Numerisches Praktikum – Numerical Practical Training

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Integration

Return by 9:15 a.m. Feb 23nd
as `.pdf` by Mail to: `cecil@mpia.de`

Free Training

- Write a program code for numerically computing a definite integral, using multiple segments (free parameters are: number of segments n, lower and upper bound a and b, step size $h = (b - a)/n$). Prepare the following three methods:

  1. Trapezium Rule
  2. Simpson 1/3 Rule
  3. Gaussian Two-Point Quadrature

  Test your programm for $f(x) = x$ ($a = 0$, $b = 2$) and $f(x) = x^2 - 3x$ ($a = -3$, $b = 6$)

Assignment for the Afternoon / Homework

- **Exercise 1, 5 points**: Trapezium Rule.
  Integrate numerically the definite integral

  $$\int_0^2 (2 + \cos(2\sqrt{x}))dx \qquad (1)$$

  using the Trapezium rule. Use $n = 2, 10, 100, 1000, 10000$, print the result.

- **Exercise 2, 5 points**: Simpson 1/3 rule.
  Integrate the definite integral of 1 using Simpson's 1/3 rule, for $n = 2, 10, 100, 1000, 10000$, print the results.

- **Exercise 3, 5 points**: Gaussian two point quadrature.
  Integrate the definite integral of 1 using the Gaussian two point quadrature, for $n = 2, 10, 100, 1000, 10000$ intervals of $[a, b]$, print the results.

- **Exercise 4, 5 points**: Accuracy and Errors.
  Evaluate the integral of 1 analytically. Compute the true error (absolute and relative) of the numerically computed integral for Trapezium, Simpson 1/3 rule and Gaussian two-point quadrature (for the $n = 2, 10, 100, 1000, 10000$) values. Put all results in a double logarithmic plot of error against n. What scaling of the error do you find?