Lecture 4.

Star formation
Part III

Lecture Universität Heidelberg WS 11/12
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Based partially on script of Prof. W. Benz

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Lecture 4 overview

1. Heating and cooling
   1.1 Coolants
   1.2 Thermodynamics during collapse
2. The isothermal sphere
3. Self similar collapse
   3.1 Accretion onto the proto-star
4. Numerical simulations of star formation
1. Heating and cooling
We have seen in the last lectures that the thermodynamics of the collapsing gas cloud (isothermal versus adiabatic behavior) are important. We therefore study now the processes that lead to heating and cooling of the cloud.

1) Heating mechanism

a) Cosmic ray absorption

\[ \Gamma_{CR} \approx 2.5 \times 10^{-3} e^{-\tau} \text{erg} / (\text{g s}) \]

\( \tau \): optical depth of the cloud

b) Compressional heating

The basic expression for contraction work is

\[ \Gamma_{GC} = -p \frac{dV}{dt} \]

for the unit mass we can use \( V = 1/\rho \) so

\[ \frac{dV}{dt} = \frac{d}{dt} \frac{1}{\rho} = \frac{1}{\rho^2} \frac{d\rho}{dt} = -\frac{1}{\rho} \frac{d\ln \rho}{dt} \]

yielding

\[ \Gamma_{GC} = \frac{p}{\rho} \frac{d\ln \rho}{dt} = \frac{kT}{\mu m_H} \frac{d\ln \rho}{dt} \]
Using the result from the T=0 K collapse we studied earlier, we can write
\[ \rho = \frac{\rho_0}{\cos^6 \alpha} \]
so
\[ \frac{d \ln \rho}{dt} = 6 \tan \alpha \frac{d \alpha}{dt} = \frac{v}{2r} \approx \sqrt{24G \rho \pi} \]

which means

\[ \Gamma_{GC} = \sqrt{24\pi G} \frac{k}{\mu m_H} T^{1/2} \rho^{1/2} \approx 1.43 \times 10^5 \ T^{1/2} \rho^{1/2} \text{ erg/(g s)} \]

2) Cooling mechanism  Collisional excitation followed by the emission of an IR photon

Due to their thermal velocity, molecules collide all the time, which can lead to an excitation of an electron. At disexcitation, a photon is radiated, taking away the energy. (We assume here that the cloud is optically thin).

\[ A + B \rightarrow A^* + B \]
\[ A^* \rightarrow A + h\nu \]

-Frequent collisions (abundant partners)
-Excitation energy comparable to or less than thermal kinetic energy
-High probability of excitation during collision
-Photon emission before the next collision
-No re-absorption of the photon (low optical depth of gas to line emission)
Cooling is in general described by a cooling function $\Lambda(T)$. Its exact value depends on the detailed chemical composition, but an order of magnitude can be estimated from the reaction rate $<\sigma v>$ multiplied by the amount of energy lost in one collision. Taking $\sigma \approx 10^{-16}$ cm$^2$, $v \approx 1$ km/s, $\Delta E \approx 0.1$ eV $\approx 10^{-13}$ erg, we obtain a typical value of the cooling function: $\Lambda(T) \approx 10^{-24}$ erg / (cm$^3$ s).

The cooling rate is then obtained by multiplying the cooling function with the abundance of both cooling species.

$$\Gamma_{CE} = \Lambda(T)n_1n_2 = <\sigma v> \Delta E n_1n_2 = \sigma \sqrt{\frac{8kT}{\pi m}} \Delta E \frac{\rho_1 \rho_2}{m_1 m_2}$$

The cooling time can be estimated (for identical species) as

$$\tau_{cool} = \frac{3/2nkT}{n^2 \Lambda}$$
1.1 Coolants
**Coolants**

To calculate the specific cooling rate, one must know the chemical composition of the gas.

The figure shows the relative abundances of a molecule M, $x(M) = n(M)/n(H_2)$, for $n(H_2) = 10^6$ cm$^{-3}$.

For $T > 500$ K, all the oxygen not locked in CO, is in the form of water.

\[ O + H_2 \rightarrow OH + H \]
\[ OH + H_2 \rightarrow H_2O + H \]
Contribution of coolants

Fractions of the total cooling rate accounted for by emission of various coolants.

Contours correspond to
20% dotted
50% dashed
70% solid

Notes

- H (and He) cannot be collisionally excited at low T. It is there a poor coolant.
- At lower densities and temperatures, CO and O are dominant.
- At high densities, water (high T) and a host of other molecules (low T) become dominant.
1.2 Thermodynamics during collapse
Inset: Basic thermodynamics

First law
\[ dU = \delta Q - pdV \]
\[ = TdS - pdV \]

\[ c = \frac{\delta Q}{\partial T} \]
\[ c_v = \left( \frac{\delta Q}{\partial T} \right)_V = \left( \frac{dU - pdV}{\partial T} \right)_V = \left( \frac{\partial U}{\partial T} \right)_V \]
\[ U = c_v T \quad \text{(id. gas)} \]

Isothermal processes
\[ dT = 0 \rightarrow dU = 0 \rightarrow \delta Q = pdV \quad pV = nRT = \text{cst} \]

Adiabatic processes
\[ \delta Q = 0 \rightarrow dS = 0 \rightarrow dU = -pdV \]
\[ dU = c_v dT = -pdV \]
\[ p = \frac{1}{V} RT \]
\[ c_v \frac{dT}{T} = -R \frac{dV}{V} \]
\[ \gamma = \frac{c_p}{c_v} \]

After some algebra, and using
\[ R = c_p - c_v \]
\[ TV^{\gamma-1} = \text{cst} \]
\[ T\rho^{1-\gamma} = \text{cst} \]
Thermodynamics during collapse

Isothermal collapse
Size: several AU

H$_2$ diss. collapse

2nd core

opt. thin in IR

opt. thick in IR

- GMC density
- star density

Bate 1998
Thermodynamics during collapse

Notes

• The collapse stars at large masses and scales of $M_J \sim 100 \, M_{\odot}$ and $10^5 \, \text{AU}$.
• At the lowest densities, we have heating by cosmic rays $\Gamma_{\text{CR}}$.
• Up to a critical density of about $10^{-13} \, \text{g/cm}^3$, the collapse is isothermal $\Gamma_{\text{GC}} = \Gamma_{\text{CE}}$.
• Above the critical density, the collapse is nearly adiabatic.
• Dissociation of molecular hydrogen ($T > 2000 \, \text{K}$) away a part of the liberated potential energy. The temperature rises slower.
• In the adiabatic phases, $M_J$ increases.
• The final fragment mass is about $0.01 \, M_{\odot}$.

The different regimes are characterized by different $\gamma$ and either isothermal or adiabatic.

Using a functional form of the EOS $p = K \rho^\gamma$ where $\gamma = \begin{cases} 
1 & \rho \leq 1.0 \times 10^{-13} \\
7/5 & 1.0 \times 10^{-13} < \rho \leq 5.7 \times 10^{-8} \\
1.15 & 5.7 \times 10^{-8} < \rho \leq 1.0 \times 10^{-3} \\
5/3 & \rho > 1.0 \times 10^{-3} 
\end{cases}$

1) Isothermal part: $K = c_s^2$ and $\gamma = 1$ as $p = \frac{k}{\mu m_H} \rho T = c^2 \rho$

2) Adiabatic parts: $T \rho^{1-\gamma} = \text{cst}$ and $p \propto \rho T$ therefore $p = K \rho^\gamma$

3) “Dissociation” part: $\gamma = 1.15$ to model both the decreasing mean molecular weight and the slow increase of temperature with density.
2. The isothermal sphere
We are now looking for solutions of the continuity, Euler and Poisson equation for a spherically symmetric sphere of isothermal gas (no more T=0 K, but finite T). Remember

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p - \nabla \phi
\]

\[
\Delta \phi = 4\pi G \rho
\]

In spherical symmetry (no \(\phi\) and \(\theta\) dependence) we have

**Gradient:**

\[
\nabla_R U = \frac{\partial U_r}{\partial r}
\]

**Divergence:**

\[
\nabla_R \cdot \mathbf{U} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 U_r \right)
\]

**Laplace:**

\[
\Delta_R U = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial U_r}{\partial r} \right)
\]

and the EOS is with T=cst

\[
p = \frac{k}{\mu m_H} \rho T = c^2 \rho
\]
Equilibrium solution

For a time independent solution, the Euler equation becomes

\[ \frac{1}{\rho} \nabla p = -\nabla \phi \]

which is nothing else than the equation of hydrostatic equilibrium. In spherical symmetry

\[ \frac{1}{\rho} \frac{dp}{dr} = \frac{c^2}{\rho} \frac{d\rho}{dr} = -\frac{d\phi}{dr} \quad \text{i.e.} \quad -c^2 \frac{d \ln \rho}{dr} = \frac{d\phi}{dr} \]

which has the solution \( \rho(r) = \rho_c e^{-\phi(r)/c^2} \)

The poisson equation \( \Delta \phi = 4\pi G\rho \) becomes

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi(r)}{dr} \right) = 4\pi G \rho(r) = 4\pi G \rho_c e^{-\phi(r)/c^2} \]

Combining this with the Euler equation, we can also eliminate \( \phi \) to get

\[ \frac{d}{dr} \left( r^2 \frac{d \ln \rho}{dr} \right) = -\frac{4\pi G}{c^2} \frac{r^2 \rho}{r^2} \quad \text{Differential equation for } \rho(r) \]
The isothermal sphere III

Singular isothermal sphere

We try a powerlaw ansatz of the form \( \rho(r) = ar^{-b} \) to find \( b = 2 \) and \( a = \frac{c^2}{2\pi G} \).

The density distribution is therefore given as

\[
\rho_{SIS}(r) = \frac{c^2}{2\pi G} \frac{1}{r^2}
\]

This is clearly an unphysical solution, since the density diverges at \( r=0 \). For this reason, this is called the singular isothermal spheres SIS. The mass is given as

\[
M_{SIS}(r) = \frac{2c^2}{G} r
\]

Bonnor-Ebert sphere

To remove this singularity, we remember

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\Phi(r)}{dr} \right) = 4\pi G \rho_c \exp \left( -\frac{\Phi(r)}{c_s^2} \right)
\]

Defining a new dependent variable replacing the potential, \( \psi \equiv \frac{\Phi}{c_s^2} \),

and a new independent variable replacing the coordinate \( r \) \( \xi \equiv \left( \frac{4\pi G \rho_c}{c_s^2} \right)^{1/2} r \) we get...
The isothermal sphere IV

the Lane-Emden equation

\[
\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\psi}{d\xi} \right) = \exp(-\psi)
\]

This is a dimensionless, scale-free formulation of the problem. We have the two boundary conditions at the center (\(\xi=0\))

\[
\psi(0) = 0 \quad \frac{d\psi(\xi)}{d\xi} \bigg|_{\xi=0} = 0
\]

Solutions are found by numerically integrating inside-out. Such solutions are called Bonnor-Ebert spheres.

Not all solutions are however stable ones. For an isothermal cloud embedded in an external pressure \(P_0\) hydrostatic solutions are possible if the mass of the cloud does not exceed the critical value

\[
M_{BE} = \frac{m_1 c_s^4}{P_0^{1/2} G^{3/2}}
\]

where \(m_1=1.18\). This corresponds to a critical ratio of the central to the surface density of 14.3. For clouds below this mass, both stable and unstable equilibria exist. Stable equilibria are characterized by a degree of central concentration which is less than the critical BE sphere. Unstable ones have a higher concentration. For infinite concentration, the solutions approach the SIS.
The isothermal sphere $V$

A cloud can be driven into instability by increasing the external pressure, or by adding matter to it.

$$M_{BE} = \frac{m_1 c_s^A}{P_0^{1/2} G^{3/2}}$$

Density distributions (Shu 1977)
3. Self-similar collapse
Self-similar collapse

The gravitational collapse of an isothermal sphere takes a self-similar form (Shu, 1977). We here give a general outline of the solution. Since power laws have no characteristic scale, if the outer boundary does not introduce a characteristic scale of pressure, the the gravitational constant and the sound speed are the only dimensional parameters of the problem. We therefore define a dimensionless variable by

\[ x = \frac{r}{c_s t} \]

The relevant equations of conservation are

\[ \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 \rho v)}{\partial r} = 0 \]

\[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{GM(r)}{r^2} \]

We now look for a similarity solution of these two equations in the form

\[ \rho(r,t) = \frac{\alpha(x)}{4\pi G t^2} \]

\[ M(r,t) = \frac{c_s^3 t}{G} m(x) \]

\[ v(r,t) = c_s u(x) \]

Inserting this into the continuity equation gives

\[ m = x^2 (x - u) \alpha \]
Self-similar collapse II

With some manipulations we can write the conservation equations as a set of coupled ODEs:

\[
\begin{align*}
[(x-u)^2 - 1] \frac{dy}{dx} &= \left[(x-u)\alpha - \frac{2}{x}\right](x-u) \\
\left[(x-u)^2 - 1\right] \frac{d\alpha}{dx} &= \left[\alpha - \frac{2}{x}(x-u)\right](x-u)
\end{align*}
\]

Solutions

1) static solution
u=0, \( \alpha = 2/x^2, m=2x \) is a static solution to this set of equations. This solution is nothing else than the singular isothermal sphere.

2) dynamic solution
To find a dynamic solution, let us imagine a small perturbation (collapse) at the center of sphere. In the dimensionless coordinates, the perturbation will have reached x=1 after a time t. All loci with x>1 will not have yet noticed the perturbation and still satisfy the equilibrium solution while loci with x<1 will be infalling.

a) asymptotic solution when t->0 that is when x->\infty
It is clear that the density remains well behaved provided that \( \alpha(x)x^2 \rightarrow 0 \) for \( x \rightarrow \infty \)
In this limit, the coupled ODEs give

\[
\begin{align*}
\alpha(x) &= \frac{A}{x^2} \\
u(x) &= \frac{A - 2}{x} \\
m(x) &\approx Ax
\end{align*}
\]
From the velocity we notice that infall exists for $A>2$ (velocity must be negative). The density is given by

$$\rho(r, 0) = \frac{Ac_s^2}{4\pi Gr^2}$$

which is very similar as the static singular sphere (but larger for $A>2$).

In other words, the gas in the exterior parts does not notice what is going on, until a expansion wave hits it with the information that the interior has collapsed. Then, it starts to collapse, too.

b) asymptotic solution when $x->0$, that is when $t->\infty$ or $r->0$

After some manipulation, one obtains

$$u = -\sqrt{\frac{2m_0}{x}} \quad \alpha = \sqrt{\frac{m_0}{2x^3}} \quad m = m_0$$

The first expression shows that the matter is traveling at the (dimensionless) free fall velocity. The last one means that at any given time, a quantity $m_0$ has collapsed to the core. Numerically, one finds $m_0=0.975$. The matter immediately above it will be moving at free fall velocity, while the matter beyond $x=1$ is still at rest. This means we see an inside out collapse.

In other words, the gas in the exterior parts does not notice what is going on, until a expansion wave hits it with the information that the interior has collapsed. Then, it starts to collapse, to.
Main limitations:
- Singular isothermal sphere is unstable and therefore unphysical as an initial condition
- No rotation, no B fields

But
- Only existing analytic model for collapse
- Demonstrates much of the physics

Self-similar collapse IV

K. Dullemond
Self-similar collapse $V$

We can also compute the mass of the proto-star as well as the accretion rate onto it ($r\to0$):

$$M(0, t) = \frac{c^3 t}{G} m_0$$

$$\frac{dM}{dt} = \frac{c^3 m_0}{G} = \text{cst}$$

The proto-star thus grows linear in time.

Typical numbers: $T=10$ K, $c=0.2$ km/s: $\frac{dM}{dt} = 2 \times 10^{-6} M_{\text{sun/yr}}$
3.1 Accretion onto the protostar
Accretion onto the protostar

1) Outer envelope: optically thin, isothermal. Expansion wave, below free fall.

2) Dust photosphere and envelope: Opaque to IR radiation. T increases towards interior, up to a point where the dust sublimates (dust destruction front).

3) Opacity gap: region of low opacity (no dust).

4) Gas again optically thick at the radiative precursor. Heated by the accretion shock below.

5) Accretion shock. Gas is brought to halt. R~10^{11} cm ~ 2 R_{\text{sun}}

6) Second hydrostatic core (protostar).

Stahler et al. 1980
At the moment, when the gas hits the accretion shock, it gets stopped. The velocity is the free fall velocity

\[ v_{ff} \approx \sqrt{\frac{2GM}{r}} \]

\[ = 280 \text{ km/s } \left( \frac{M_*}{1M_\odot} \right)^{1/2} \left( \frac{R_*}{5R_\odot} \right)^{-1/2} \]

The kinetic energy liberated per second, which is the accretional luminosity, is

\[ L_{acc} = \frac{1}{2} \dot{M} v_{ff}^2 = \frac{GM_* \dot{M}}{R_*} \]

\[ = 61L_\odot \left( \frac{\dot{M}}{10^{-5}M_\odot/\text{yr}} \right) \left( \frac{M_*}{1M_\odot} \right) \left( \frac{R_*}{5R_\odot} \right)^{-1} \]

This accretional luminosity is the main energy source of a protostar, at least for low and intermediate mass protostars. The accretion shock produces hard radiation, which gets absorbed both in the settling region and in the radiative precursor. The effective temperature is

\[ L_{acc} \approx 4\pi R_*^2 \sigma T_{eff}^4 \]

For typical values, we find \( T_{eff} \approx 7300 \text{ K} \).
Accretion onto the protostar III

At the shock (which is the surface of the protostar), the deceleration causes a ram pressure.

\[ P_{\text{ram}} = \rho v_{ff}^2 \]

With \( \rho = \frac{\dot{M}}{4\pi R^2 v_{ff}} \) we have

\[ P_{\text{ram}} = \frac{\dot{M} v_{ff}}{4\pi R^2} = \frac{\dot{M}}{4\pi R^2} \sqrt{\frac{2GM_*}{R_*^5}} \]

This is one of the two necessary outer boundary conditions to calculate the internal structure of the protostar itself.

The second one is the temperature, which contains the contribution from the internal luminosity of the protostar (contraction, nuclear fusion), and the accretional luminosity.
4. Numerical simulations of star formation
Numerical simulations

There is a large body of literature concerning the collapse of gas clouds. Various initial cloud shapes and initial density structures have been investigated. The effects of different EOS, radiation transport, and of magnetic fields have been studied. In the absence of the latter, these numerical simulations have shown that the key initial parameters are given by:

1) The initial thermal energy content: \[ \alpha = \frac{E_{\text{therm}}}{|E_{\text{pot}}|} \]
2) The initial rotational energy content: \[ \beta = \frac{E_{\text{rot}}}{|E_{\text{pot}}|} \]
3) The exponent n of the initial power law density distribution \[ \rho \propto r^{-n} \]

For \( \beta > 0.274 \), a spherical cloud is dynamically unstable to fragmentation.


- Initial molecular cloud is rotating. First hydrostatic core rotating so fast that it is flattened.
- After a short time it goes from begin round to a bar-shaped object (dynamical instability).
- The ends of the bar rotate slower creating spiral arms. These create gravitational torques that transfer angular momentum from the centre of the object into the ends of the arms. The result is that some gas forms a large disc while, in the centre, the density and temperature increase rapidly.
- Molecular hydrogen dissociates at the centre and the second collapse to form the star occurs.
- The calculation stops just after the star forms.
### Numerical simulations II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_0$</td>
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</tr>
<tr>
<td>$\rho(r)$</td>
<td>cst</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>$10^{-11}$ 1/s</td>
</tr>
<tr>
<td>$T$</td>
<td>10 K</td>
</tr>
<tr>
<td>$\alpha$</td>
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</tr>
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<td>$\beta$</td>
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<tr>
<td>Radiation</td>
<td>not included</td>
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<tr>
<td>Magnetic fields</td>
<td>not included</td>
</tr>
<tr>
<td>Numerical method</td>
<td>SPH</td>
</tr>
<tr>
<td>Star</td>
<td>central object</td>
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<tr>
<td>N. particles</td>
<td>300'000</td>
</tr>
<tr>
<td>Stop</td>
<td>1.03 free fall time</td>
</tr>
</tbody>
</table>

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**Collapse of a Molecular Cloud Core to Stellar Densities:**

**Rotational Instability of the First Hydrostatic Core**

**Matthew R. Bate**

MPI für Astronomie, Heidelberg, Germany

Institute of Astronomy, Cambridge, U.K.

October 1998
Fig. 3.— The state of the system at the end of the calculation. The six panels on the left give the density and velocity in the plane perpendicular to the rotation axis and through the stellar core. The six panels on the right give the density and velocity in a section down the rotation axis. In each case, the six consecutive panels give the structure on a spatial scale that is 10 times smaller than the previous panel to resolve structure from 3000 AU to \( \approx 0.2 R_\odot \). The remnant of the first hydrostatic core (now a disc with spiral structure), the inner circumstellar disc, and the stellar core are all clearly visible. The distance scale (in AU), velocity scale (in km s\(^{-1}\)), and logarithm of the minimum density (in g cm\(^{-3}\)) are given under each panel. The maximum density is 0.03 g cm\(^{-3}\) and the logarithm of density is plotted with contours every 0.5 dex.
Simulation outcome

- Isothermal collapse. First core forms with mass $\approx 0.01 \, M_\odot$ and radius $\approx 7 \, \text{AU}$. Rapidly rotating, oblate, and has $\beta \approx 0.34 > 0.274$, therefore dynamically unstable to the growth of non-axisymmetric perturbations.
- At $t \approx 1.023 \, t_{\text{ff}}$, after about 3 rotations, the first core becomes violently bar-unstable and forms trailing spiral arms. This leads to a rapid increase in maximum density as angular momentum is removed from the central regions of the first core (now a disc with spiral structure) by gravitational torques ($t > 1.015 \, t_{\text{ff}}$).
- When the maximum temperature reaches 2000 K, molecular hydrogen begins to dissociate, resulting in a rapid second collapse to stellar densities ($t = 1.030 \, t_{\text{ff}}$).
- The collapse is again halted at a density of $\approx 0.007 \, \text{g cm}^{-3}$ with the formation of the second hydrostatic, or stellar, core. The initial mass and radius of the stellar core are $\approx 0.0015 \, M_\odot$ and $\approx 0.8 \, R_\odot$, respectively.
- Finally, an inner circumstellar disc begins to form around the stellar object, within the region undergoing second collapse. The calculation is stopped when the stellar object has a mass of $\approx 0.004 \, M_\odot$, the inner circumstellar disc has extended out to $\approx 0.1 \, \text{AU}$, and the outer disc (the remnant of the first hydrostatic core) contains $\approx 0.08 \, M_\odot$ and extends out to $\approx 60 \, \text{AU}$.
- $\beta < 0.274$ in the region undergoing the second collapse: no formation of a close binary. Angular momentum removed by spiral arms (gravitational torques).
Numerical simulations V

Example 2:
Bate 2009: Hydrodynamic 3D simulation of a stellar cluster

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial condition</th>
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</thead>
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<td>500 $M_{\text{sun}}$</td>
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<tr>
<td>$\nu$</td>
<td>supersonic turbulent</td>
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<td>$T$</td>
<td>10 K</td>
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<td>$T_{ff}$</td>
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<tr>
<td>Numerical method</td>
<td>SPH</td>
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<tr>
<td>Stars and BD</td>
<td>sink particles</td>
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<tr>
<td>Accretion radius</td>
<td>5 AU</td>
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<tr>
<td>Min. binary sep.</td>
<td>1 AU</td>
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<tr>
<td>EOS</td>
<td>piecewise polytropic</td>
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<tr>
<td>N particles</td>
<td>35 Mio</td>
</tr>
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</table>

Models the collapse and fragmentation of a 500 solar mass cloud. The calculation produces a cluster containing more than 1250 stars and brown dwarfs to allow comparison with clusters such as the Orion Trapezium Cluster.
0 yr: We begin with such a gas cloud, 2.6 light-years across, and containing 500 times the mass of the Sun. The images measure 1 pc (3.2 lightyears across).

38,000 yr: Clouds of interstellar gas are seen to be very turbulent with supersonic motions.

76,000 yr: As the calculation proceeds, the turbulent motions in the cloud form shock waves that slowly damp the supersonic motions.

152,000 yr: When enough energy has been lost in some regions of the simulation, gravity can pull the gas together to form dense "cores".

190,000 yr: The formation of stars and brown dwarfs begins in the dense cores. As the stars and brown dwarfs interact with each other, many are ejected from the cloud.

The cloud and star cluster at the end of simulation (which covers 210,000 years so far). Some stars and brown dwarfs have been ejected to large distances from the regions of dense gas in which the star formation occurs.
Column density through the star cluster formation calculation.
Main simulation results

1) Since all sink particles (and thus stars/BD) are created from pressure-supported fragments, their initial masses are just a few Jupiter masses, as given by the opacity limit for fragmentation. Subsequently, they may accrete large amounts of material to become higher-mass brown dwarfs ($< 75 \, M_{\text{Jupiter}}$) or stars ($> 75 \, M_{\text{Jupiter}}$), but all the stars and brown dwarfs begin as these low-mass pressure-supported fragments.

2) The IMF originates from competition between accretion and ejection which terminates the accretion and sets an object’s final mass. Stars and brown dwarfs form the same way, with similar accretion rates from the molecular cloud, but stars accrete for longer than brown dwarfs before undergoing the dynamical interactions that terminate their accretion.

3) The calculations produce an IMF with a similar form to the observed IMF, including a Salpeter-type slope at the high-mass end but they over-produce brown dwarfs. It is likely due to the absence of radiative feedback and/or magnetic fields in the calculation.
Further reading


Bate, M. R. Collapse of a Molecular Cloud Core to Stellar Densities: The First Three-Dimensional Calculations, 1998  http://www.astro.ex.ac.uk/people/mbate/
Questions?

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