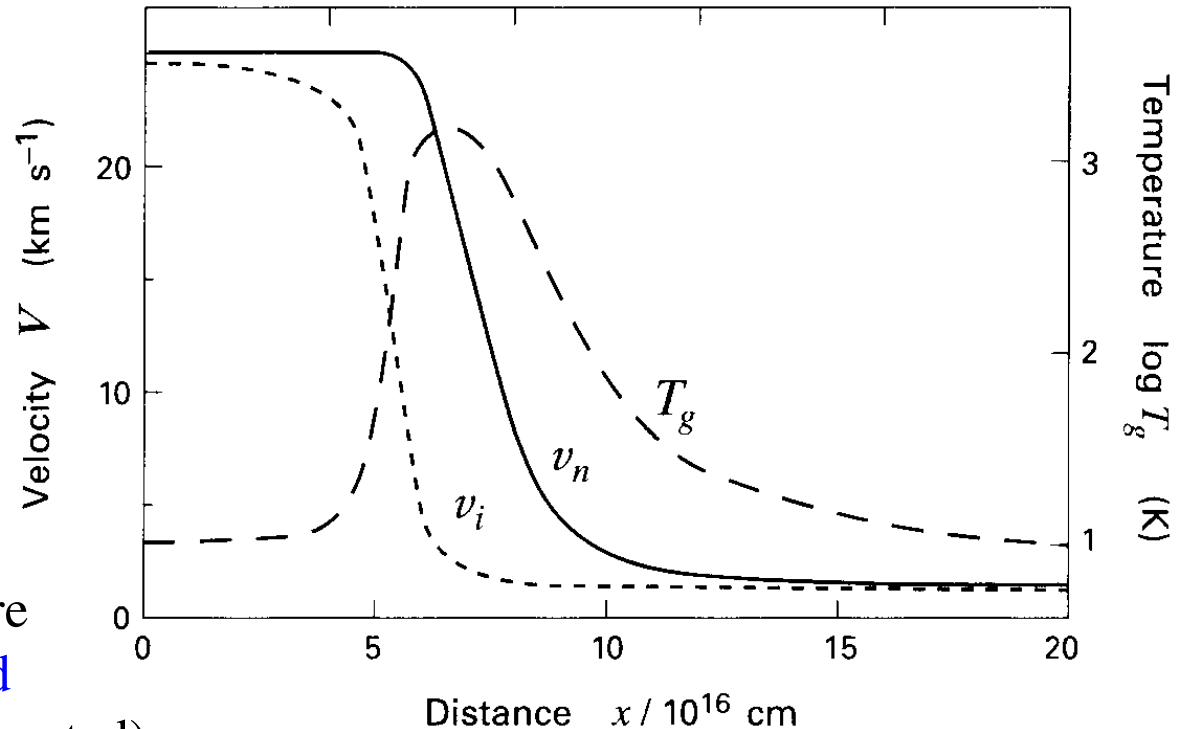
## Structure of jet head



## Shocks in jets and outflows

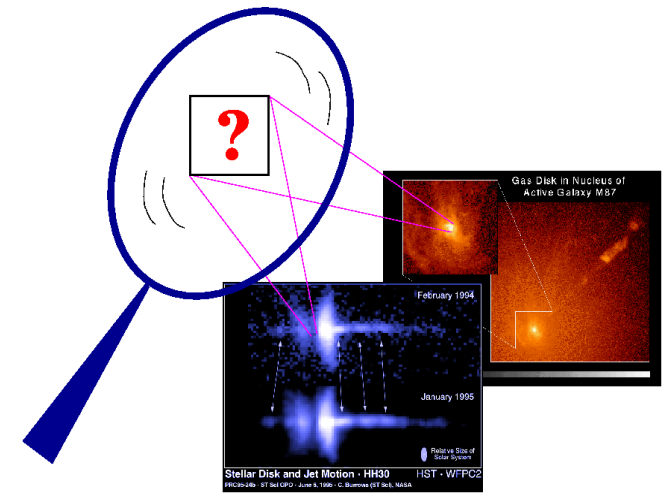
### MHD shocks – C-shocks

- > magnetic shocks:  
compress both field & gas
- > field compression absorbs momentum
- > upstream gas pressure / temperature lower than for un-magnetised fluid (e.g. molecular dissociation is prevented)
- > partially ionised gases:  
neutral and charged particles may have different speed (e.g. ions braked by magnetic field, later brake neutrals by collisions / friction / viscosity)
- > weak field: neutral matter undergoes J-shock to subsonic velocity
  - > magnetic precursor: increasing field strength, deceleration of ionised gas
  - > ion-neutral collisions increase precursor temperature/sound speed -> J-shock weakens
- > strong field: strong precursor, no viscous shock at all, as fluid density & temperature increase smoothly, velocity may remain supersonic



**C-shock in molecular cloud, numerical results for pre-shock  $B = 100 \mu\text{G}$ . Ions decelerate prior to neutrals  
Un-magnetic case would result in post-shock  $T = 34.000 \text{ K}$ .**

# Outflows & Jets: Theory & Observations



## 10.10 Introduction & Overview ("H.B." & C.F.)

17.10 Definitions, parameters, basic observations (H.B.)

24.10 Basic theoretical concepts & models I (C.F.): Astrophysical models, MHD

31.10 Basic theoretical concepts & models II (C.F.): MHD, derivations, applications

07.11 Observational properties of accretion disks (H.B.)

14.11 Accretion, accretion disk theory and jet launching (C.F.)

21.11 Outflow-disk connection, outflow entrainment (H.B.)

28.11 Outflow-ISM interaction, outflow chemistry (H.B.)

05.12 Theory of outflow interactions; Instabilities (C.F.)

**12.12 Outflows from massive star-forming regions (H.B.)**

19.12 Radiation processes - 1 (C.F.)

26.12 and 02.01 *Christmas and New Year's break*

09.01 Radiation processes - 2 (H.B.)

16.01 Observations of AGN jets (C.F.)

23.01 Some aspects of AGN jet theory (C.F.)

30.01 Summary, Outlook, Questions (H.B. & C.F.)