Hamiltonian model of capture into mean motion resonance

Alex Mustill
& Mark Wyatt
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Mean motion resonance

- Occurs when ratio of two bodies' mean motions is close to two integers, e.g., 2:1, 3:2 (1st order); 3:1, 5:3 (2nd order)
- Resonant argument $j\lambda - (j-k)\lambda_{pl} - k\varpi$ librates
- Resonance can be entered through convergent migration, e.g. ...
Mean motion resonance

- Planet migration in PPD (e.g., Kley+05)
- Planetesimal capture by outward migration of planets (e.g., Wyatt 03, Reche+08)
- Capture of dust drifting under PR-drag (e.g., Kuchner & Holman 03)
- Dynamical changes during post-MS evolution (e.g., Dong+10, Perets 10)
Mean motion resonance

- What are conditions for resonance capture to occur?
- What is change in $e$ if capture does not occur?
- (What is the resulting libration amplitude, if capture occurs?)
Hamiltonian model

- Take leading-order term from disturbing function, in limit of one body being of negligible mass, and non-dimensionalise

\[ \mathcal{H} = J^2 + \beta J + (-1)^k J^{k/2} \cos k\theta \]

Keplerian  Resonant
Hamiltonian model

- Take leading-order term from disturbing function, in limit of one body being of negligible mass, and non-dimensionalise
- \( \mathcal{H} = J^2 + \beta J + (-1)^k J^k/2 \cos k\theta \)
- Momentum \( J \) proportional to \( e^2 \)
- \( \beta \) measures distance to resonance
- Migration imposed by changing \( \beta \): this varies semi-major axis and does not change \( e \)
Hamiltonian model

- Two free parameters:
  - $J_0$, initial momentum (particle eccentricity)
  - $d\beta/dt$, migration rate
- All resonances of same order, whether external or internal, for any planet parameters ($m_{pl}$, $m_*$, $a_{pl}$, but fix $e_{pl} = 0$), reduce to same dimensionless form

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Hamiltonian model

- Two free parameters:
  - $J_0$ initial momentum (particle eccentricity)
  - $d\beta/dt$ migration rate
- We randomise over initial angle—talk about capture probability in a deterministic system
Hamiltonian model

• Adiabatic case of low migration rate well-understood (e.g., Henrard 1982, Murray & Dermott 1999)
• Some work on fast migration (Quillen 2006)
• I extend Quillen (2006) to more finely sample parameter space, and derive libration behaviour and eccentricity changes as well as capture probability
• Goal is to have a large grid of outcomes that can be quickly applied to any situation
Numerical integrations

- Capture probabilities determined numerically
Numerical integrations

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Numerical integrations

- Eccentricity jumps
- Red contours: $\langle e_f \rangle / e_i$
- Yellow contours: $(\langle e_f \rangle - e_i) / \sigma_e$

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Validation

- Comparison to full N-body integration (Wyatt 2003)
Two massive planets

• Above relied on assuming one planet is a test particle.
• Accuracy needs to be tested for comparable mass planets.
Validation—two massive planets

- Outer planet migrating into 3:2 resonance
- Hamiltonian model vs. N-body
- One-planet Hamiltonian model accurate to ~50%
Two-planet Hamiltonian model

- More free parameters: this plot for two equal mass planets and 3:2 resonance
- Critical migration rate slightly higher than one-planet model
Applications

• Resonance capture during planet formation
• Planetesimal capture by outward migration of planets
• Capture of dust drifting under PR-drag – potential for Martian dust ring
Planet formation

• Type I/II migration can lead to resonance capture
Planet formation

- Classical Type I too fast to capture into 2\textsuperscript{nd} order resonances
Planet formation

- Will see lower mass planets in higher $j$ resonances (e.g., 6:5 not 2:1)
Summary

- Resonance capture treated with a Hamiltonian model. Very general and encompasses many scenarios
- Extends analytic work to high migration rate
- Capture probability low at high migration rates and high eccentricities
- One-planet model accurate at disparate mass ratios; needs adjusting for comparable mass ratios
Future work

- Further work on two-planet Hamiltonian
- Eccentricity damping
- Particle lifetimes in resonance, temporary capture
- Generation of synthetic debris discs
  - planet migrating outwards—extrasolar discs and Neptune
  - dust migrating inwards—Mars
Extras...
Numerical integrations

- Libration amplitudes
Validation

- Comparison to full N-body integration (Wyatt 2003)
Planet formation

- High eccentricity at capture leads to high libration amplitude
- So an observed high libration amplitude (>~60°; e.g., HD 128311) can be explained if planet eccentricities are fairly high in the disc
- Such eccentricities would be ~0.1 for a Jupiter-mass planet, ~0.01 for an Earth mass planet
- Possible? (Moorhead & Adams 08) or not (Bitsch & Kley 10)
Planetesimal discs

- Planets migrating outwards can impose structure on a debris disc (*e.g.*, Wyatt 03)

- As particle eccentricity rises, disc structures get washed out as capture probability decreases and libration amplitude grows (Reche+08)

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Planetesimal discs

- Libration amplitudes and capture probabilities from Hamiltonian model
Zodiacal cloud

- Dust migrating under PR drag can be captured into external resonances
- Earth has resonant ring; Mars' not detected yet, if it exists
- Kuchner et al. (2000) give observational upper limit for Mars' trailing fractional overdensity of 18% of Earth's
Zodiacal cloud

- Dust migrating under PR drag can be captured into external resonances

Dermott et al 94
Zodiacal cloud

- Capture probabilities for Earth and Mars:
Zodiacal cloud

• Capture probabilities for Earth and Mars:
  • Martian capture probabilities $<\sim25\%$ Terrestrial capture probabilities
  • Consistent with observational upper limits (Kuchner et al 2000)
  • Probability may be further reduced by extra resonant terms (Quillen 2006)