Numerisches Praktikum – Numerical Practical Training

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Random numbers

Return by 9:15 a.m. tomorrow

Free Training / Homework

- **Exercise 1, 6 points:** The infamous `randu`.
  Write a program code with the (not-so) random number generator randu following the recurrence:

  $$I_{j+1} = (65539\ I_j) \left(\text{mod } 2^{31}\right)$$

  (see Lecture for details). Then to obtain a random number x_i drawn from the interval $(0, 1)$ use the normalisation $x_i = u_i/2^{31}$. Create some 100 000 random number, starting with an initial seed $u_0 = 1$ and plot the consecutive triples (x_i, x_{i+1}, x_{i+2}) in a 3-dimensional plot, e.g., using the `splot` command from gnuplot. Count the number of 2D planes by viewing the data in different projections. What is the number of planes for `randu`?

- **Exercise 2, 8 points:** Transformation method.
  Write a program code to generate random numbers with an exponential probability distribution function (PDF) $\rho(y) = e^{-y}$ in the interval $y_{\min} = 0$ to $y_{\max} = 5$ using the transformation:

  $$y = -\ln(1 - x) \quad \Leftrightarrow \quad x = e^{-y}.$$

  Use an random number generator with uniform PDF of your choice or the one offered in the Lecture. Show that your resulting distribution of random numbers indeed follows an exponential one.

- **Exercise 3, 6 points:** Monte-Carlo Integration.
  Approximate the value of π using the Monte-Carlo technique by integrating the area of a square with side length a and a circle of radius $1/2a$. Use the equation:

  $$\pi = 4\frac{A_c}{A_s} \approx 4\frac{N_c}{N_s},$$

  where A_c and A_s is the area of the square and the circle, respectively. How does the precision of the result scale with the number of points used in the integration?