Planet Formation
lecture by Roy van Boekel (MPIA)

suggested reading:
“LECTURE NOTES ON THE FORMATION AND EARLY EVOLUTION OF PLANETARY SYSTEMS”

by Philip J. Armitage

with material from
Kees Dullemond, Christoph Mordasini, Til Birnstiel
the scales

- From ~1 \mu m
- To ~13,000 km
- Or ~30,000,000,000 km (~100 AU)
- To ~140,000 km
star formation vs. planet formation

- ~spherical collapse, “own gravity” dominant
- rotation, shed very much angular momentum
- cooling
- (elemental) composition ~interstellar/solar
- scales: ~0.1 pc → ~1 \( R_{\text{Sun}} \)

- stellar gravity usually dominant
- Keplerian shear, much less net angular momentum loss
- cooling!
- composition can (locally) be highly enriched in heavy elements/“dust”
- scales: <1 to ~several AU → ~1 \( R_{\text{Earth}} \) to ~1 \( R_{\text{Jupiter}} \)
Minimum Mass Solar Nebula

“What is the minimum amount of disk material to make the solar system planets?” Basic idea:

1. Consider the disk region from which each planet (given current mass / location) would accrete material.

2. Consider the amount of refractory elements in each planet (add volatiles/ices beyond iceline).

3. Assume disk bulk composition is solar; add H+He gas accordingly.

Result (Hayashi 1981):

\[
\Sigma = 1.7 \times 10^3 \left( \frac{r}{\text{AU}} \right)^{-3/2} \text{ g cm}^{-2}.
\]

\( \Sigma \) is gas surface density (~total dens., H+He gas ~99% of mass)

This estimate relies on several assumptions and is only an approximate result.
Minimum Mass Solar Nebula

Hayashi (1981)

Fig. 1. Distribution of dust mass and ranges of planetary perturbation. The ordinate is the surface density $\rho$ of dust mass multiplied by the distance $r$. The ranges of planetary perturbation, indicated by the segments in the lower region of the diagram, correspond to $a \pm (7h+ea)$ where $a$, $e$ and $h$ are the semi-major axis, the eccentricity and the Hill radius of the present planets.
Minimum Mass Solar Nebula

more recent rendition, ~equivalent to previous plot

Ruden (2000)
(Giant) Planet Formation

Two main theories of planet formation:

(1) Core accretion: formation of solid core (terrestrial planet), if core sufficiently massive (~8 M⊕) subsequent accretion of gas (gas giant planet).

(2) Gravitational Instability: direct collapse from gas phase (only gas giant planets, relatively far out in the disk)
Gravitation Instability (GI)

Main idea:

(1) instability causes initial over-density, subject to self-gravity

(2) if these “clumps” can get rid of potential energy faster than pressure and differential rotation smooth them out again, they can collapse

(3) fast process; planet composition ~ (local) disk bulk composition

Gravitation Instability (GI)

Safronov-Toomre Criterion for disk stability against isothermal collapse in keplerian disk:

\[ Q = \frac{c_s \Omega}{\pi G \Sigma} > Q_{\text{crit}} \approx 1 \]  

(213)

\( Q \) is “Toomre parameter”, \( c_s \) is sound speed \([\text{cm s}^{-1}]\), \( \Omega \) is orbital frequency \([\text{rad s}^{-1}]\), \( G = \text{gravitational constant} \quad [\text{cm}^3 \text{ g}^{-1} \text{ s}^{-2}]\), \( \Sigma \) is surface density \([\text{g cm}^{-2}]\).

Disk will stable against fragmentation where \( Q > Q_{\text{crit}} \). Typical value \( Q_{\text{crit}} \approx 1 \). Having \( Q < Q_{\text{crit}} \) is necessary but not sufficient condition for fragmentation;
Gravitation Instability (GI)

Example:
$h/r = 0.05$ at 10 AU around solar-type star;
$h/r = \frac{c_s}{v_\phi} \rightarrow$ sound speed $c_s \approx 0.5 \text{ km s}^{-1}$. ($v_\phi$ is orbital velocity)

To get $Q < 1$ we require $\Sigma > 1500 \text{ g cm}^{-2}$.

Compare to “minimum mass solar nebula”: $\Sigma \approx 54 \text{ g cm}^{-2} @ 10 \text{ AU}$ (using normalization of Hayashi (1981))

$\rightarrow$ GI works only for very massive disks
GI: resulting planets

spatial scale most unstable to collapse:

\[ \lambda_{\text{crit}} = \frac{2c_s^2}{G\Sigma} \]

resulting planet mass if such a disk region would collapse (for our example with \( \Sigma = 1500 \text{ g cm}^{-2} \), at 10 AU with \( c_s = 0.5 \text{ km s}^{-1} \)):

\[ M_p \sim \pi \Sigma \lambda_{\text{crit}}^2 \sim \frac{4\pi c_s^4}{G^2 \Sigma} \sim 6 M_J \]  \hspace{1cm} (215)

\( \rightarrow \) GI produces very massive planets. Works better in outer disk
Importance of cooling
(additional requirement on top of Toomre criterion)

Collapsing fragment —> release of gravitational energy, needs to be radiated away sufficiently quickly for collapse to proceed.

Cooling time:

\[ t_{\text{cool}} = \frac{U}{2\sigma T_{\text{disk}}^4} \quad (217) \]

where U is the thermal energy content of the disk per unit surface area. For an ideal gas EOS (\( \gamma = 5/3 \)) we get:

- \( t_{\text{cool}} \approx 3\Omega^{-1} \) — the disk fragments.
- \( t_{\text{cool}} \approx 3\Omega^{-1} \) — disk reaches a steady state in which heating due to dissipation of gravitational turbulence balances cooling.
Is this (GI) how it goes?

(1) vast majority of planets (solar system, exoplanets discovered mainly with transit & radial-velocity techniques) are less massive than GI predicts

(2) In solar system, gas/ice giants are enriched in heavy elements; hard to reconcile with GI.

(3) some “direct imaging” observations found (young) planets at large distances. Possibly these formed through GI (“jury is still out”).
core-accretion (CA)

Basic idea:

(1) solid “refractory” material comes together to form larger bodies. Beyond “snowline” this includes (water-) ice.

(2) bodies grow by low-velocity collisions, until they get so large that their mutual gravity becomes important for the dynamics.

(3) gravity-assisted growth, initially in a “run-away” fashion, later in an “oligarchic” fashion, to rocky/icy planets

(4) if rocky/icy core reaches sufficient mass (surface gravity) to retain H+He gas (5-10 M_{earth}) \textit{while the disk is still gas-rich}, then gas accretion ensues. If \sim 30 M_{earth} is reached, run-away gas accretion forms a \sim Jupiter mass planet
Formation of gas giants
(if sufficient gas is present)

- gas giants (~100,000 km)
- Earth-sized (~10,000 km)
- protoplanets
- planetesimals (~km)
- dust (μm)

C. Mordasini
The long road from dust to planets

1 μm → 1 mm → 1 m → 1 km → 1000 km

First growth phase

Gravity keeps/pulls bodies together

Gas is accreted

Covers 13 orders of magnitude in size = 40 (!!) orders of magnitude in mass

C. Dullemond
initial growth

Assumed initial situation: small dust particles, e.g. 0.1 μm, mixed homogeneously with gas. (ignores dust growth in cloud/core/collapse phase).

(1) small dust grains well coupled to gas; small relative motions (brownian motion dominant until ~1 μm, \(\Delta v \approx 100 \) μm/s; turbulence-driven motions dominant for larger grains, with \(\Delta v\) up to several 10 m/s for ~cm particles

(2) “touch and stick” at low \(\Delta v\) (\(\approx 1\) m/s for pure silicates, \(\approx 10\) m/s if particles are icy), destruction / erosion at high \(\Delta v\)

(3) vertical component of gravity, “dust settling”, accelerates growth. **Also radial drift \(\rightarrow\) problem!**
The basic picture of the early stage of planet formation (growth from dust to km sized planetesimals) is the following:

- Dust grains condense, coagulate and gradually decouple from the gas.
- The dust grains settle into a thin mid-plane layer in the disk.
- Planetesimals form by continued coagulation (two body collisions) or a self-gravitational instability of the dust (or a combination of the two) in the dense mid-plane layer.

After a few settling time scales, an equilibrium will be reached in which the downward flux from sedimentation is balanced by the upward mass flux from diffusion. In this case, the time derivative of the advection-diffusion equation becomes zero and the equation can be integrated analytically. Assuming a vertically isothermal disk and constant diffusivity, Fromang & Nelson (2009) derive the dust density distribution as:

$$\rho(z) = \rho_0 \exp(-z^2 / 2H_g^2),$$

where a subscript of 0 denotes mid-plane values. Close to the mid-plane, this profile indeed approaches a Gaussian profile with the scale height derived above.
gas-dust coupling

\[ \frac{t_{\text{stop}}}{t_{\text{orb}}} = t_{\text{stop}} \cdot \Omega_K \equiv \text{St} \quad \text{(Stokes number)} \]

\[ \begin{align*} 
\text{St} &\ll 1 \quad \text{i.e.} \quad \tau_{\text{fric}} \ll \tau_{\text{orb}} \\
\text{St} &\sim 1 \quad \text{i.e.} \quad \tau_{\text{fric}} \sim \tau_{\text{orb}} \\
\text{St} &\gg 1 \quad \text{i.e.} \quad \tau_{\text{fric}} \gg \tau_{\text{orb}}
\end{align*} \]

It’s (mostly) not size that matters - it’s the Stokes number!

T. Birnstiel
gas-dust coupling

“Epstein” drag regime, particle radius $a << \lambda$, where $\lambda$ is mean free path of gas molecule.

$$F_{\text{drag}} = -\frac{4\pi}{3} \rho_g a^2 v_{\text{th}} v$$

$\rho_g$ is gas density, $v$ is the velocity of the particle relative to the bulk motion of the gas, $v_{\text{th}}$ is the thermal velocity of the gas, given by:

$$v_{\text{therm}}^2 = \frac{8kT}{\pi \mu m_H}$$

where $\mu$ is the mean molecular weight in AMU (typically $\mu=2.3$) and $m_H$ is the mass of a hydrogen atom.
(1) Gas pressure radially decreasing (in continuous disk).

(2) Pressure gradient force supports gas disk (but not the dust)

(3) Force Equilibrium leads to *sub-Keplerian gas rotation*

(4) Head-wind removes dust angular momentum

(5) Orbital decay;
relative particle velocities

- very small particles ~completely coupled to gas (they go wherever the gas goes), Stokes $\ll 1$; small $\Delta v$.

- very large particles ~completely de-coupled from gas (they “don’t care” about the gas), Stokes $\gg 1$; small $\Delta v$ (all $\sim$keplerian), unless gravitational stirring becomes important (later on)

- Intermediate range where particles are semi-coupled (gas affects velocities differently for different sizes), Stokes $\sim 1$; large $\Delta v$. Relative velocity several 10 m/s.
“meter-sized barrier”

(note: more “barriers” exist; e.g. “bouncing barrier”)

two-fold “barrier” for reaching “boulder” sizes (large enough to ~de-couple from gas)

(1) drift: In a MMSN, the decay time for particle with maximum drift velocity (“Stokes parameter” ~1) from:
   1 AU : ~200 yrs (~3 m particle)
   5 AU: ~1,400 yr (~1 m particle)
   100 AU: ~30,000 yrs (~10 cm particle)

(note: numbers are for specific set of assumptions, only indicative of order of magnitude).

(2) fragmentation relative velocities up to ~several 10 m/s around St~1, destructive collisions.
overcome m-sized barrier

(1) **Gravitational Instability planetesimal formation:** if dust settles in very thin disk \((H_{\text{dust}}/H_{\text{gas}} \sim 10^{-4})\) that is also nearly perfectly free of turbulence (relative velocities \(<\sim 10 \text{ cm/s at 1 AU}\)), then dust disk may fragment into clumps that collapse under own gravity ("Goldreich & Ward mechanism"). **Considered unlikely** (turbulence prohibits these circumstances to be reached).

(2) **gravo-turbulent planetesimal formation:** the turbulence itself causes local enhancements of dust density, vortices can "trap" dust
why dust piles up in regions of high (gas) density
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why dust piles up in regions of high (gas) density
gravo-turbulent planetesimal formation

(1) MRI turbulence

(2) particles gather in overdensities, local enhancements in solids;

(3) gravity of concentrated dust sustains overdensity, attract more solids

(4) simulations produce \( \sim \) few to \( \sim 35 \) Ceres mass bodies in just 13 orbits

Johansen, Klahr, Henning
further growth

(1) particles are $\gg 1$ m, all move on $\sim$keplerian orbits.

(2) velocities are "damped" by gas $\rightarrow$ low $\Delta v$

(3) growth by low velocity collisions, "mergers". Slow, "orderly" growth

(4) initially gravity of growing bodies is not important, but once bodies reach $\sim$km sizes, "gravitational focusing" starts to become important and growth goes faster.
further growth (II)

(5) gravity-assisted, accelerated growth. For “dynamically cold” disk, growth rate \( \frac{dM}{dt} \propto M^{4/3} \) \( \rightarrow \) largest bodies grow fastest, run-away growth where “winner takes all”

(6) largest body accretes most planetesimals within its gravitational sphere of influence.

(7) largest bodies start to gravitationally disturb (“excite”) smaller ones, planetesimal disk gets “dynamically hot” and gravitational focusing is less effective.

(8) “oligarchic growth” where dozens of \( \gtrsim 0.1 \) \( M_{\text{Earth}} \) bodies dominate their local environment and slowly grow further

(9) complex N-body interactions; many “oligarchs” merge in “giant impacts” (last one in our case is thought to have resulted in Earth-Moon system.)
On to final planets

- gas giants (~100,000 km)
- Earth-sized (~10,000 km)
- protoplanets
- planetesimals (~km)
- dust (μm)

Time (years): $10^4$-$10^5$, $10^5$-$10^7$, $10^7$-$10^8$

- runaway gas accretion
- oligarchic growth
- dust sticking
- Self. Gravity
- late stages giant impacts

C. Mordasini
Formation of a Gas Giant Planet


C. Dullemond
Formation of a Gas Giant Planet

Growth by accretion of planetesimals until the local supply runs out (isolation mass).


C. Dullemond
Formation of a Gas Giant Planet

Slow accretion of gas (slow, because the gas must *radiatively cool*, before new gas can be added). *Speed is limited by opacities.*

The added gas increases the mass, and thereby the size of the feeding zone. Hence: New solids are accreted.

If planet migrates, it can sweep up more solids, accelerating this phase.


C. Dullemond
Formation of a Gas Giant Planet

Once $M_{\text{gas}} > M_{\text{solid}}$, the core instability sets in: accelerating accretion of more and more gas.

C. Dullemond

A hydrostatic envelope smoothly connecting core with disk no longer exists. Planet envelope detaches from the disk.
Formation of a Gas Giant Planet

Something ends the gas accretion phase, for example: strong gap opening. „Normal“ planet evolution starts.


C. Dullemond
Is this (CA) how it goes?

CA is a complex, “multi stage” problem, still only partially understood theoretically and many phases poorly/not constrained observationally. But: huge progress in modeling; “barriers” resulting from over-simplification go away if better physical models are applied.

(1) CA can produce rocky planets

(2) CA can yield strongly enriched planets (GI much less so)

(3) CA can yield ice giants (GI cannot do this)

Core Accretion is the currently favored scenario. Not much doubt that the basic idea is right. Unclear whether GI, in addition, is at work in outer regions of massive disks
migrate

(forming) planets interact gravitationally with the disk (and other planets), and may move from where they form(ed), sometimes a lot

(1) type I migration: relatively low-mass planets (e.g. \(~1 \, M_{\text{earth}}\)) do not significantly alter surface density profile \(\Sigma(R)\) but material concentrates asymmetrically in resonances and exerts torque causing migration

(2) type II migration: high-mass planets (\(~1 \, M_{\text{jup}}\)) open gaps and launch strong spiral arms that exert torque.

(3) Planet-planet interaction can significantly alter orbits of planets on timescales of \(>>1\) orbit