Essentials of
Radio and (Sub-)Millimeter Astronomy

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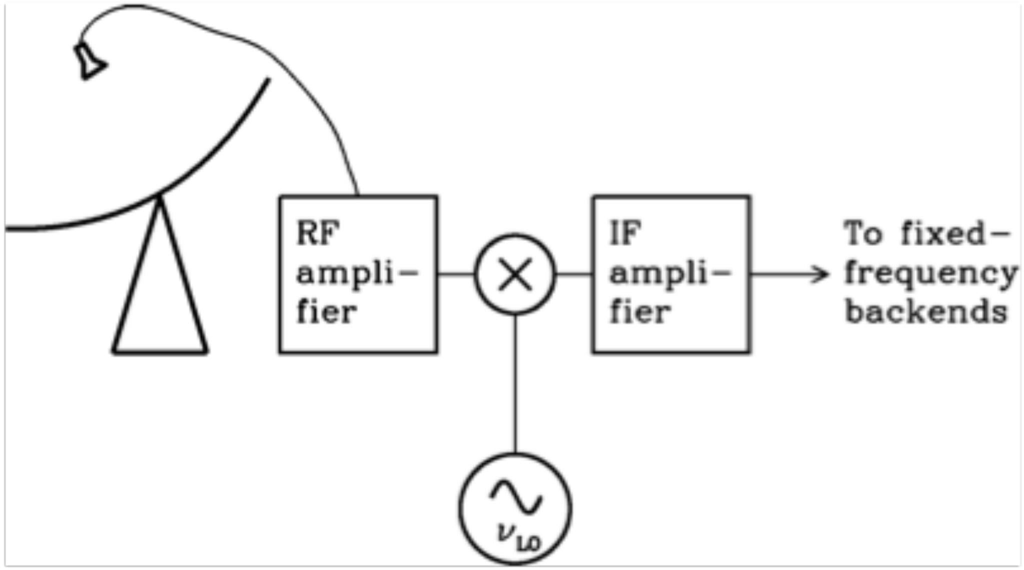
Heterodyne Receivers

The simple block diagram that we used for radiometers is more complicated in a real system.

Radio telescopes almost always use some form of heterodyne mixing to manipulate the incoming signal before running it through most of the electronics of the system.

The block diagram for such a system looks like:



Where the operative “heterodyne” part is the cross and the squiggle: the incoming signal, after amplification, is mixed with a local oscillator of a known frequency. The two waves interfere with one another and produce a signal with frequency equal to the difference of the incoming radio light and the local oscillator.

Heterodyne Receivers

The two waves interfere with one another and produce a signal with frequency equal to the difference of the incoming radio light and the local oscillator.

$$v_{\mathrm{RF}}(t) = \cos 2\pi f_{\mathrm{RF}} t,$$

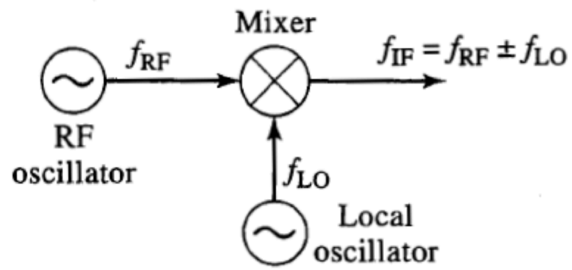
(radio) frequency being observed

$$v_{\mathrm{LO}}(t) = \cos 2\pi f_{\mathrm{LO}} t.$$

Local Oscillator - This can be set by the observer

That is, once the signals are mixed you get the multiplication of the two:

$$\begin{aligned} v_{\mathrm{IF}}(t) &= K\, v_{\mathrm{RF}}(t) v_{\mathrm{LO}}(t) = K \cos 2\pi f_{\mathrm{RF}} t \cos 2\pi f_{\mathrm{LO}} t \\ &= \frac{K}{2}[\cos 2\pi (f_{\mathrm{RF}} - f_{\mathrm{LO}})t + \cos 2\pi (f_{\mathrm{RF}} + f_{\mathrm{LO}})t] \end{aligned}$$

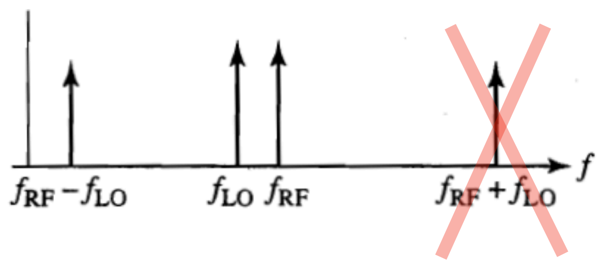


One term has frequency of the ***difference*** *in original frequencies.*

One term has frequency of the ***sum*** *in original frequencies.*

$$f_{\mathrm{IF}} = f_{\mathrm{RF}} - f_{\mathrm{LO}}$$

IF: intermediate frequency — can be selected by electronics / filtering



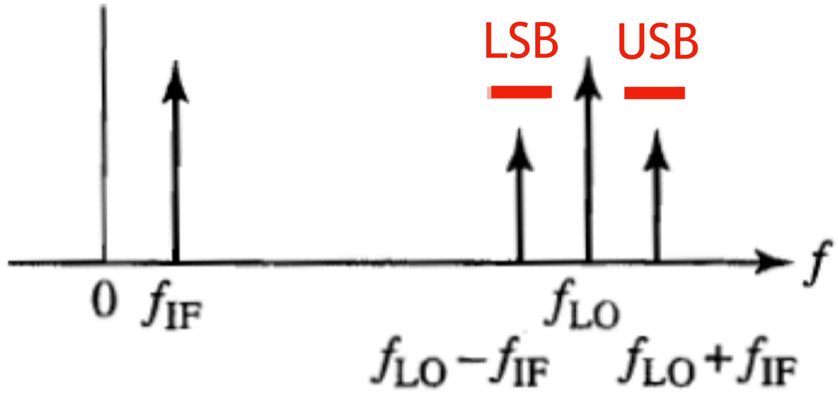
Heterodyne Receivers

Note: ‘image frequency’ is also mixed down to the same IF!

$$f_{\mathrm{IM}} = f_{\mathrm{LO}} - f_{\mathrm{IF}}.$$

$$f_{\mathrm{RF}} = f_{\mathrm{LO}} + f_{\mathrm{IF}},$$

from: $f_{\mathrm{IF}} = f_{\mathrm{RF}} - f_{\mathrm{LO}}$



In the old days this was a problem - as one could not tell whether lines were observed in the upper or lower sidesband (USB or LSB).

Single sideband receiver (SSB): only one sideband makes it through systrem.
The other (image) sideband received is rejected

Dual sideband receiver (DSB): both sidebands are superposed.

Sideband seperating receiver (2SB): both sidebands are recorded separately - this is achieved, eg. Using 2 LOs, and shifting one LO by 90 degrees. This Became achievable in the early 2000s.

Currently most receivers (ALMA etc) are 2SB receivers.

Heterodyne Receivers

A heterodyne receiver mixes the signal and filters it down to a lower, intermediate frequency set by the electronics. Why is this useful?

(1) Generically, it’s easier to work with lower frequencies.

(2) It means that by tuning a single local oscillator, the input frequency for any of a wide range of radio frequencies can be fed into the electronics at a controlled **intermediate frequency (IF)**.

(3) That means you can build **one set of electronics** to process this IF (so-called “backends”) that can be used across the spectrum.

Lecture 6

Basics of Interferometry

slides: Essential Radio Astronomy by NRAO (Condon & Ransom)
+ lectures by Ohio State University Professor Adam Leroy
Dr. Jason Hessels (ASTRON)
Prof. David Wilner (Harvard)

https://www.astron.nl/astrowiki/lib/exe/fetch.php?media=ra_uva:2017:ra_uva_lecture6_2017.pdf

The Nobel Prize in Physics 1974

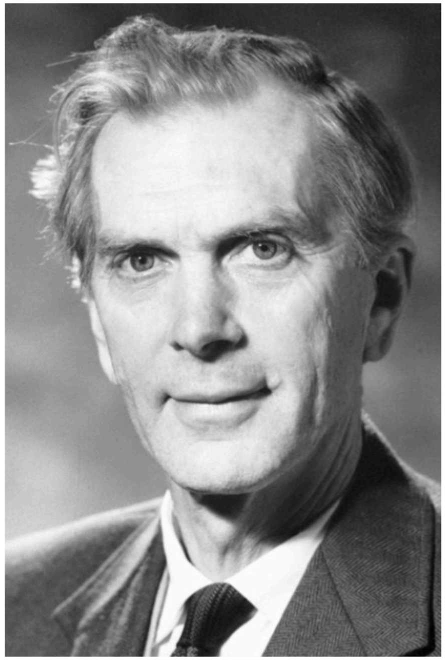


Photo from the Nobel Foundation archive.

Sir Martin Ryle

Prize share: 1/2

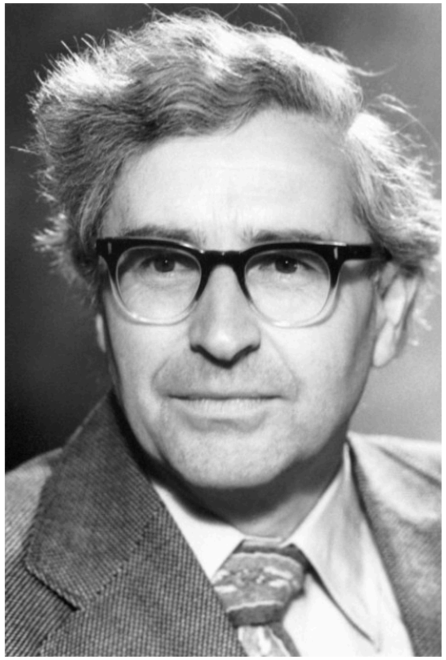


Photo from the Nobel Foundation archive.

Antony Hewish

Prize share: 1/2

The Nobel Prize in Physics 1974 was awarded jointly to Sir Martin Ryle and Antony Hewish "for their pioneering research in radio astrophysics: Ryle for his observations and inventions, in particular of the aperture synthesis technique, and Hewish for his decisive role in the discovery of pulsars."

https://www.nobelprize.org

Martin Ryle,

first Nobel Prize for astronomy (1974)

inspired by Bragg’s X-ray crystallography

Lecture Outline

- Quest for resolution
- Terminology
- Basic radio interferometer and correlator
- Visibilities and the uv-plane
- Basics of making an image
- Calibration issues
- Other consideration

The Quest for Resolution

We want sub-arcsecond resolution (cf. optical, X-ray)

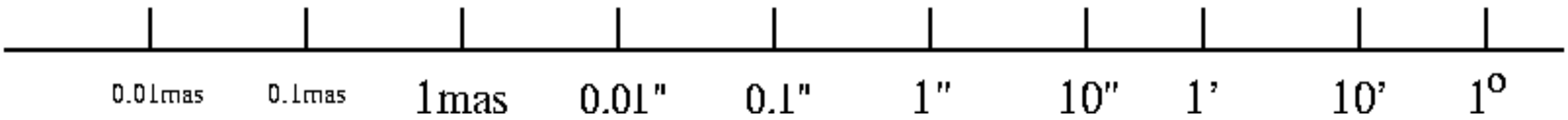
$$\Theta_{\rm rad} \propto \frac{\lambda}{D}$$

Unlike large, ground-based optical telescopes (atm. limits!), radio telescopes are always diffraction limited.

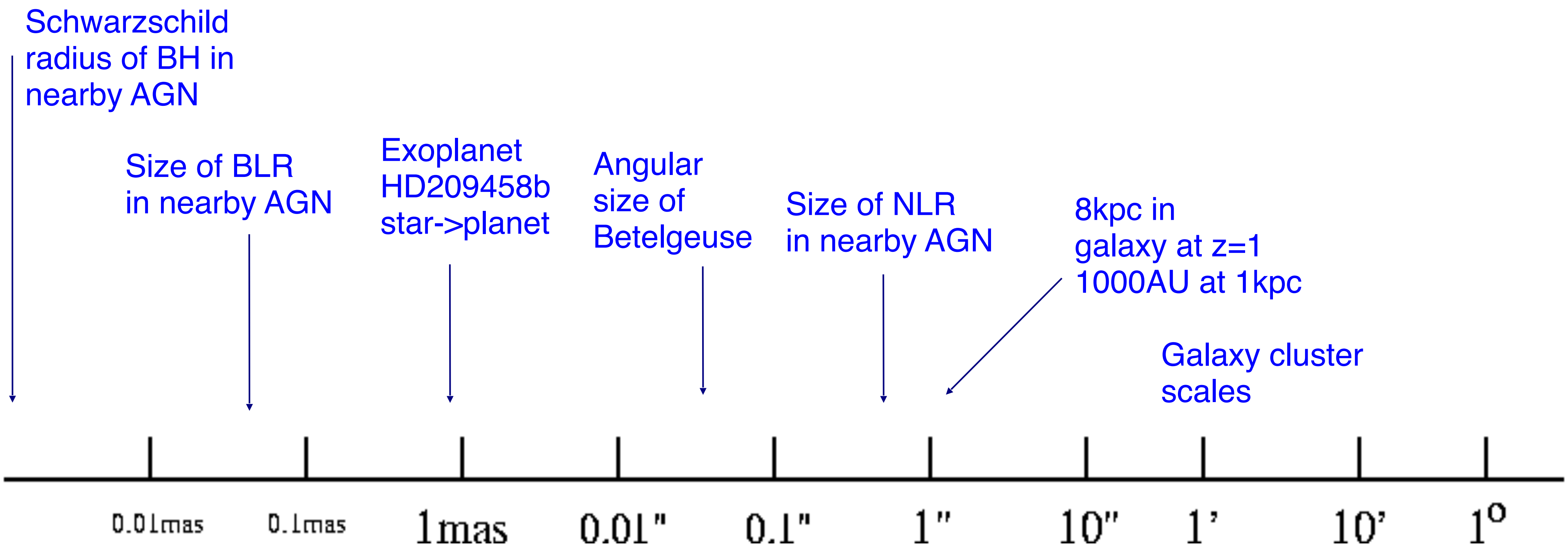
$$\Theta_{\rm arcsec} \sim 2\,\frac{\lambda_{\rm cm}}{D_{\rm km}}$$

So to get 1 arcsec resolution at 21cm requires a 42km diameter!

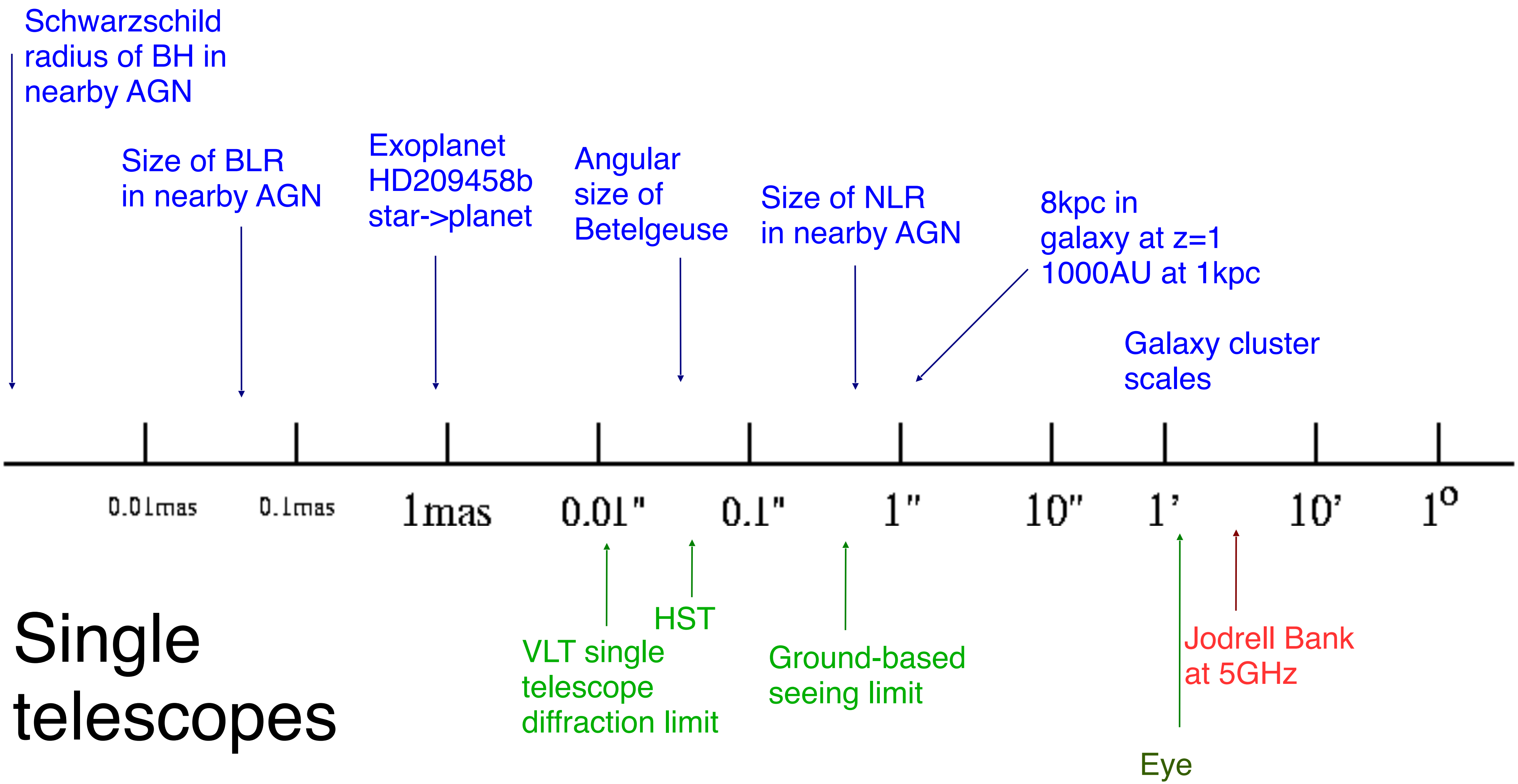
The Quest for Resolution



The Quest for Resolution

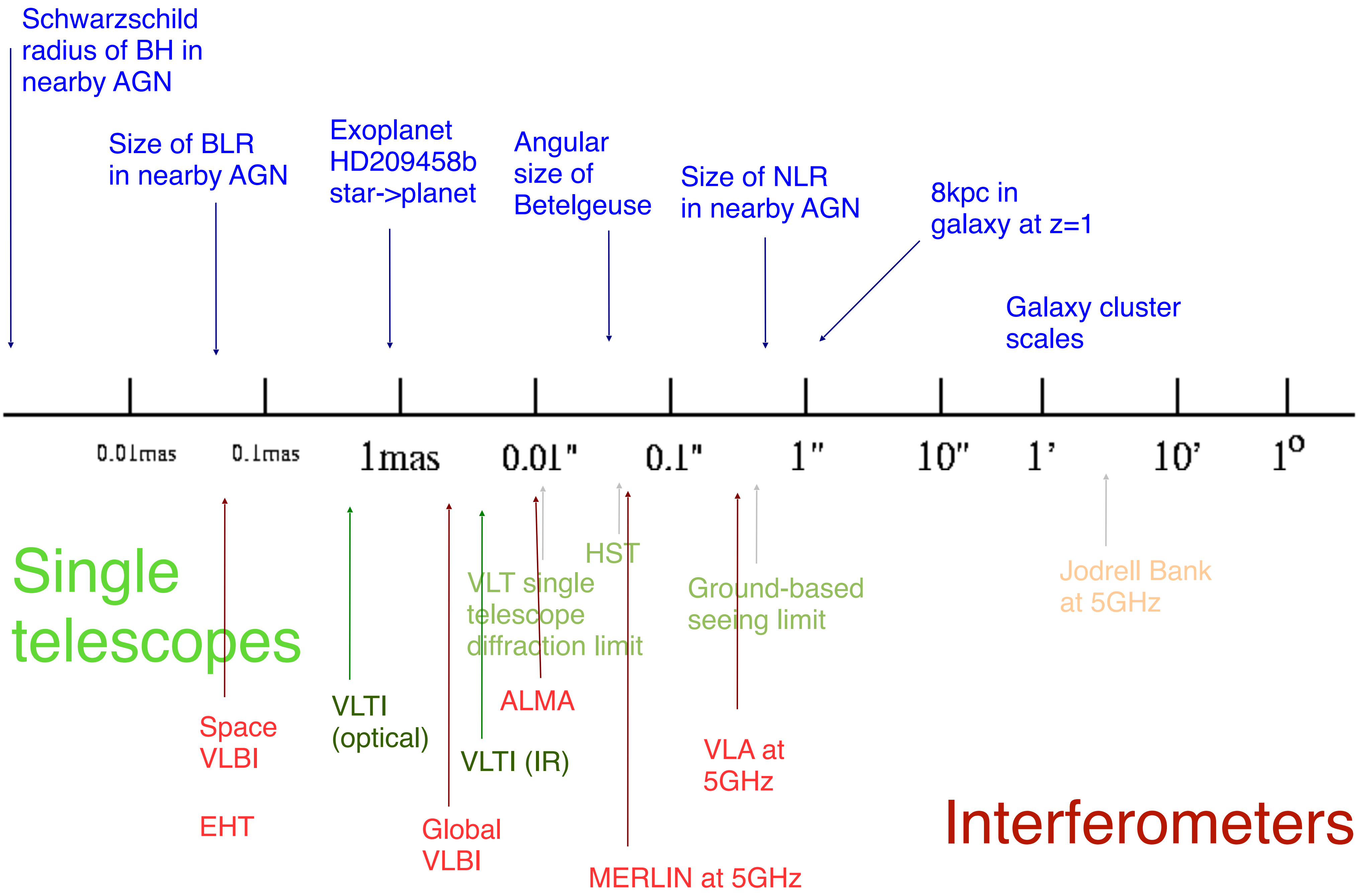


The Quest for Resolution



Single telescopes

The Quest for Resolution



The Quest for Resolution

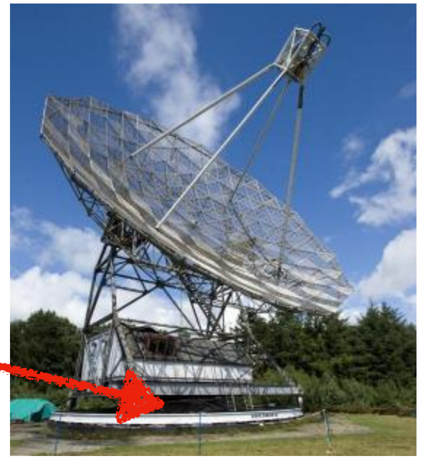
We don't have to build a single 42-km-wide radio dish

Aperture Synthesis

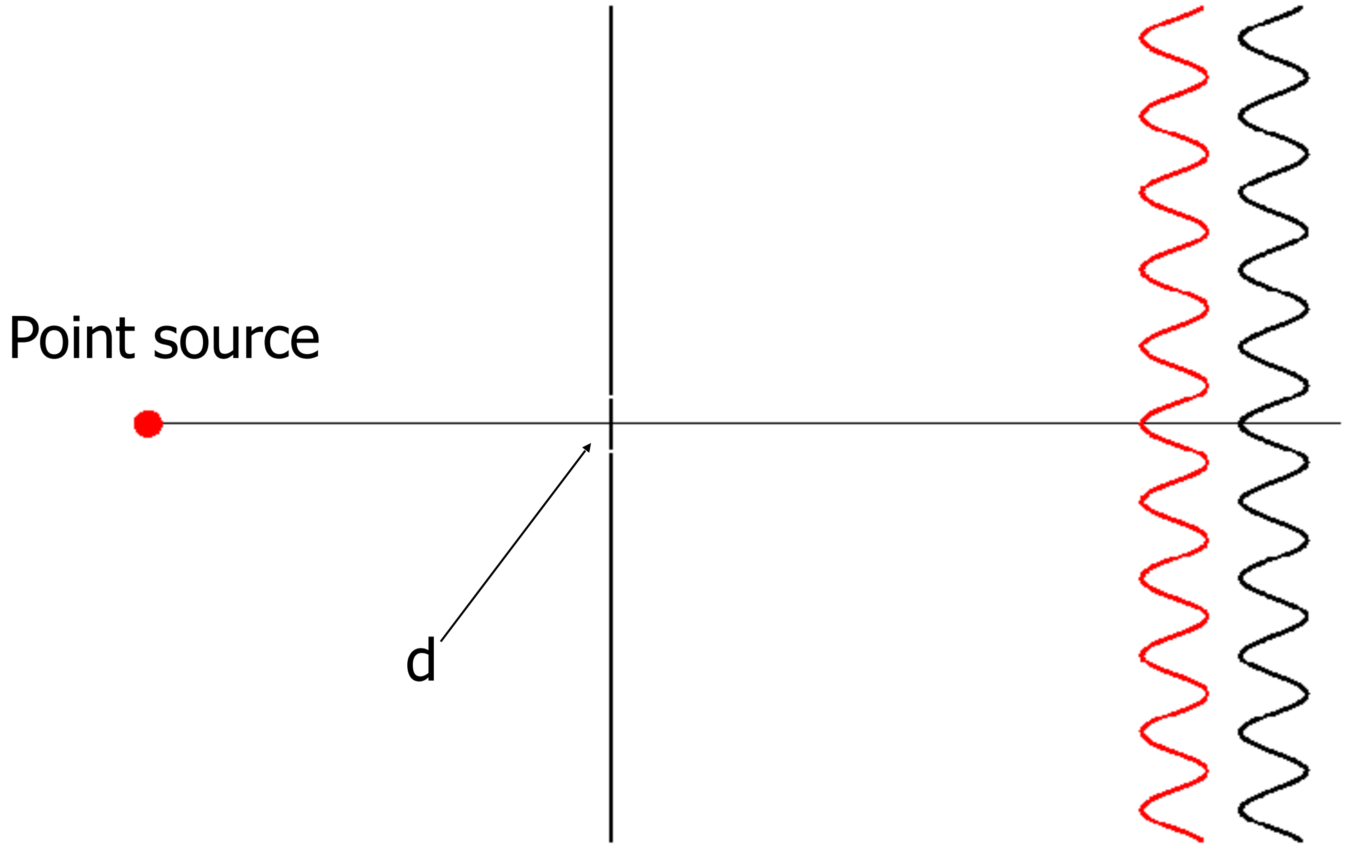
$$\Theta \propto \frac{\lambda}{D} \propto \frac{\lambda}{B}$$

B = 'baseline'.   (distance between telescopes)

but note: collecting area MUCH lower!



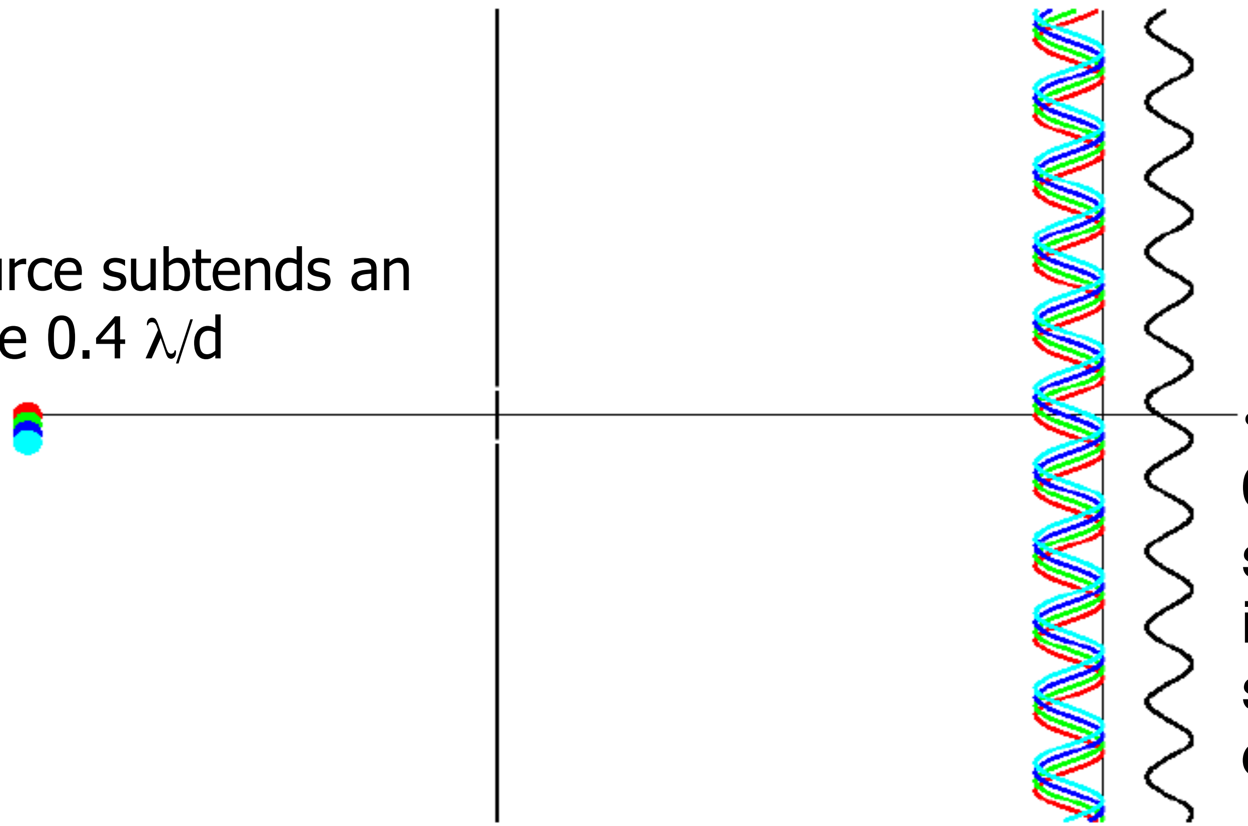
Young's double-slit experiment



•Fringes of separation λ/d

Young's double-slit experiment

Larger source



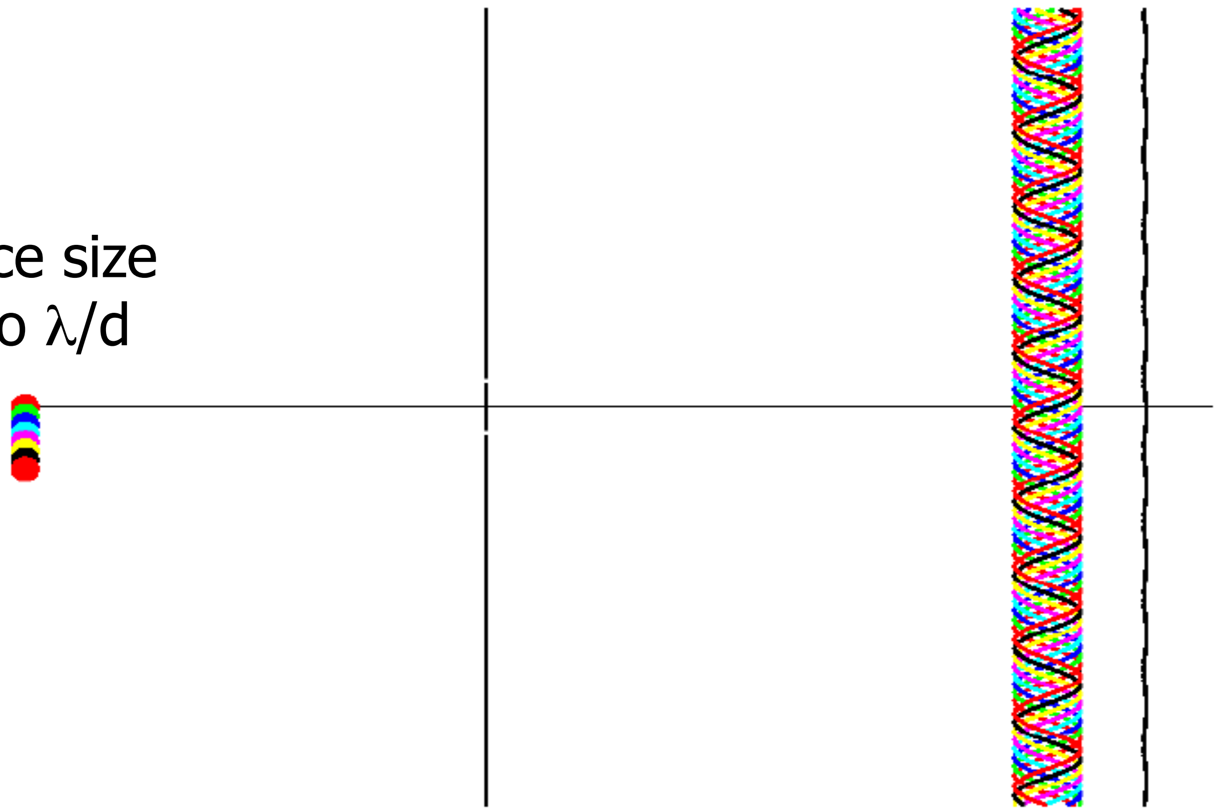
•Source subtends an angle 0.4 λ/d

•Fringes move by 0.4 λ/d. Incoherent sources -> add intensities, fringes start to add destructively

Young's double-slit experiment

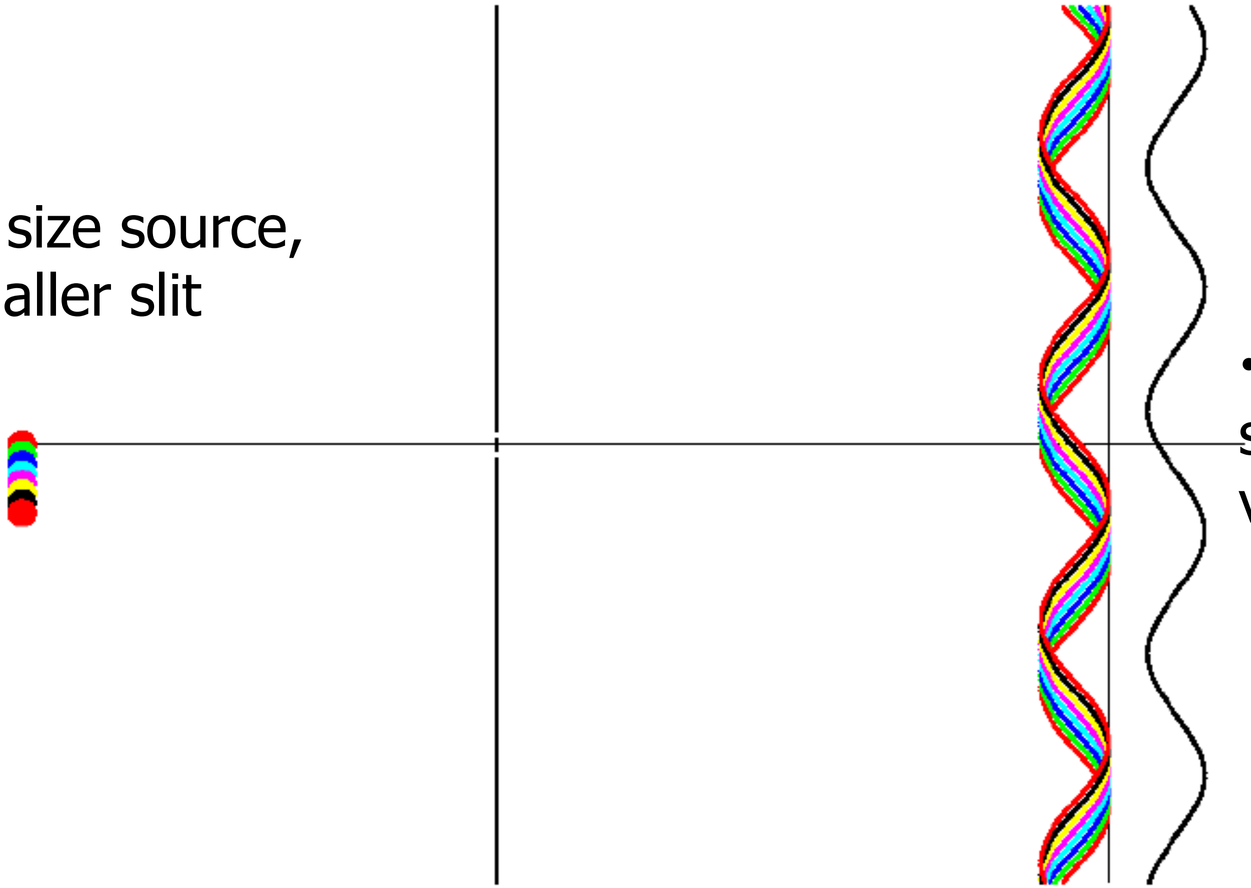
Still larger source

•Source size gets to λ/d



•No fringes remain (cancellation). Little fringing seen for larger sources than λ/d either.

Effect of slit size

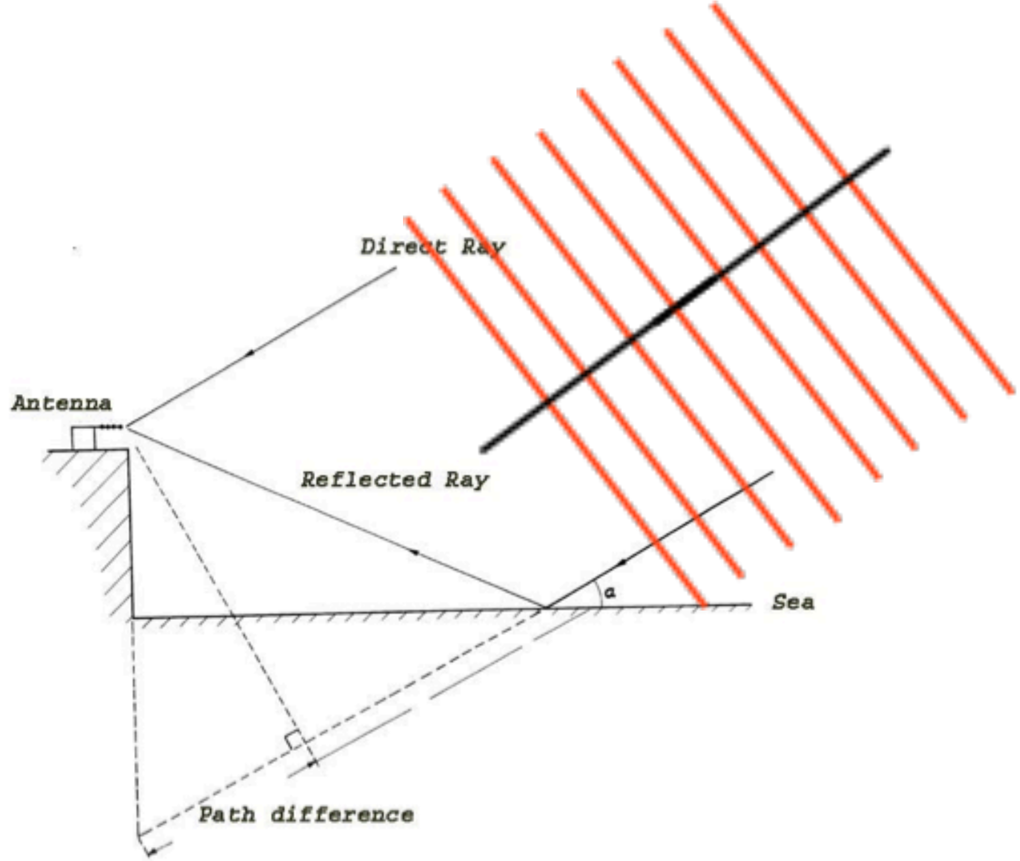


- Same size source, but smaller slit

- Increased fringe spacing, so fringes visible again

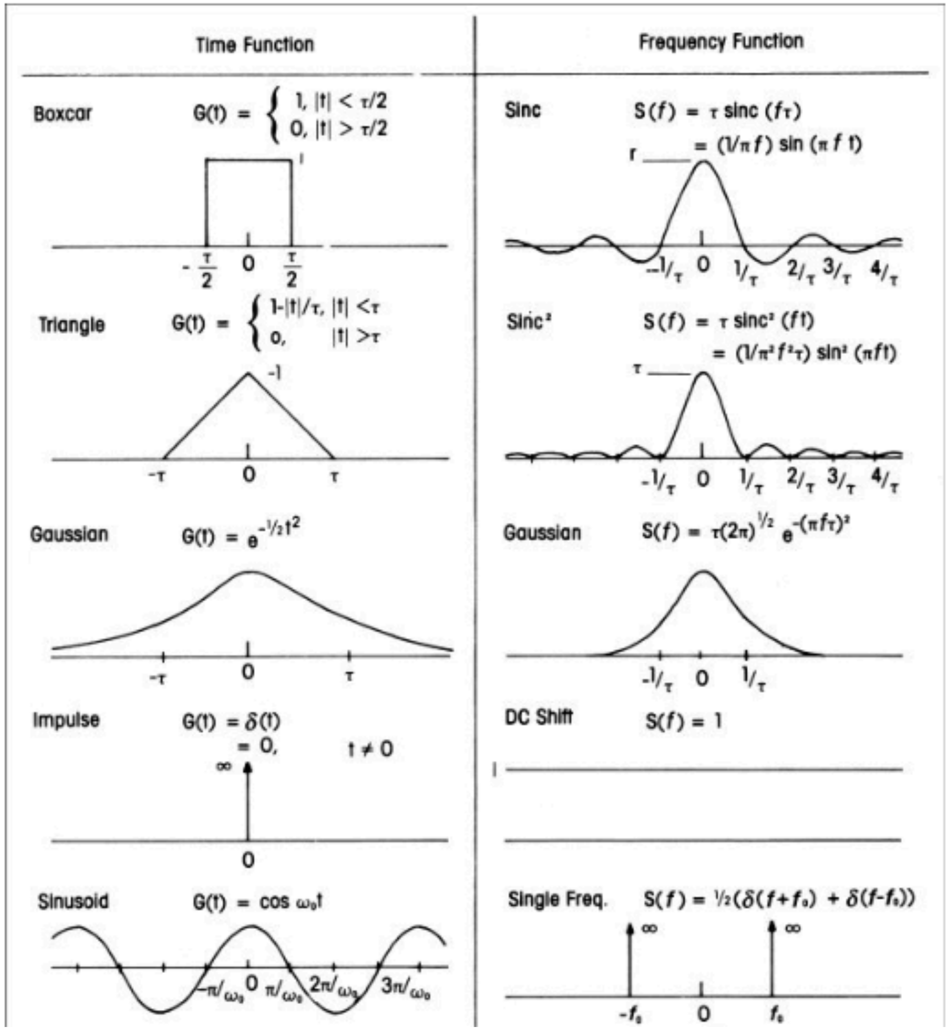
Sea interferometry

mid 1940: Dover Heights, near Sydney, Australia



Note: Path difference

Reminder: Fourier Transforms



Fourier Transform

$$\mathcal{F}(s) = \int_{-\infty}^{\infty} f(x)\, e^{-j2\pi s x}\, dx$$

FFT is an expedient way to calculate $\mathcal{F}(s)$ for a subset of regularly sampled datasets

Reminder: Fourier Transforms

- a few properties of the Fourier transform $g(x) \xrightarrow{\mathcal{F}} G(s)$

  An **addition** in one domain is an addition in the other

  $$g(x) + h(x) = G(s) + H(s)$$

  A **multiplication** in one domain is a convolution in the other

  $$g(x) = h(x) * k(x) \qquad G(s) = H(s)K(s)$$

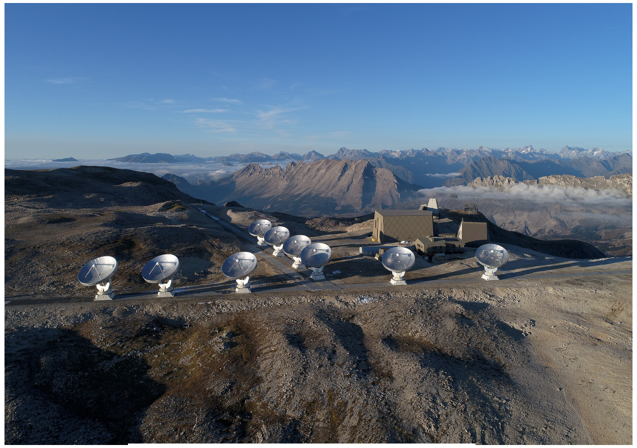
  Scaling: large in one domain is small in the other

  $$g(\alpha x) = \alpha^{-1} G(s/\alpha)$$

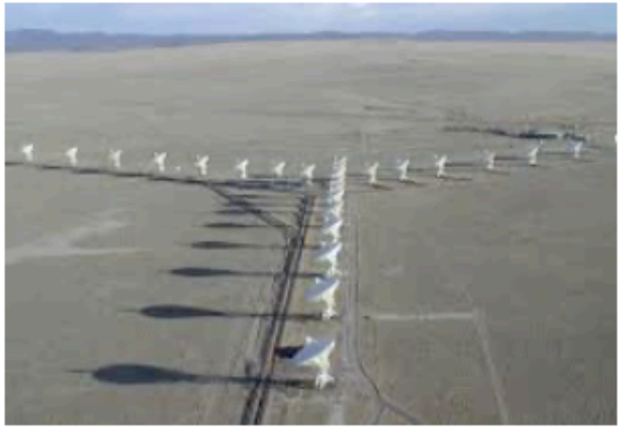
  An **offset** in one domain is a phase shift in the other

  $$g(x - x_0) = G(s)e^{i2\pi x_0 s}$$

Modern Radio Interferometers



NOEMA



JVLA

LOFAR

ALMA

Meerkat

GMRT

Modern Radio Interferometers

$$N_{baselines} = N_{elements}(N_{elements}-1)/2$$

Want to compare signal **amplitude** and **phase** between these telescopes.

VLA: 27 antennas, 351 baselines

ALMA: 45 antennas, 990 baselines

Modern Radio Interferometers

- The concept of a "baseline" is really important.

• Baselines have different "lengths" (distance between a pair of dishes/antennas) and they have different orientation with respect to the sky (RA,DEC).

- Note that the orientation of these baselines with respect to the sky changes during an observation.

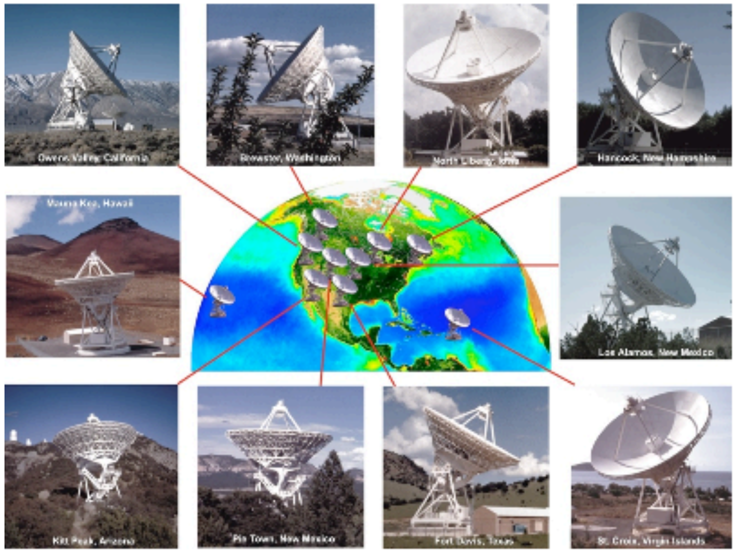
• This change in orientation complicates the picture but also provides a powerful way to accurately image the sky.

• The number of baselines (telescope pairs) is related to the number of elements in the array through 0.5 x N(N-1)

Very Long Baseline Interferometry

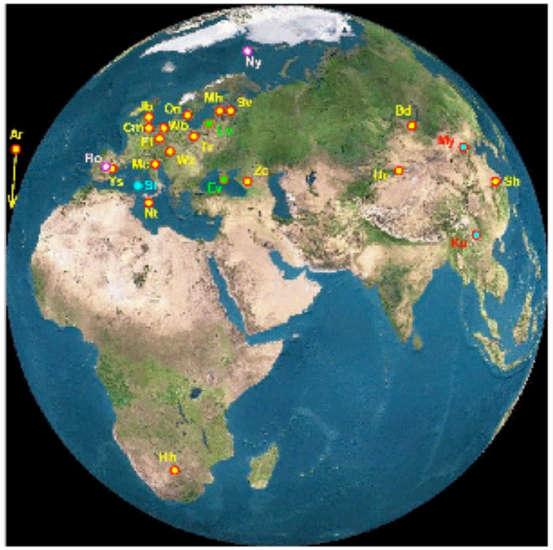
>1000-km baselines

Data traditionally recorded locally and shipped to correlator (though moving more and more towards real-time)



VLBA, USA

EVN, Europe

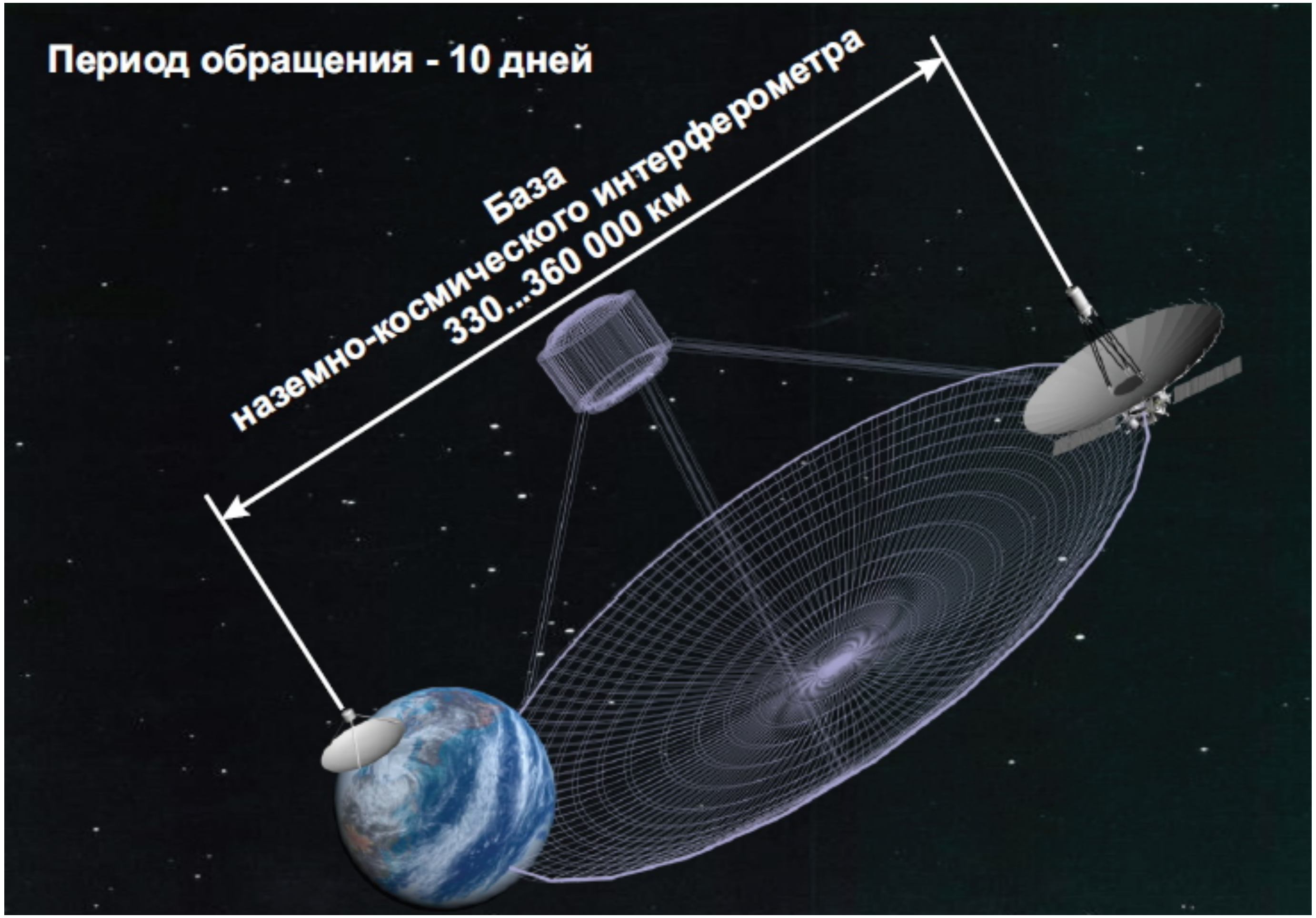


Global VLBI

Very Long Baseline Interferometry

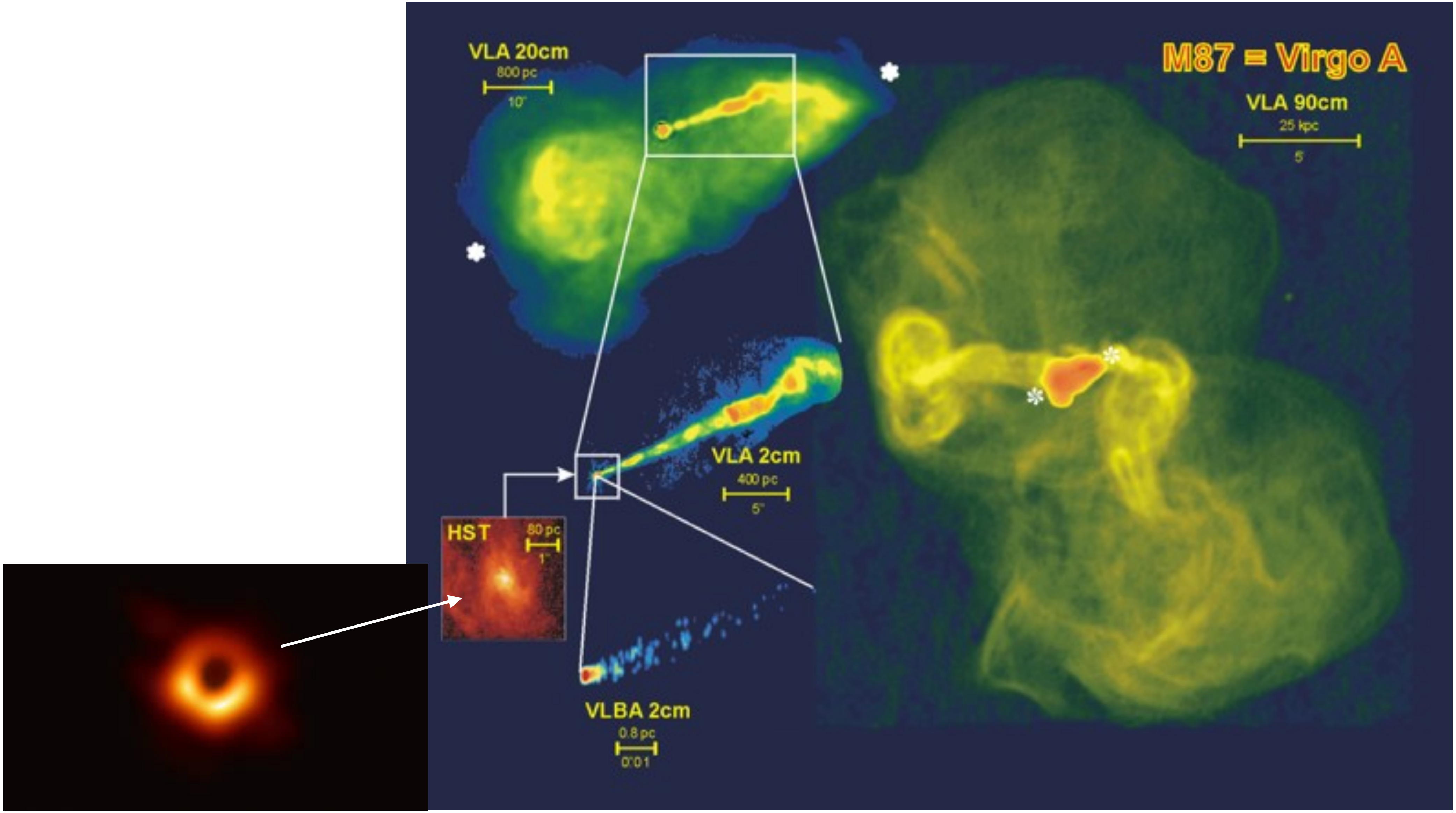
100.000-km baselines (!)

RadioAstron (Spekr-R): 10m radio telescope



http://www.asc.rssi.ru/radioastron/

The quest for resolution



Terminology

"Specific Intensity" or "Brightness"

Energy per unit time, area, frequency, and solid angle

$$[I(\vec{s}, \nu, t)] = \mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}\ ster.^{-1}}$$

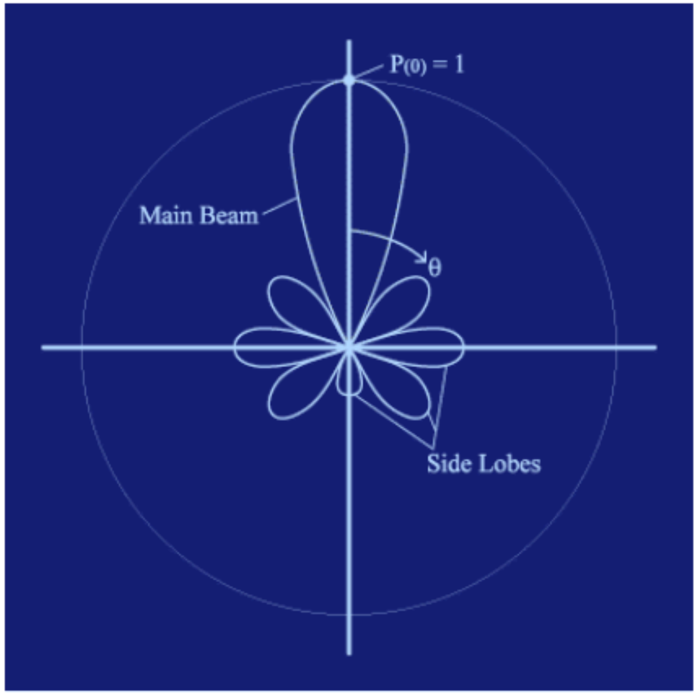
"Specific Flux Density" - Integrate over entire area of the source

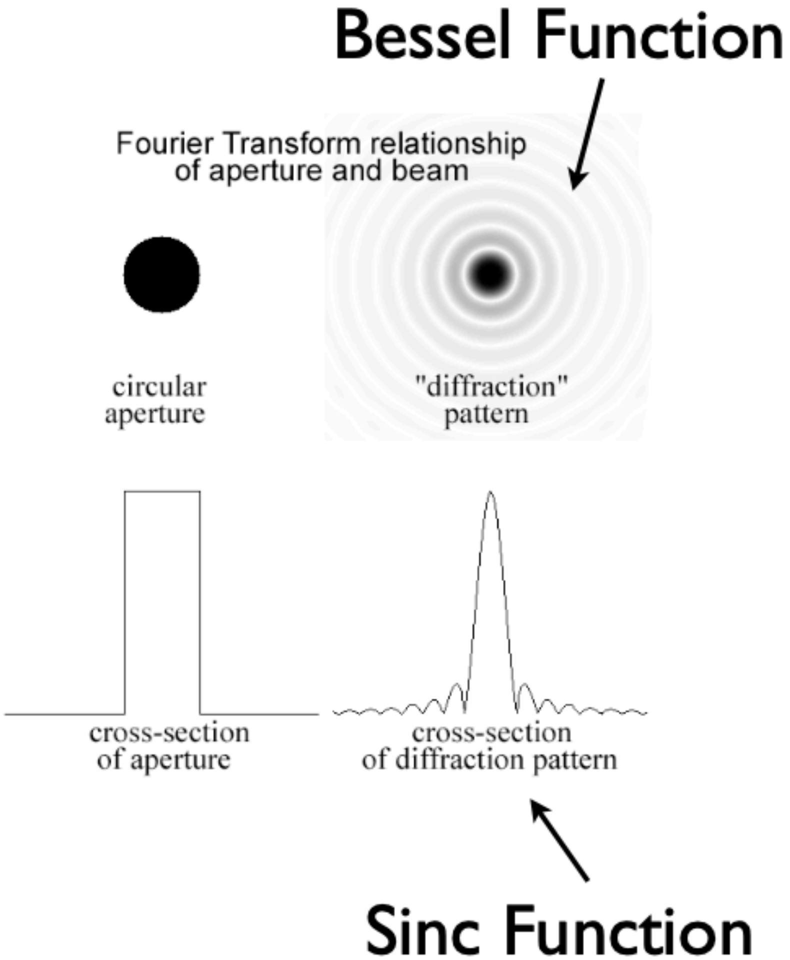
$$S = \int I(\vec{s}, \nu, t) d\Omega$$

$$[S(\nu, t)] = \mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$$

Units: Janskys: $1\mathrm{Jy} = 10^{-23}\mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$

Radio Telescope Field of View

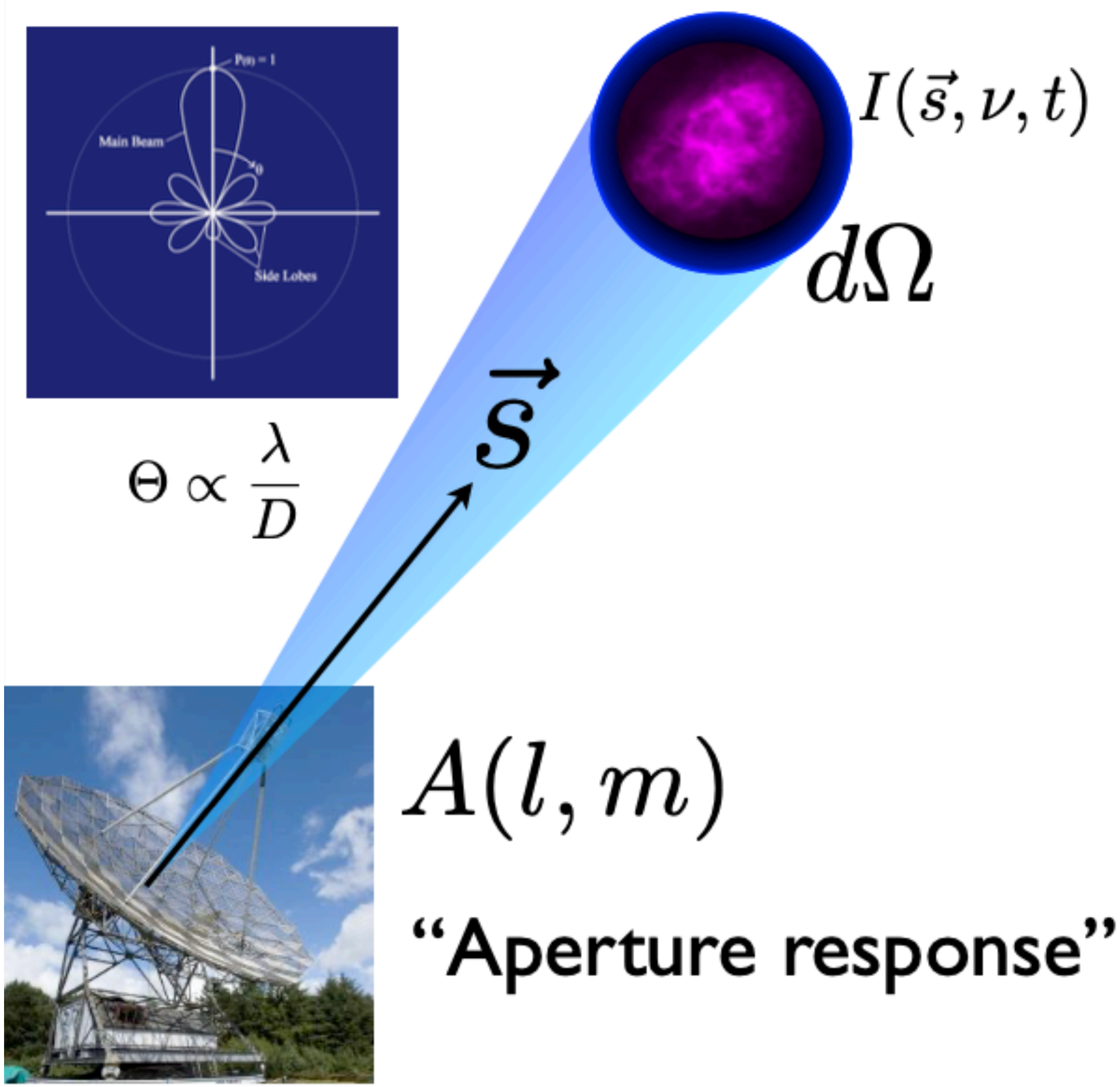




Fourier transform relationship between the aperture and the beam

Terminology

"**Primary beam**" (main beam of individual elements)

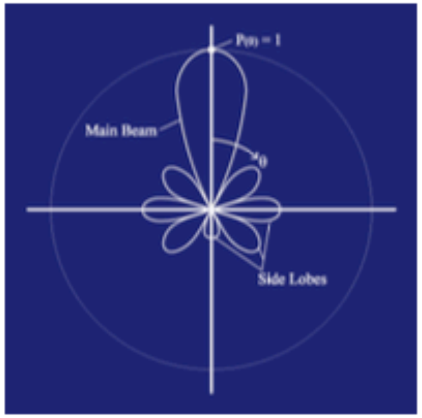


This determines the total FoV, whereas the **synthesized beam** determines the resolution within that FoV.

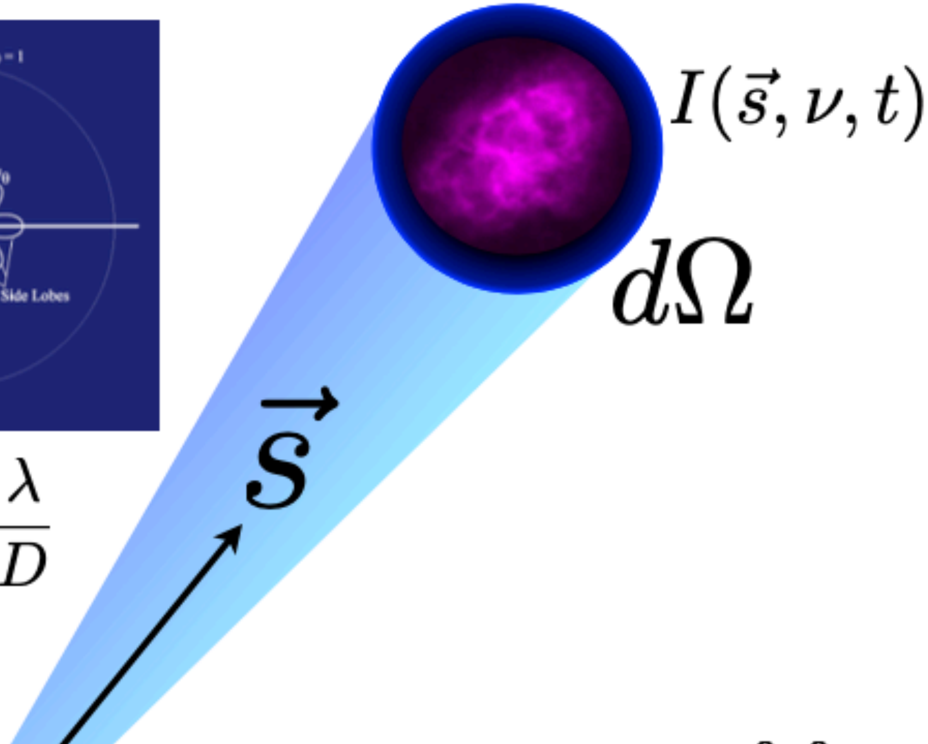
The aperture has a response as a function of direction and frequency.

Terminology

"Primary beam" (main beam of individual elements)



$$\Theta \propto \frac{\lambda}{D}$$



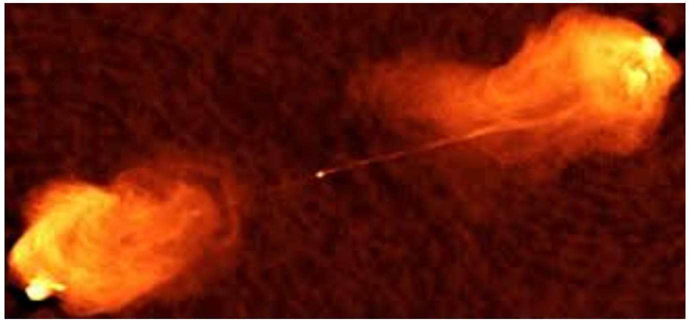
$$P(\nu, t) = \iint I(\vec{s}, \nu)\ A(\vec{s}, \nu)\ d\nu d\Omega$$

received power

Emitted brightness

Aperture response

Radio interferometric imaging



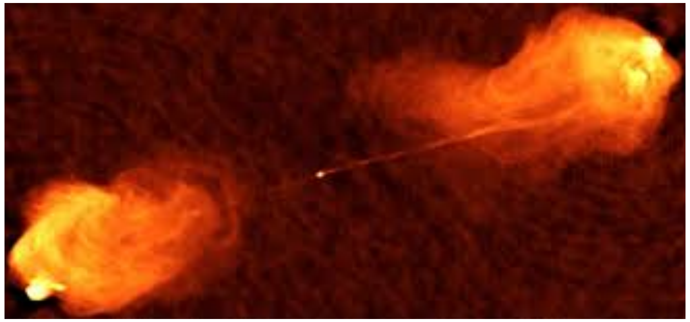
$I(\vec{s}, \nu, t)$

How do we relate the brightness of the radio sky to the power received by the antennas?

How do we turn this into a radio image?

$P_0(\nu, t)$ $P_1(\nu, t)$ $P_2(\nu, t)$

Radio interferometric imaging



$I(\vec{s}, \nu, t)$

How do we relate the brightness of the radio sky to the power received by the antennas?

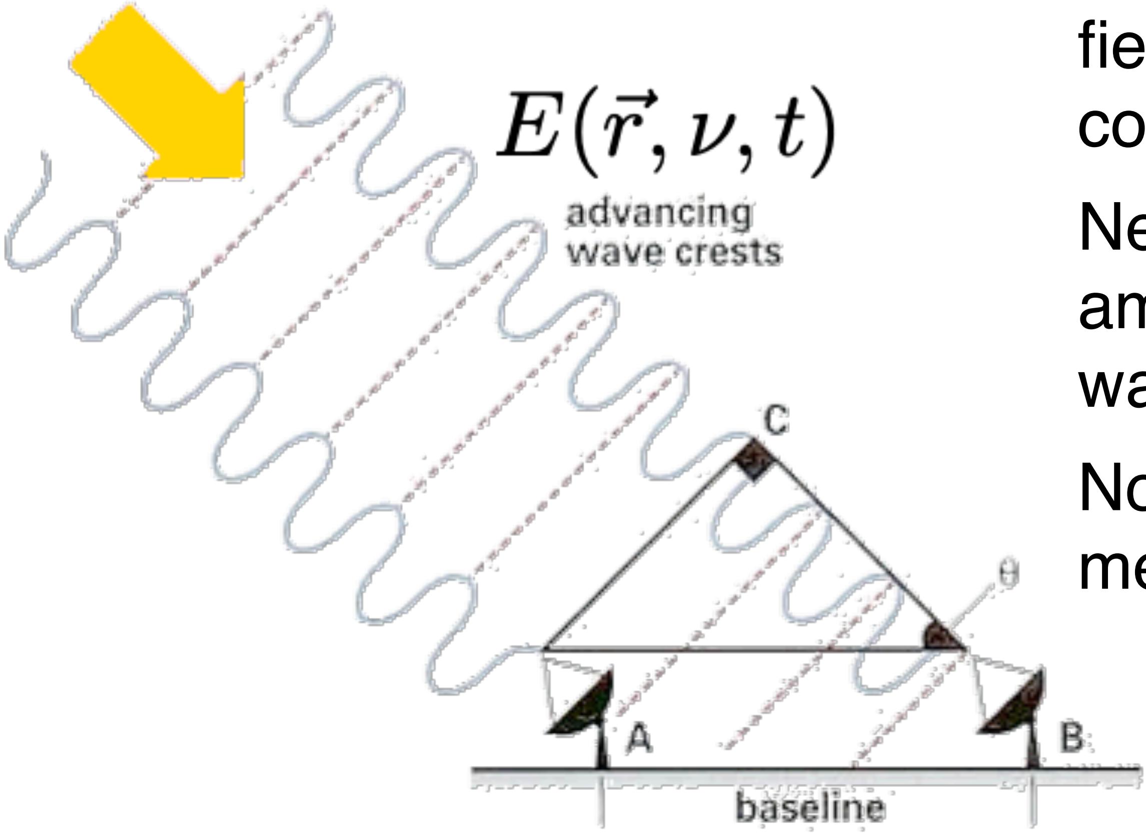
How do we turn this into a radio image?

$P_0(\nu, t)$ $P_1(\nu, t)$ $P_2(\nu, t)$

Need to correlate the received electric field (signal) at various geographically separate locations.

Each element gives a signal **amplitude** and **phase**.

Simple Interferometer



$$V_0(\nu, t) \qquad V_1(\nu, t)$$

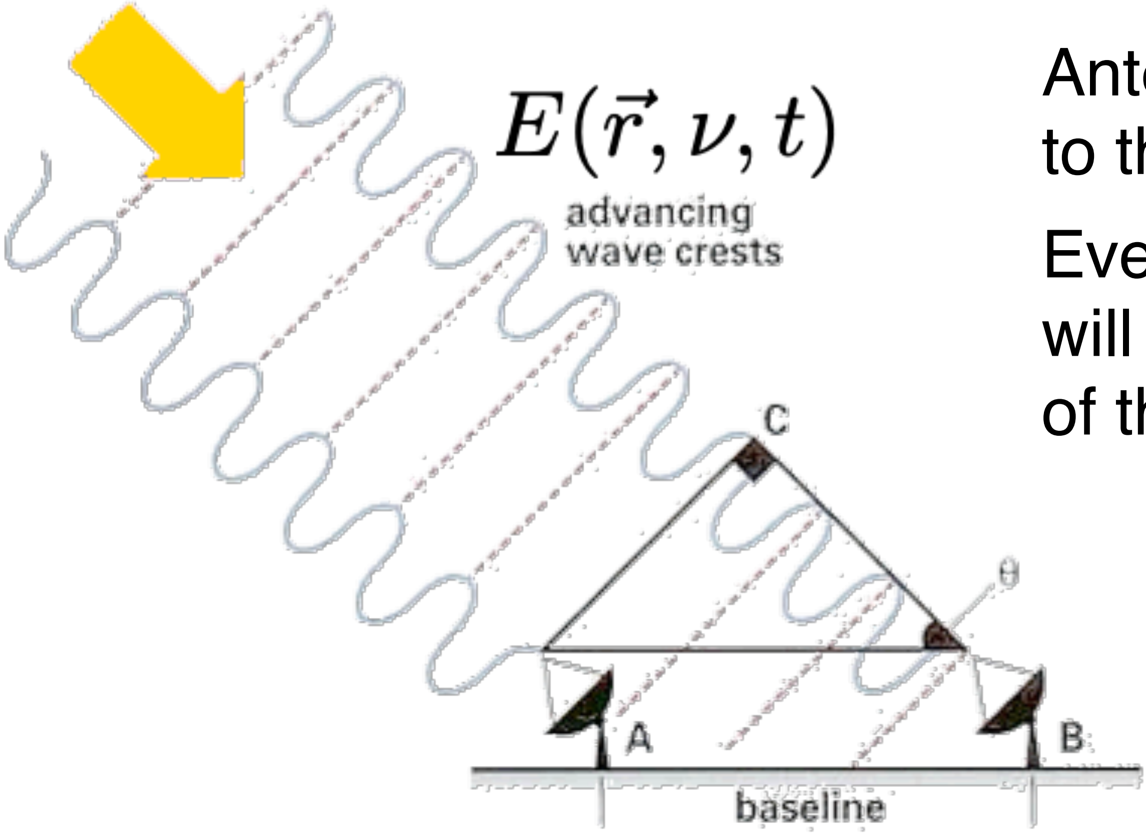
Sample an incoming electric field at various points and convert to voltages.

Need to register both amplitude and phase of the wave.

Note: single-dish ‘power measurement’ does not work

$$\cancel{P(\nu,t) \propto V^2(\nu,t)}$$ ~~power ~voltage squared~~

Simple Interferometer



$$E(\vec{r}, \nu, t)$$

$$V_0(\nu, t) \qquad V_1(\nu, t)$$

Antenna adds additional power to the signal.

Even a bright source of ~1Jy will still constitute only ~0.5% of the output power.

This signal is buried in noise but signal is correlated between antennas and noise is not

$\cancel{P(\nu, t) \propto V^2(\nu, t)}$ ~~power ~voltage squared~~

Simple Interferometer

Interferometer is fixed w.r.t. the sky (an instant in time).

Quasi-monochromatic waves. (single frequency)

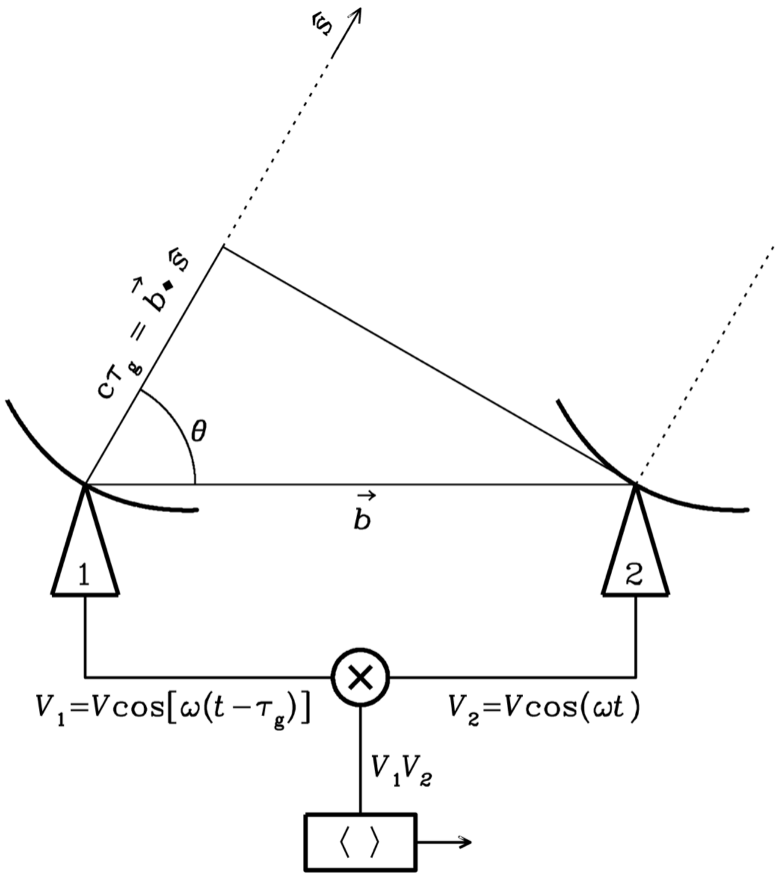
Interferometer directly measures the sky frequency

Single polarization.

No distortions from ionosphere.

Identical elements and perfect electronics.

Simple Interferometer



note: geometric delay removed through electronics
we are interested in delays due to different positions on sky

$$\tau_g = \frac{\vec{b} \cdot \hat{s}}{c}.$$

voltages:

$$V_1 = V\cos[\omega\,(t - \tau_g)]$$

$$V_2 = V\cos(\omega t).$$

multiplication: $$V_1V_2 = V^2\cos[\omega\,(t-\tau_g)]\cos(\omega t) = \left(\frac{V^2}{2}\right)[\cos(2\omega t - \omega\tau_g) + \cos(\omega\tau_g)]$$

$$\cos x \cos y = [\cos(x+y) + \cos(x-y)]/2$$

time averaging: $$R = \langle V_1V_2\rangle = \left(\frac{V^2}{2}\right)\cos(\omega\tau_g).$$

for $(\Delta t \gg (2\omega)^{-1})$

$\cos(2\omega t - \omega\tau_g)$-> 0

Slightly Extended Sources and the Complex Correlator

consider slightly extended sources as a sum of point sources,

$$R_c = \int I(\hat{s}) \cos(2\pi\nu \vec{b}\cdot\hat{s}/c)\, d\Omega$$

The response *Rc* depends on the received field strength and orientation of the baseline w.r.t. the source.

Doesn't depend on observation epoch, distance to source, or incoming signal phase (source is in the far field).

This cosine response seems problematic though: we don't want to have a ripple pattern of sensitivity. Uniform response over the **whole** mapped sky would be way better.

Slightly Extended Sources and the Complex Correlator

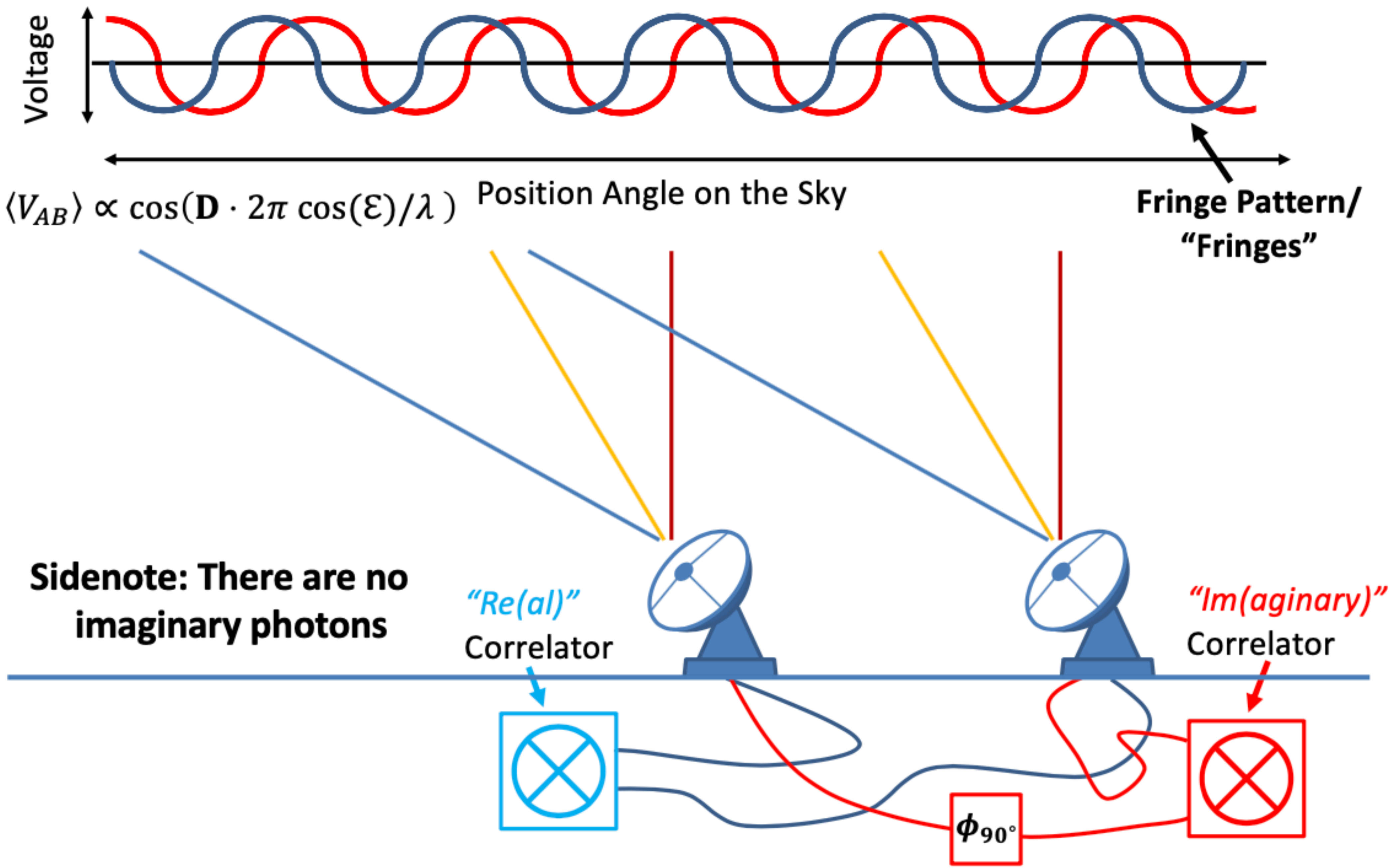
$$R_c = \int I(\hat{s}) \cos(2\pi\nu \vec{b} \cdot \hat{s}/c) d\Omega$$

…this is why we add a second correlator (sin-correlator) that has a 90 degree phase shift (adding 90 degree shift into the output of one antenna)

$$R_S = \int I(\hat{s}) \sin(2\pi\nu \vec{b} \cdot \hat{s}/c) d\Omega$$

R_C samples the **even** part of I(s), R_S samples the **odd** part of I(s)

another way to picture this



Sidenote: There are no imaginary photons

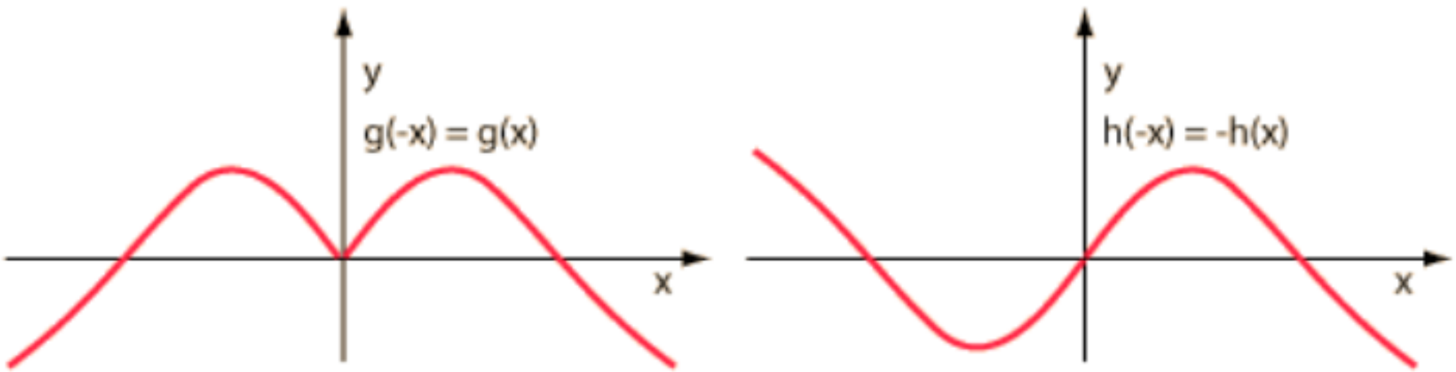
Reminder: even / odd functions.

Any real function can be decomposed into an even and odd part:

$$I(x,y) = I_E(x,y) + I_O(x,y)$$

such that:

$$I_E(x,y) = \frac{I(x,y) + I(-x,-y)}{2} = I_E(-x,-y)$$
$$I_O(x,y) = \frac{I(x,y) - I(-x,-y)}{2} = -I_O(-x,-y)$$



Visibilities

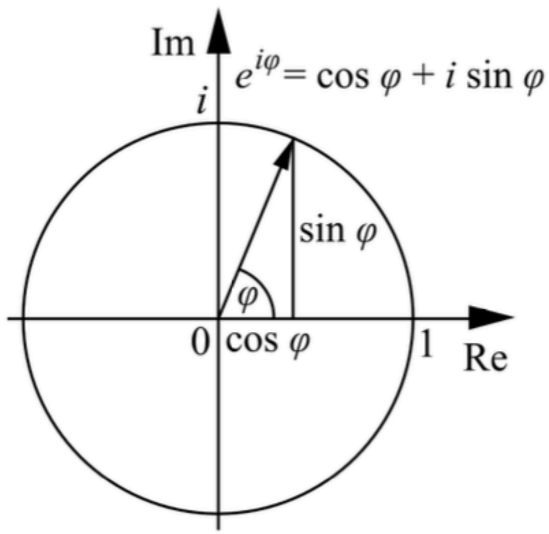
Define a complex function called the "visibility", which contains all the information for that baseline:

$$V = R_C - iR_S = Ae^{-i\phi} \qquad A = \sqrt{R_C^2 - R_S^2} \qquad \phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

Phase

Amplitude

Recall Euler's Formula



$$e^{i\phi} = \cos\phi + i\sin\phi.$$

Visibilities

Define a complex function called the “visibility”, which contains all the information for that baseline:

$$V = R_C - iR_S = Ae^{-i\phi} \qquad A = \sqrt{R_C^2 - R_S^2} \qquad \phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

$$V_\nu(\vec{b}) = R_C - iR_S = \iint I_\nu(\vec{s})e^{-2\pi i\nu\vec{b}\cdot\vec{s}/c}d\Omega$$

- *Complete* relation between the interferometer response and the source brightness.
- **This is a 2-D Fourier relation**

reminder: Fourier Transforms

FT from the time domain to the frequency domain:

$$F(v) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i v t} dt$$ is the Fourier transform of *f(t)*

$$f(t) = \int_{-\infty}^{\infty} F(v) e^{2\pi i v t} dv$$ is the inverse FT of *F(v)*

Basically, the idea is that any function can be expressed as the sum of a series of sines and cosines of varying amplitude and phase.

In other words, f(t) can be built up from the spectral distribution F(v) which is the power at frequency v.

See “The Fourier Transform and its applications” by Ronald Bracewell.

The fourier transform of a function e.g. f is often denoted as F (f) and the inverse is F^{-1} (f).

Complex correlator

- **Correlator**: machine to produce both the real and imaginary part of the visibilities (i.e. *Rc* and *Rs*).
- Effectively casts two sets of sinusoids on the sky, offset by 90deg

Complex correlator

- **Correlator**: machine to produce both the real and imaginary part of the visibilities (i.e. *Rc* and *Rs*).
- Effectively casts two sets of sinusoids on the sky, offset by 90deg

All Systems Go for Highest Altitude Supercomputer

ALMA correlator turns many antennas into one giant telescope

21 December 2012

1979 VLA correlator

the fastest supercomputer at the time!

(faster than a CRAY at the time, but limited in operations)

SKA testing on Summit supercomputer

building the correlator remains a major challenge in the era of large-N interferometers

... need to be smart

Oak Ridge National Laboratory (ORNL)

paper from 1983

PROCEEDINGS OF THE IEEE, VOL. 71, NO. 11, NOVEMBER 1983

The Very Large Array: Design and Performance of a Modern Synthesis Radio Telescope

PETER J. NAPIER, A. RICHARD THOMPSON, SENIOR MEMBER, IEEE, AND RONALD D. EKERS

Invited Paper

Abstract—Since its development in the 1960's, the technique of obtaining high-resolution radio images of astronomical objects using Fourier synthesis has advanced sufficiently so that today such images often provide better angular resolution than is obtainable with the largest optical telescopes. A synthesis array measures the Fourier transform of the observed brightness distribution by cross-correlating the signals from antennas separated by distances up to tens of kilometers. The antennas must be equipped with low-noise receiving systems and connected together by phase-stable transmission links. Wide-bandwidth digital correlators are used to perform the cross correlation. The data-reduction algorithms and computing system play a critical role in determining the quality of the images produced by the array.

The Very Large Array (VLA) synthesis telescope, recently constructed in New Mexico, consists of twenty-seven 25-m-diameter antennas arranged in a Y-shaped array. Each arm of the Y is approximately 21 km long and the antennas can be moved to various positions on the arms by a rail-mounted transporter. The antennas are equipped with cryogenically cooled receiving systems and are interconnected by low-loss, TE_{01}-mode, large-diameter waveguide. The cross-correlation products for each of the 351 pair combinations of antennas are measured for 4 IF signals by a 50-MHz bandwidth digital correlator.

In this paper we discuss the design of synthesis arrays in general, and describe the design and performance of the VLA in particular, under the seven headings: array geometry design, sensitivity considerations, phase stability requirements, signal transmission system, delay and correlator system, control system, and data-reduction requirements. In each section, we review the underlying instrumental requirements and provide details of how the VLA was designed to meet them. Recently developed data-reduction algorithms provide effective ways of correcting synthesis images for the effects of missing Fourier components and instrumental and atmospheric amplitude and phase errors. The power of these algorithms is demonstrated using actual VLA images.

to be identified without ambiguity in another. Furthermore, intercomparison of structural detail calls for comparable angular resolution. Until recent times, the angular resolution of large, ground-based optical telescopes, which is usually limited to about 1″ by atmospheric effects, was unapproached in other parts of the spectrum. At centimeter wavelengths, however, advances in imaging techniques based on interferometry have led to the design of arrays with resolution finer than 1″ and capable of synthesizing images with 10^6 to 10^7 resolution elements. The technique used has been variously described as Fourier synthesis, aperture synthesis, and earth-rotation synthesis, and was demonstrated by Christiansen and Warburton [1], and Ryle [2], and developed by Ryle at Cambridge. The Very Large Array (VLA), construction of which was completed at the end of 1980 at a cost of $78M (1977 dollars) is the latest and most powerful synthesis array to come into operation. It is routinely producing radio images with an angular resolution as fine as a few tenths of a second of arc.

In a Fourier synthesis array, signals from different antennas are combined in pairs in voltage-multiplier circuits, the outputs of which are averaged over time for periods of the order of a few seconds. The combination of a multiplying and time averaging circuit is usually termed a correlator. As is well known, combining signals from spaced antennas results in a reception pattern which is characterized by quasi-sinusoidal fringes, the angular spacing of which depends upon the spacing of the antennas in

paper from 1983

TABLE I

PRINCIPAL PARAMETERS OF CURRENTLY OPERATIONAL SYNTHESIS ARRAYS

Common Name	Institution	Frequency (Ghz)	T=Total no. elements (M)=no. movable d=size (m) T (M)	d	Total Geometrical Collecting Area (m^2)	Array Size and Geometry (km)	λ/d for Longest Baseline Resolution at Highest Frequency (arc sec)	References*
Owens Valley Centimeter Interferometer	Caltech, USA	0.6-10.7	2 (2) 1 (1)	27 40	2401	0.5 N/S, 1.2 E/W	4.6	13
Owens Valley Millimeter Interferometer	Caltech, USA	88-120	3 (3)	10.4	85	0.43 T	1.2	18
Culgoora Radioheliograph	CSIRO, Australia	0.04, 0.08, 0.16, 0.327	96	13	12742	3 circular	63	19
Australia Telescope	CSIRO, Australia	0.3 - 44	7 (6) 1	22 64	5800	6 E.W. 300 Irregular	.01	20
Penticton Interferometer	Dominion RAO, Canada	0.408, 1.4	4 (2)	9	254	0.6 E/W	74	21
Gauribidanur Decameter Wave Telescope	Raman Res. Inst. Indian Inst. Astphys.	0.035	1000 dipoles		$250\lambda^2$	T, 1.5 E/W, 0.5 N/S	1260	22
IRAM Array	IRAM, France	70-375	3 (3)	15	530	0.39, Approx. T	0.4	23
One-Mile Telescope	Mullard RAO, U.K.	0.408, 1.42 2.7, 5.0	3 (1)	18.3	790	1.5 E/W	.8	24
Half-Mile Telescope	Mullard RAO, U.K.	1.42	4 (2)	9	254	0.73 E/W	60	25
5 km Telescope	Mullard RAO, U.K.	2.7, 5.0, 15.4, 32	4 (4)	13	1062	4.6 E/W	0.7	26
5 km, 151 MHz Telescope	Mullard RAO, U.K.	0.151	60	4 element Yagi	2000	5.0 E/W	60	27
VLA	NRAO, USA	1.4, 5, 14.4, 23	27 (27)	25	13200	21, 21, 19 Y	0.07	7
Green Bank Interferometer	NRAO, USA	2.7, 8.1	3 (2) 2	26 14	1750	35 Irregular	0.2	28,29
Westerbork Synthesis Radiotelescope(WSRT)	Netherlands Foundation for Radio Astronomy	0.608, 0.327, 1.4, 5.0	14 (4)	25	6872	2.8 E/W	4.4	30
Ooty Radiotelescope	Tata Institute, India	0.327	4 1 5	132 × 30 100 × 9 25 × 9	17865	4.7 Irregular	40	31
Nobeyama Array	Tokyo Astrophys. Obs., Japan	22, 115	5 (5)	10	392	.68 T	.8	32
Bologna Cross	University of Bologna, Italy	0.408	6 8	20 × 95 24 × 80	26800	E/W, 0.4 N/S	150	33
Hat Creek Millimeter Interferometer	University of California, USA	80-115	3 (3)	6	85	.3 E/W, .2 N/S	1.8	34
MERLIN	University of Manchester, U.K.	0.408, 1.66, 5.0, 22	1 2 3	76 15 25	6362	134 Irregular	.02	35
Clark Lake Array	University of Maryland, USA	0.0015 - 0.0125	720	Log Spirals	$250\lambda^2$	3.0 T	160	36,37
Fleurs Synthesis Telescope	University of Sydney (E.E.), Australia	1.4	64 6	6 14	2700	1.6EW, 0.8NS, 3.6 Irregular	12	38,39
Molonglo Obs. Synthesis Telescope	University of Sydney (Physics), Australia	0.843	88	11.6×17.7	18068	1.6 E/W	4.6	40
UTRAO Interferometer	University of Texas, USA	0.335, 0.365, 0.380	5	Helix Array	650	3.58 E/W, 3.38 N/S	45	41

(In many cases the listed references describe the original instrumental parameters only, and values given in the table have been supplemented through recent personal communications.)

1983 - 9 years after award of Nobel Prize!

Visibilities - moving to 2D

Of course, a more useful interferometer includes more than just two antennas and hence more than just 1 baseline.

- Also, ideally these will be distributed in 2-D.

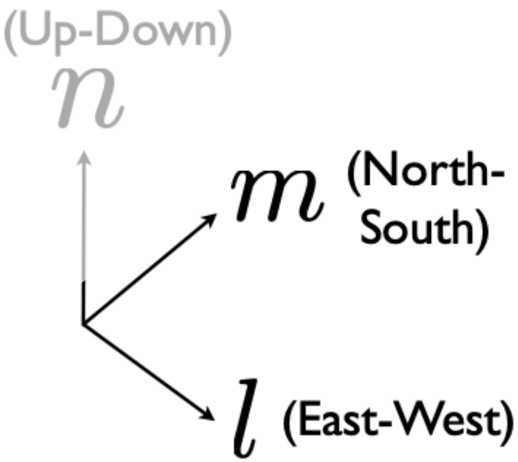
- for an N-element interferometric array, there are N(N-1)/2 independent baselines.

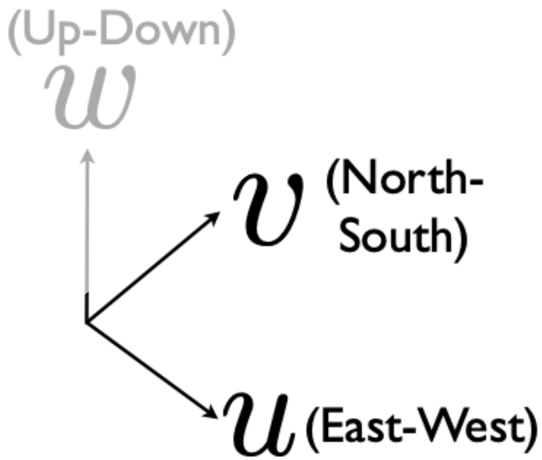
uv plane

"(l,m,n) and (u,v,w) coordinates"

Components of the source direction (unit) vector

Components of the baseline vector





up-down vector always set to be pointing at telescope

Assume all interferometric elements are in a plane - note: this does not hold at long wavelengths

$$\vec{b} = (\lambda u, \lambda v, \lambda w) = (\lambda u, \lambda v, 0)$$

Response of an Interferometer

$$V(u,v) = \int\int I\,(l,m)e^{-i2\pi(ul+vm)}dldm$$

V(u,v), the complex visibility function, is the 2D Fourier transform of I(l,m) the sky brightness distribution (for an incoherent source, small field of view, far field, quasi-monochromatic, etc.)

- *V(u=0,v=0)* is the integral of *I(l,m)dldm* = total flux density

- *I(l,m)* is real: *V(-u,-v) = V*(u,v)*   *where * = complex conjugate*
  - get two visibilities for one measurement

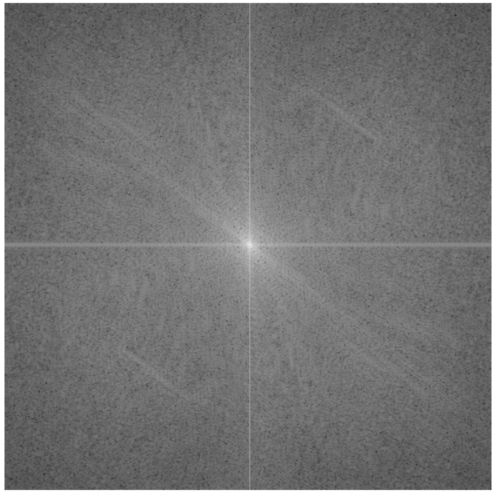
Visibilities

- each *V(u,v)* is a complex quantity
  - expressed as *(real, imaginary)* or *(amplitude, phase)*

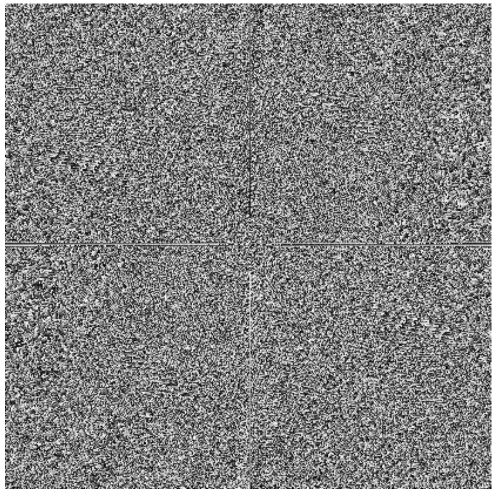
I(l,m)

$$\xrightarrow{\mathcal{F}}$$

V(u,v) amplitude



V(u,v) phase

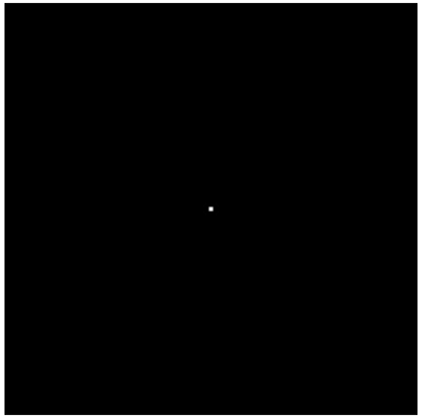


each V(u,v) contains information everywhere in the image I(l,m)

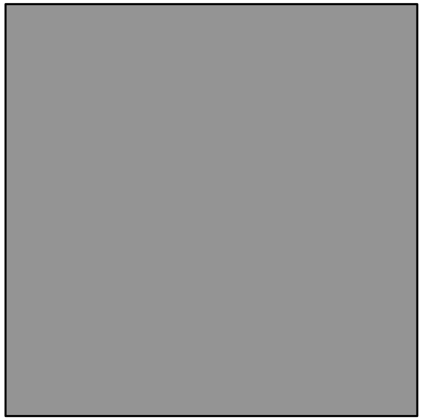
Some 2D Fourier Transform Pairs

I(l,m) *V(u,v) amplitude*

δ function



$\xrightarrow{\mathcal{F}}$

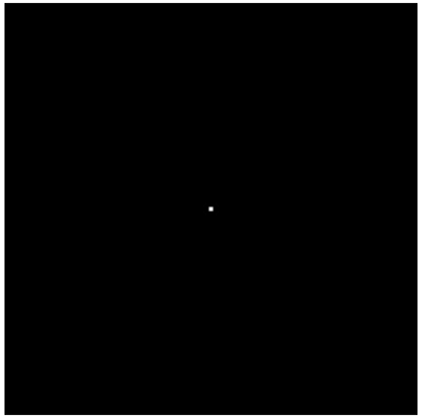
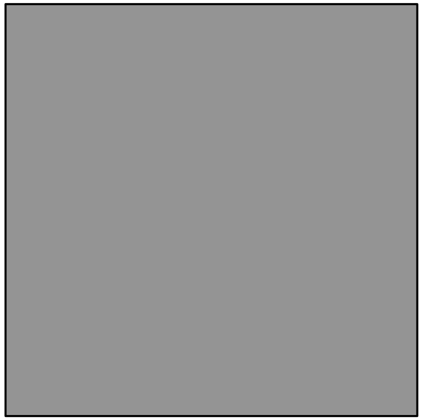


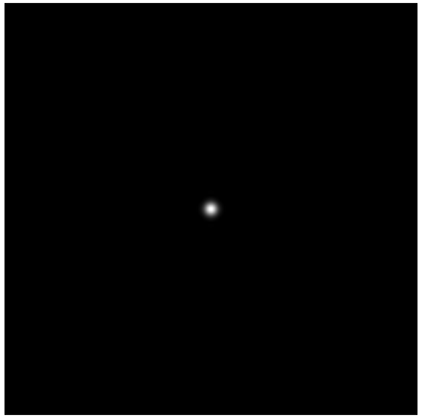
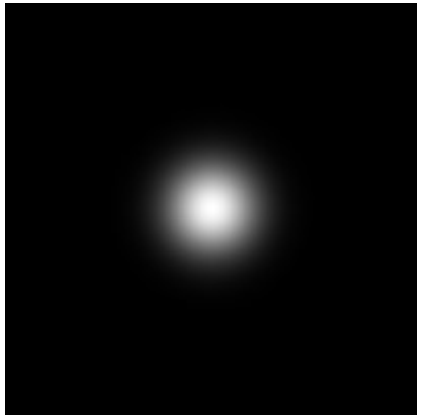
constant

narrow features transform into wide features (and vice-versa)

Some 2D Fourier Transform Pairs

I(l,m) *V(u,v) amplitude*

δ function  $\xrightarrow{\mathcal{F}}$  constant

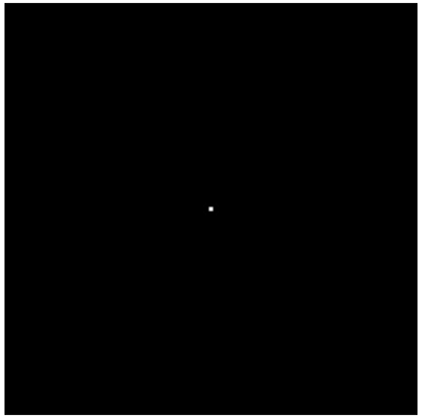
Gaussian  $\xrightarrow{\mathcal{F}}$  Gaussian

narrow features transform into wide features (and vice-versa)

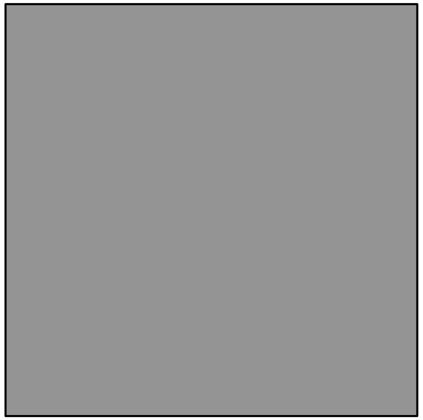
Some 2D Fourier Transform Pairs

I(l,m) *V(u,v) amplitude*

δ function

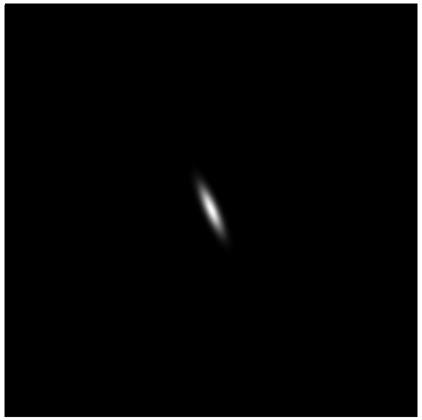


$\xrightarrow{\mathcal{F}}$

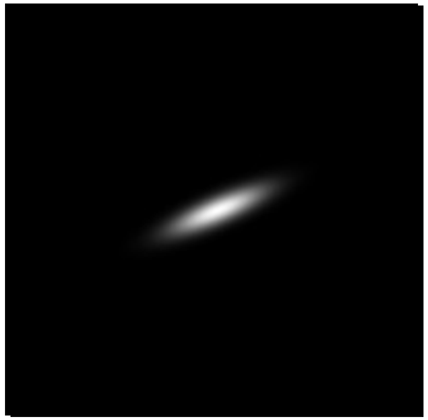


constant

elliptical Gaussian



$\xrightarrow{\mathcal{F}}$



elliptical Gaussian

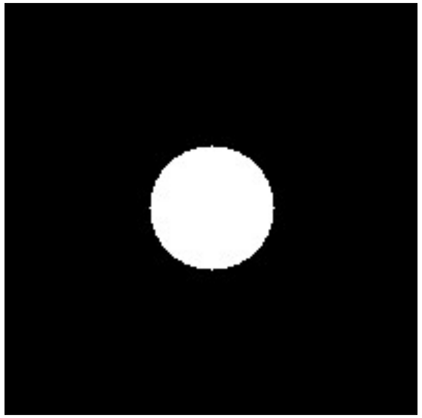
narrow features transform into wide features (and vice-versa)

Some 2D Fourier Transform Pairs

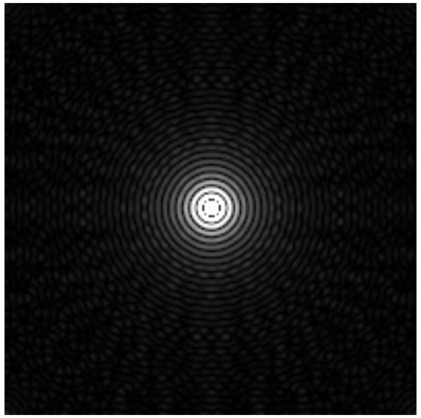
T(l,m)

V(u,v) amplitude

uniform disk



$\xrightarrow{\mathcal{F}}$

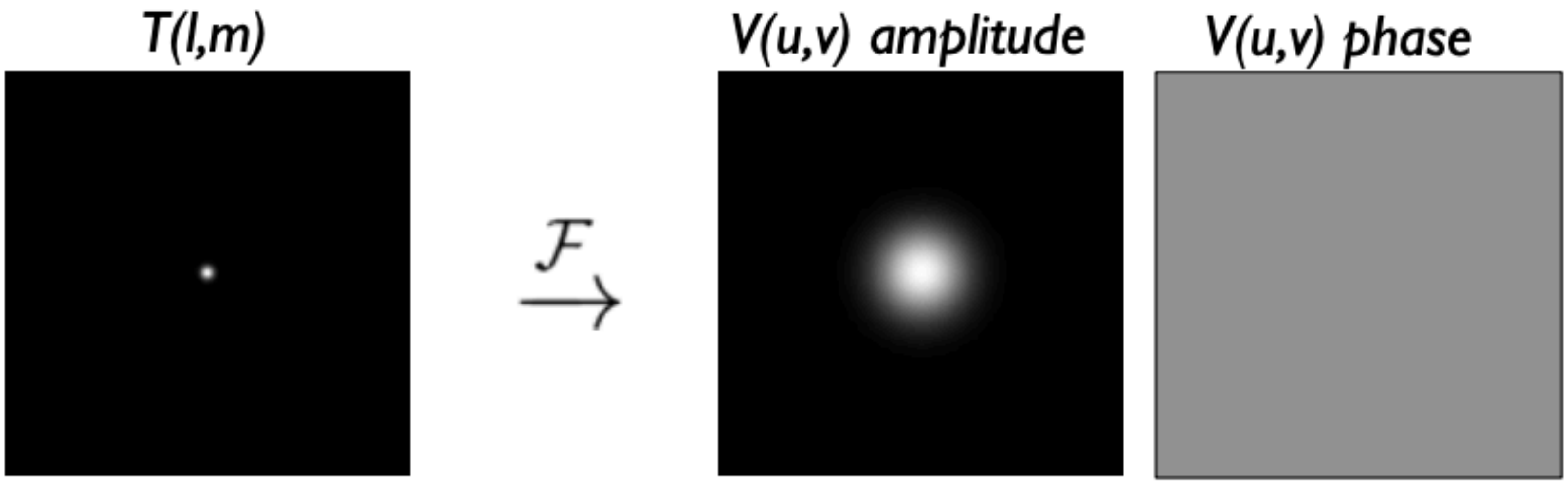


Bessel function

anything sharp in one domain generally oscillates ("rings") in the other

Amplitude and Phase

amplitude tells ‘how much’ of a certain spatial frequency
phase tells ‘where’ this spatial frequency is located



Amplitude and Phase

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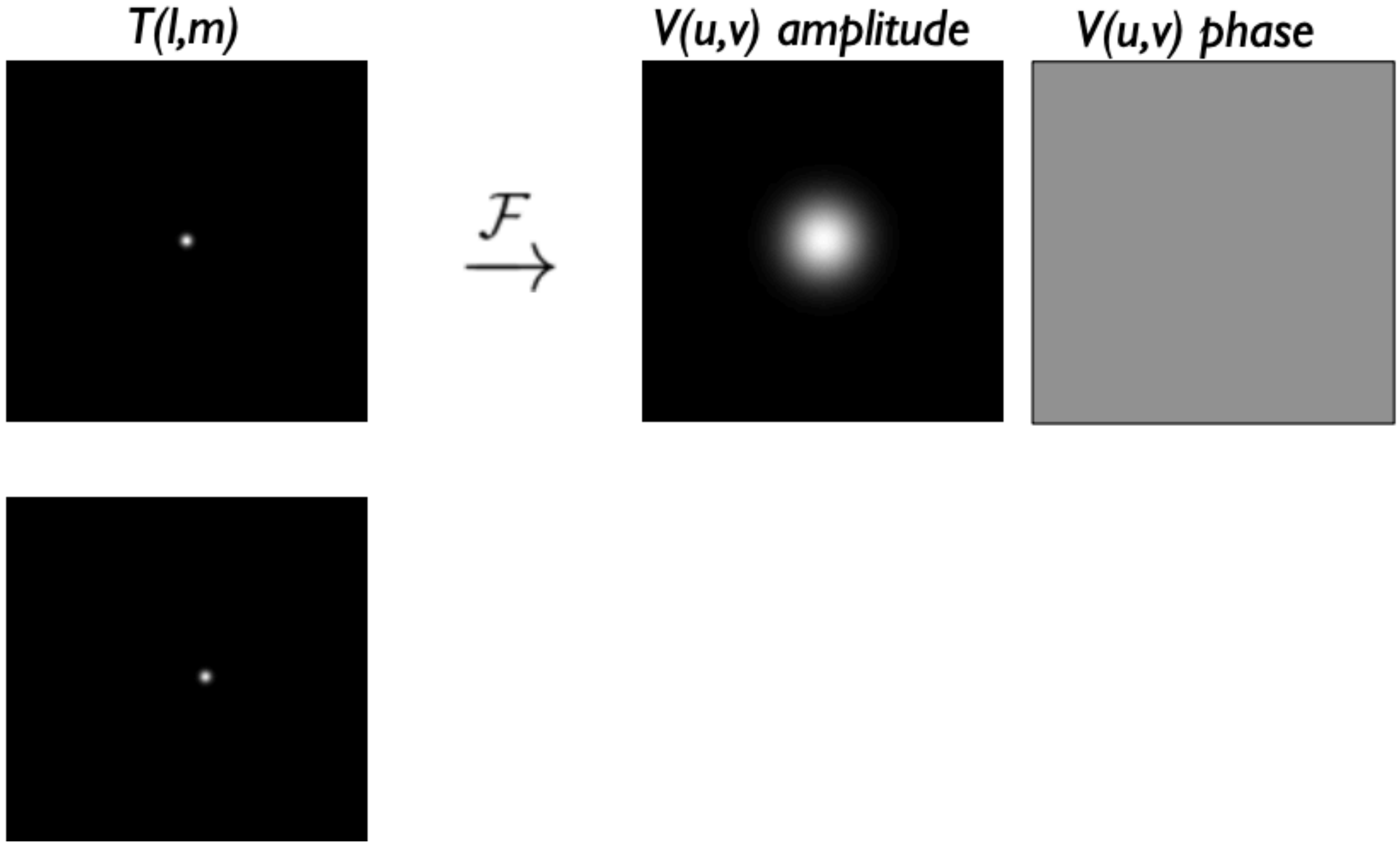

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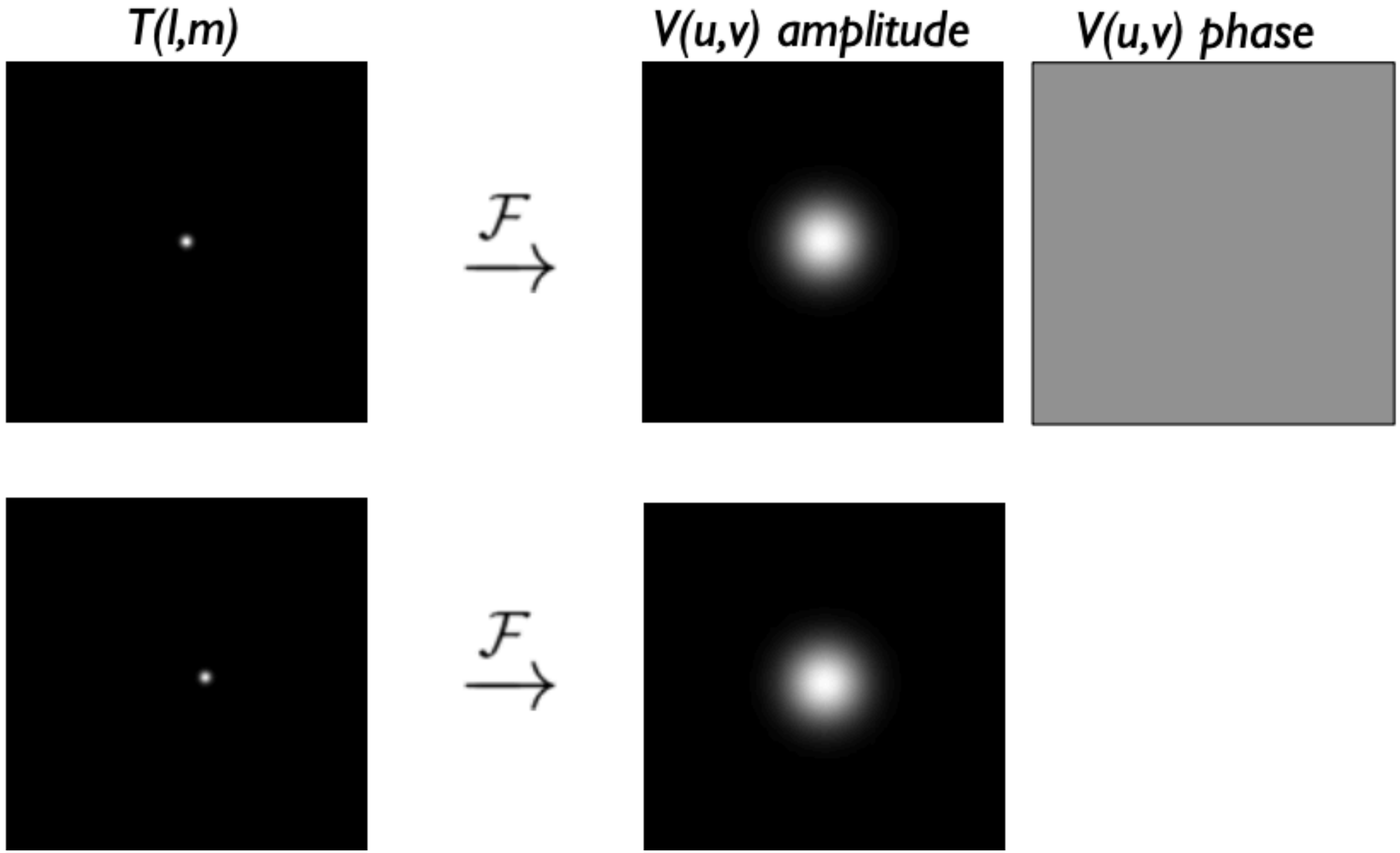

Amplitude and Phase

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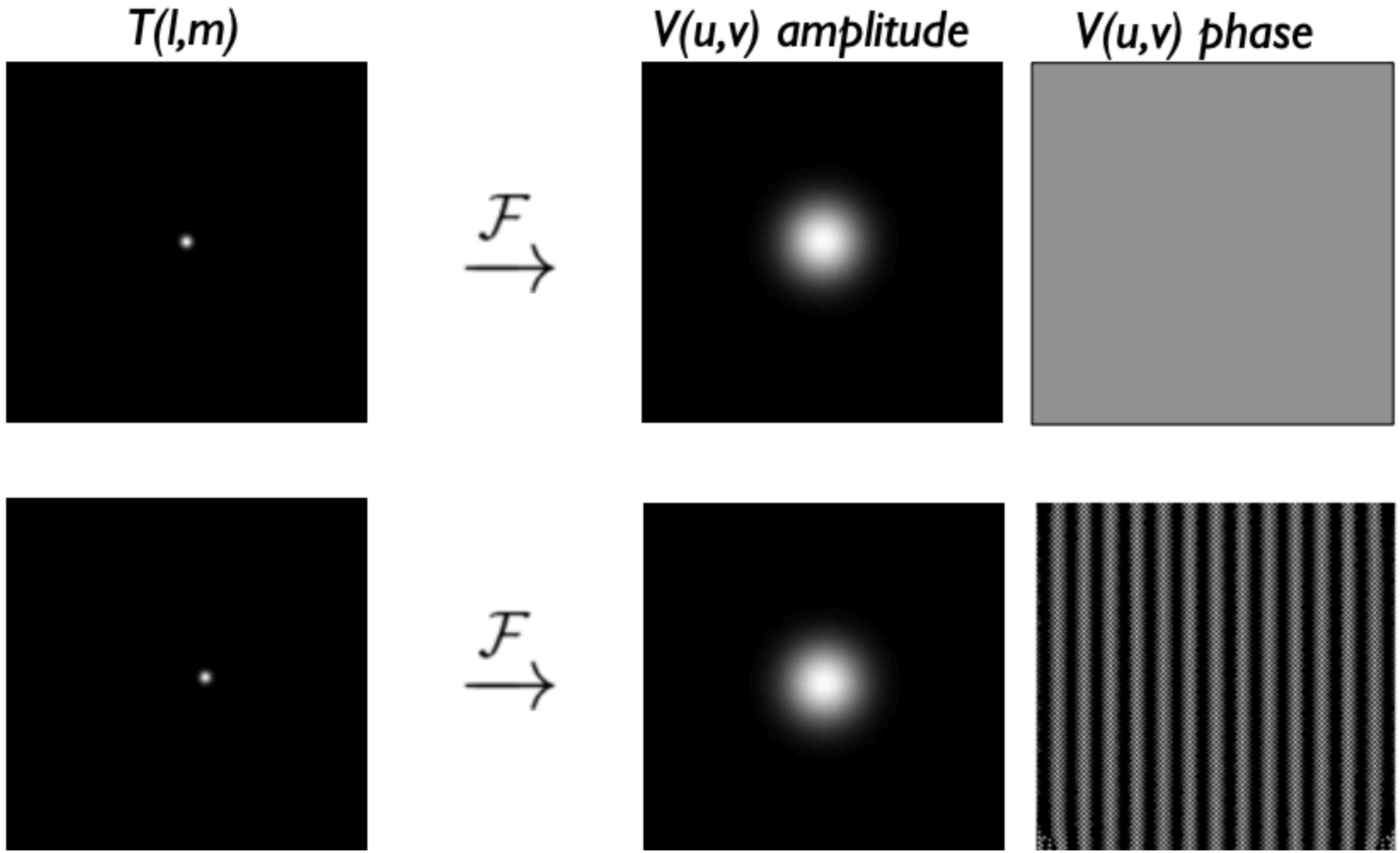

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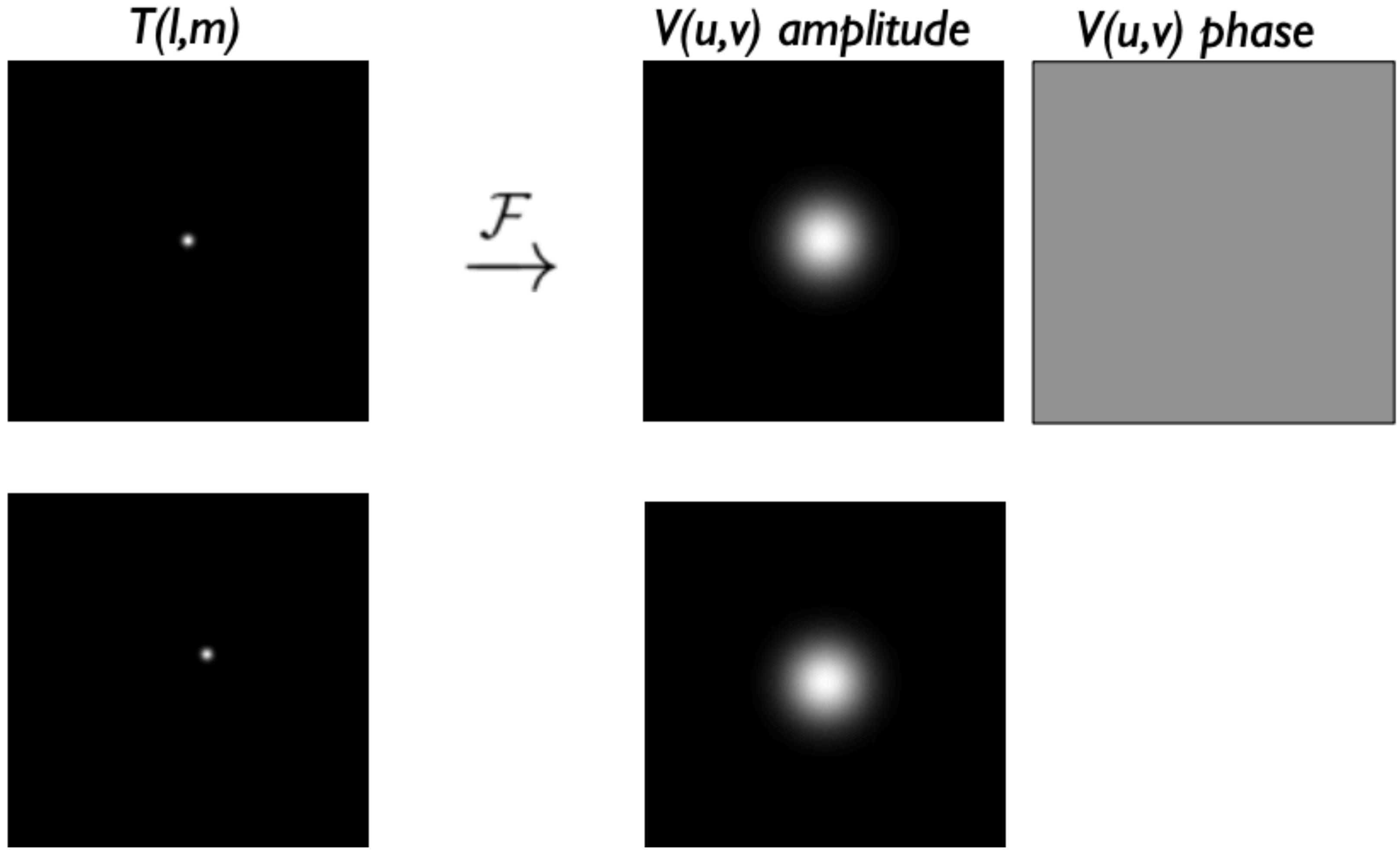

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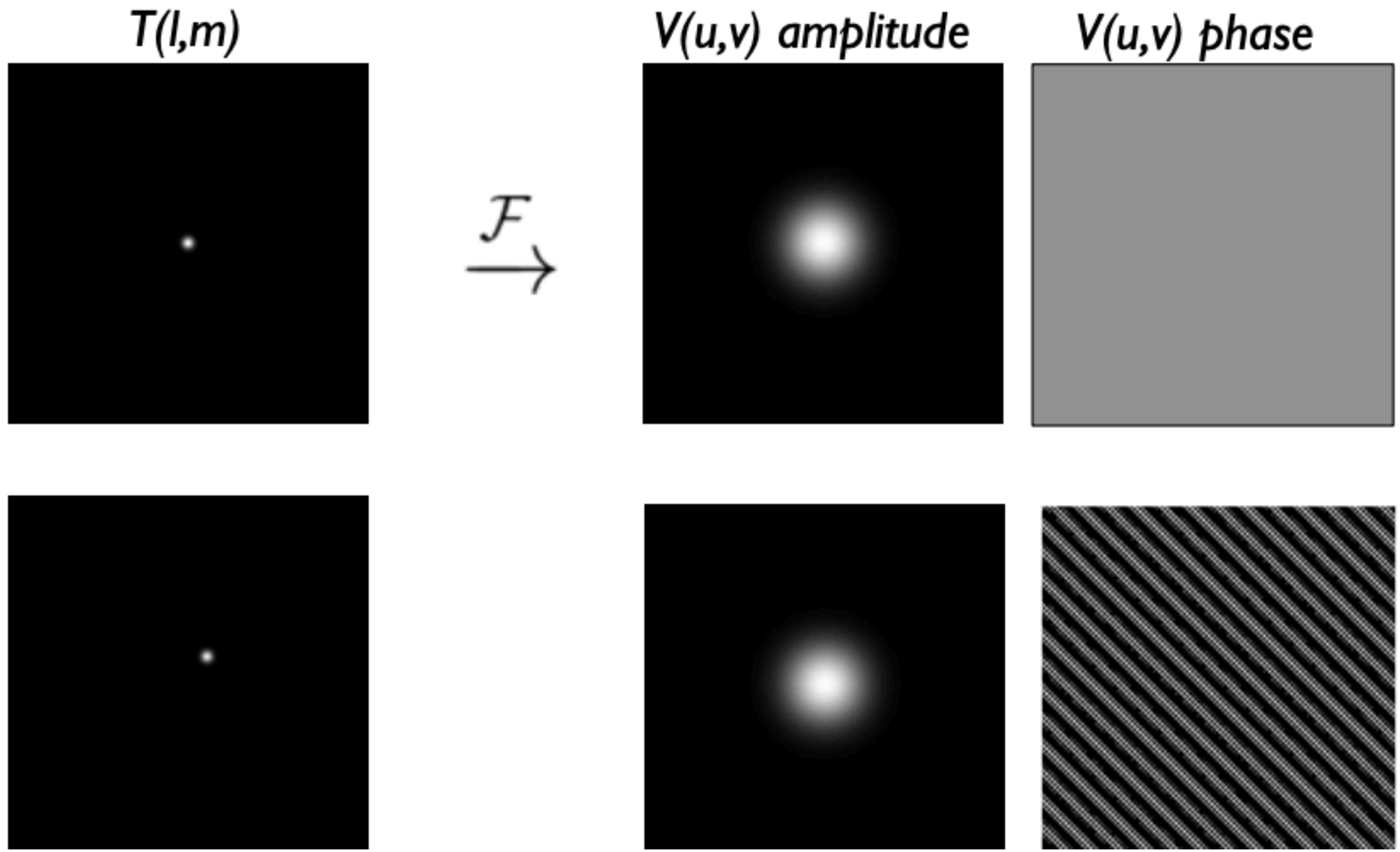
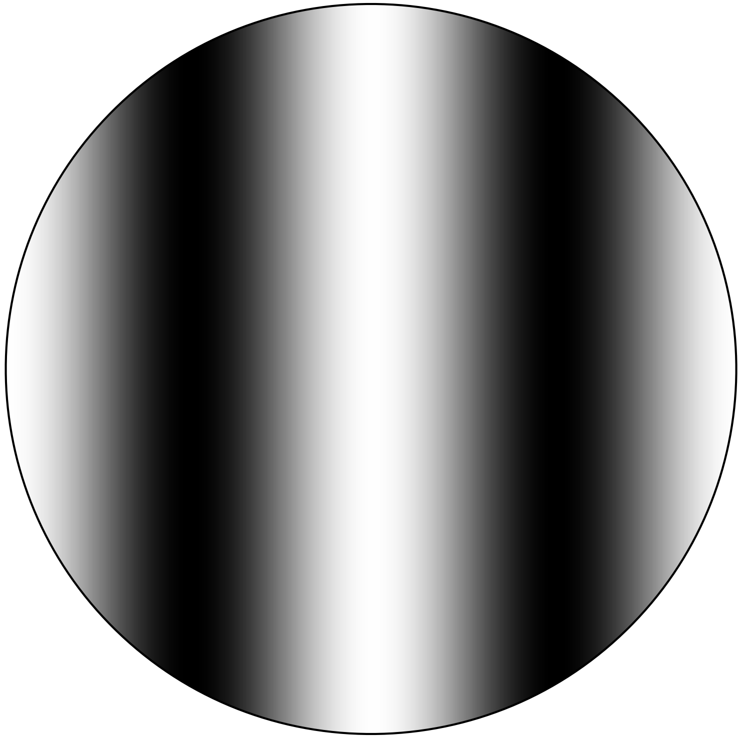
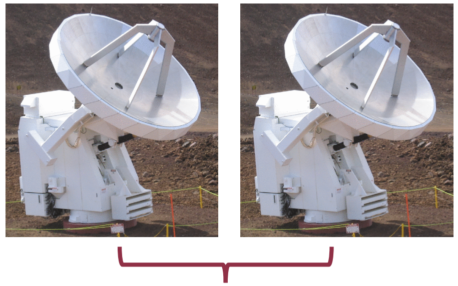


Amplitude and Phase



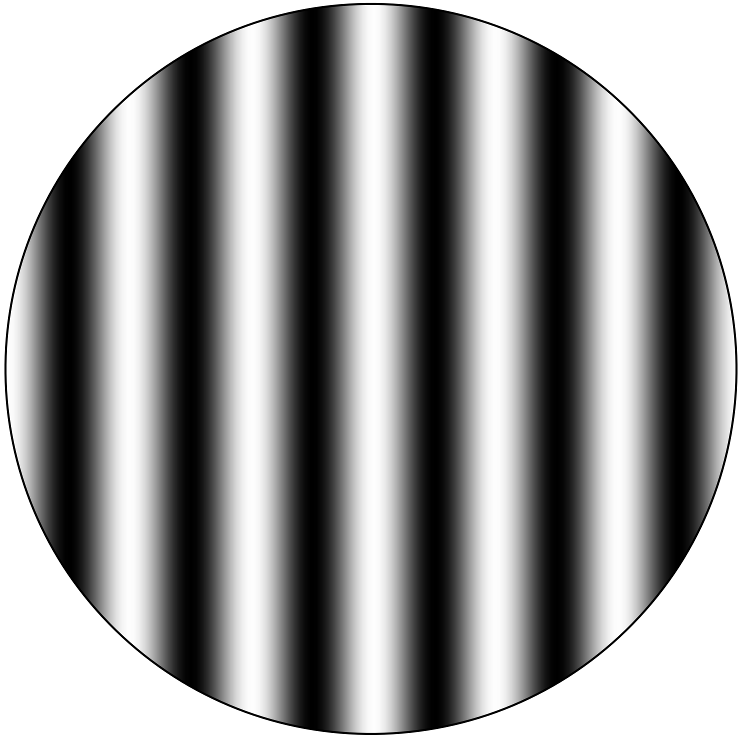
compact source



short baseline

wide fringe pattern

Amplitude and Phase

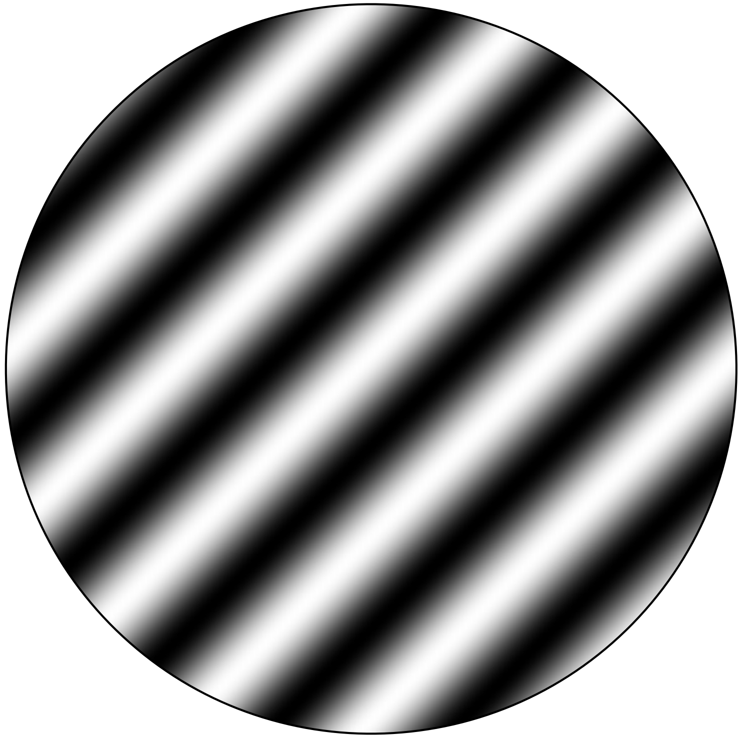


compact source

long baseline

narrow fringe pattern

Amplitude and Phase



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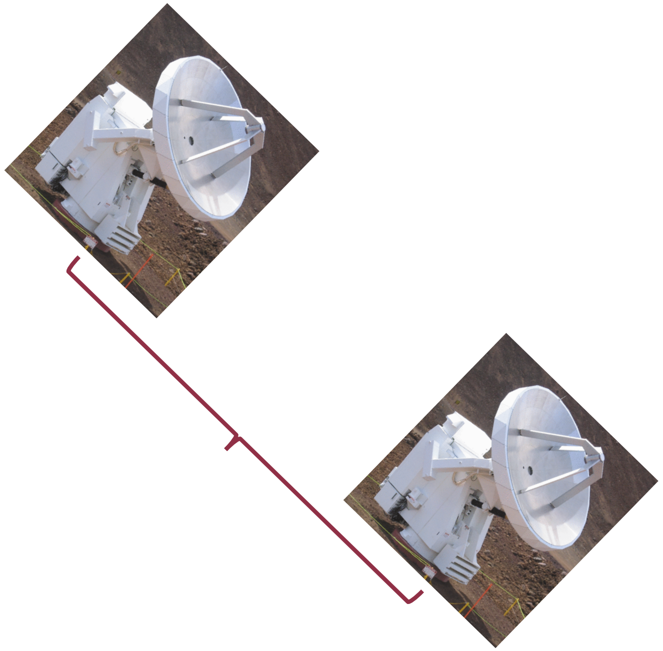
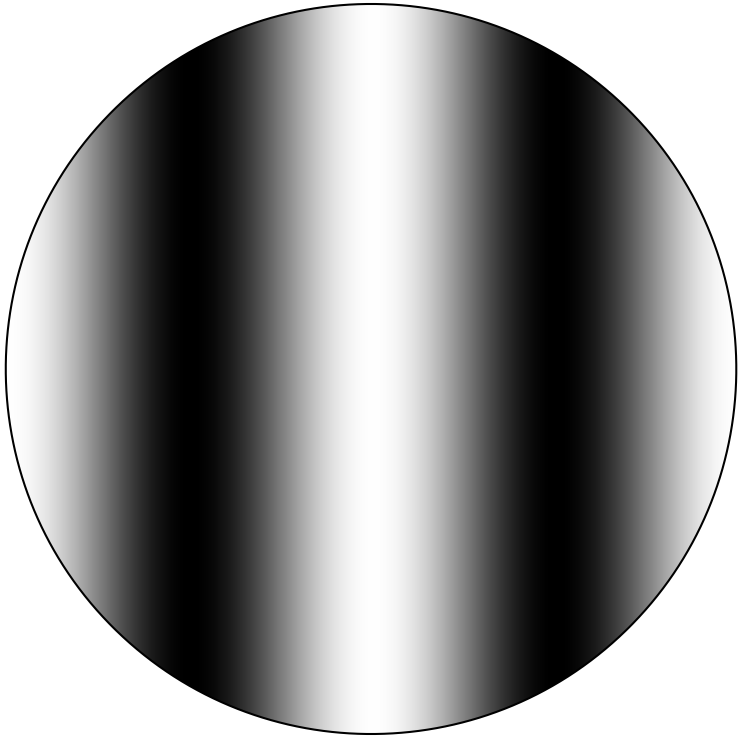


long baseline

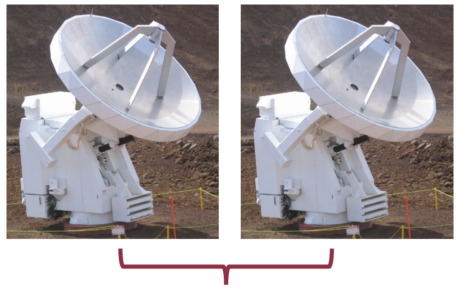
narrow fringe pattern

different orientation

Amplitude and Phase



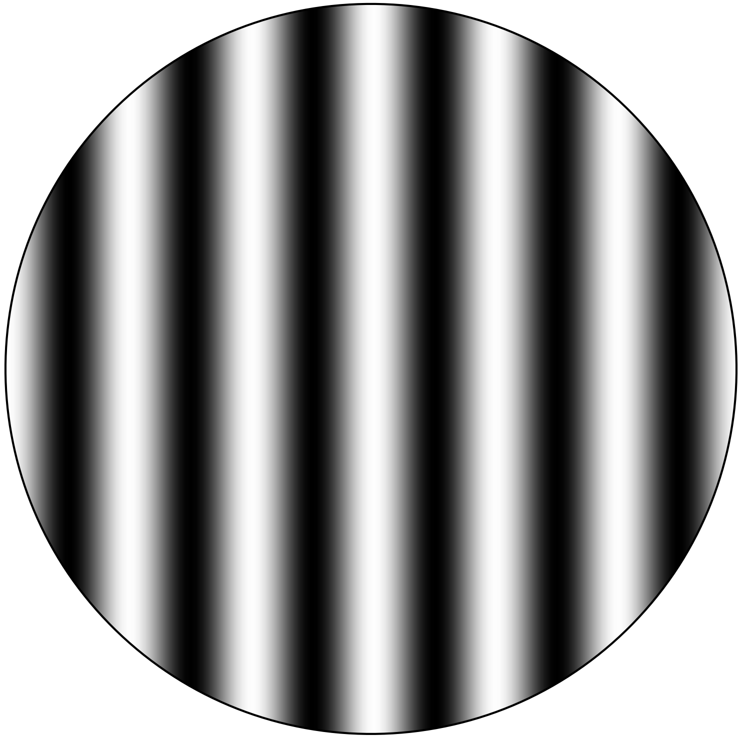
extended source



short baseline

wide fringe pattern

Amplitude and Phase

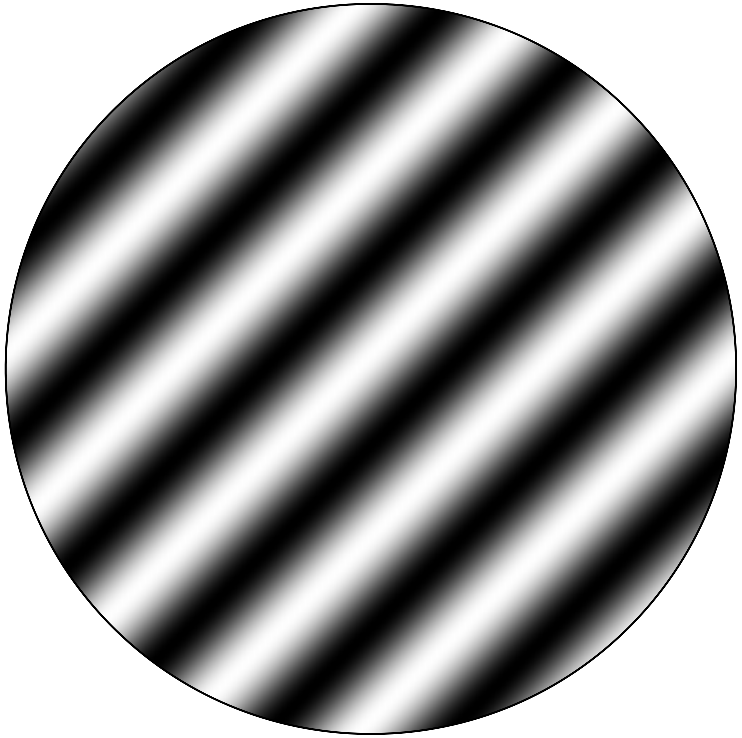


extended source

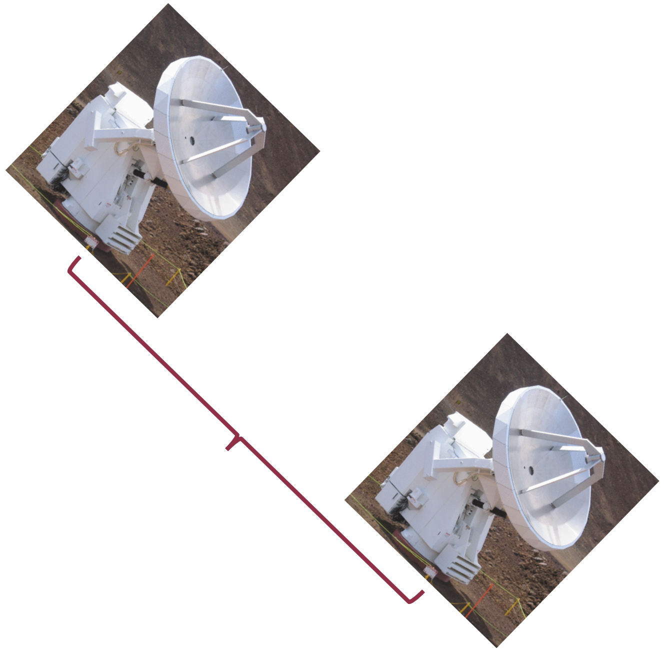
long baseline

narrow fringe pattern

Amplitude and Phase



extended source



long baseline

narrow fringe pattern

different orientation

Aperture Synthesis

basic idea: sample *V(u,v)* at enough *(u,v)* points using distributed small apertures to synthesize a large aperture of size (u_{max},v_{max})

use **more antennas** for more samples

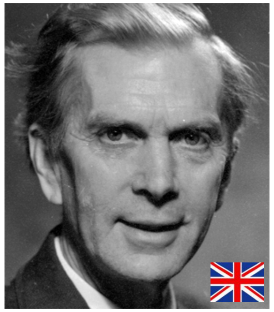
- one pair of antennas = one baseline
  = two *(u,v)* samples at one time
- N antennas = N(N-1) samples at one time
- reconfigure physical layout of N antennas for more
  (as long as source structure doesn't change with time)

use **Earth rotation** for more samples

- baseline length/orientation relative to sky change with time

use **more wavelengths** for more samples

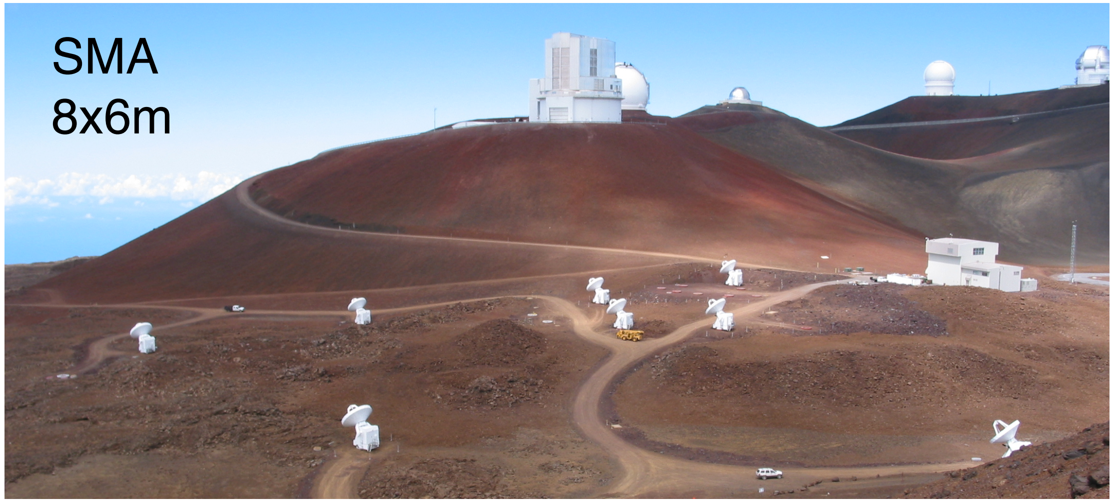
- *u* and *v* are measured in *wavelengths*
- "multi-frequency synthesis" for continuum imaging:
  determine structure at some fiducial wavelength
  and the change with wavelength, e.g. Taylor expansion



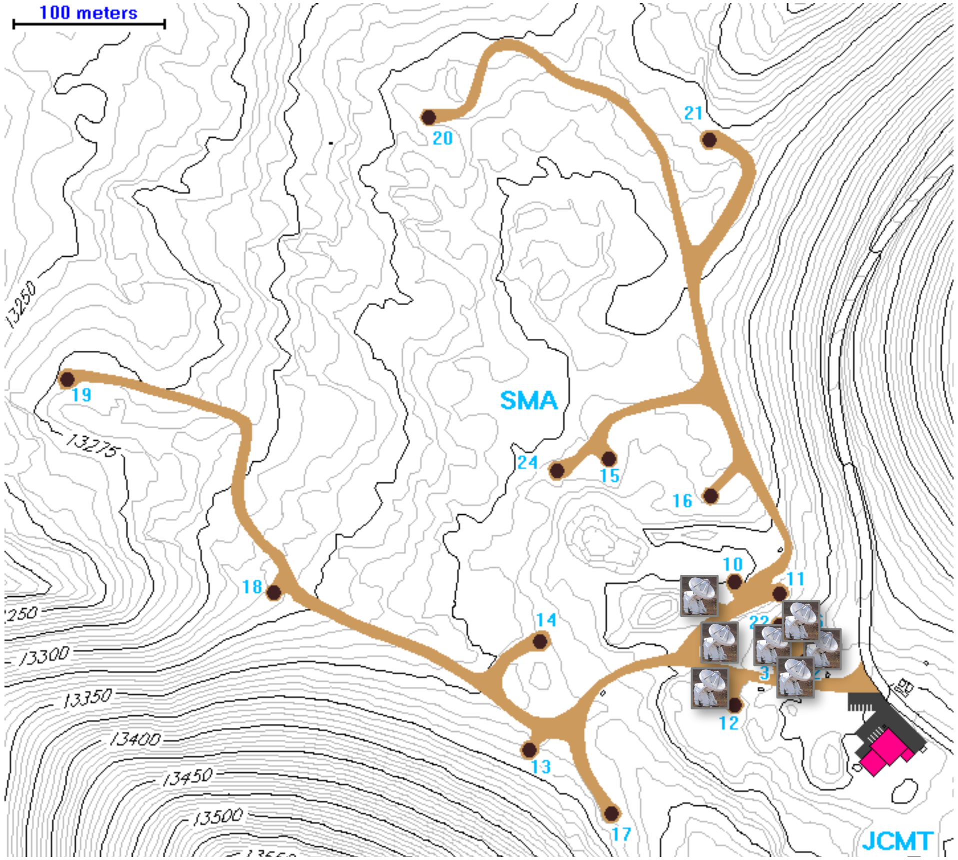
Sir Martin Ryle
1918-1984

1974 Nobel Prize
in Physics

Aperture Synthesis: wavelengths 1mm and below



Example SMA (u,v) Plane Sampling



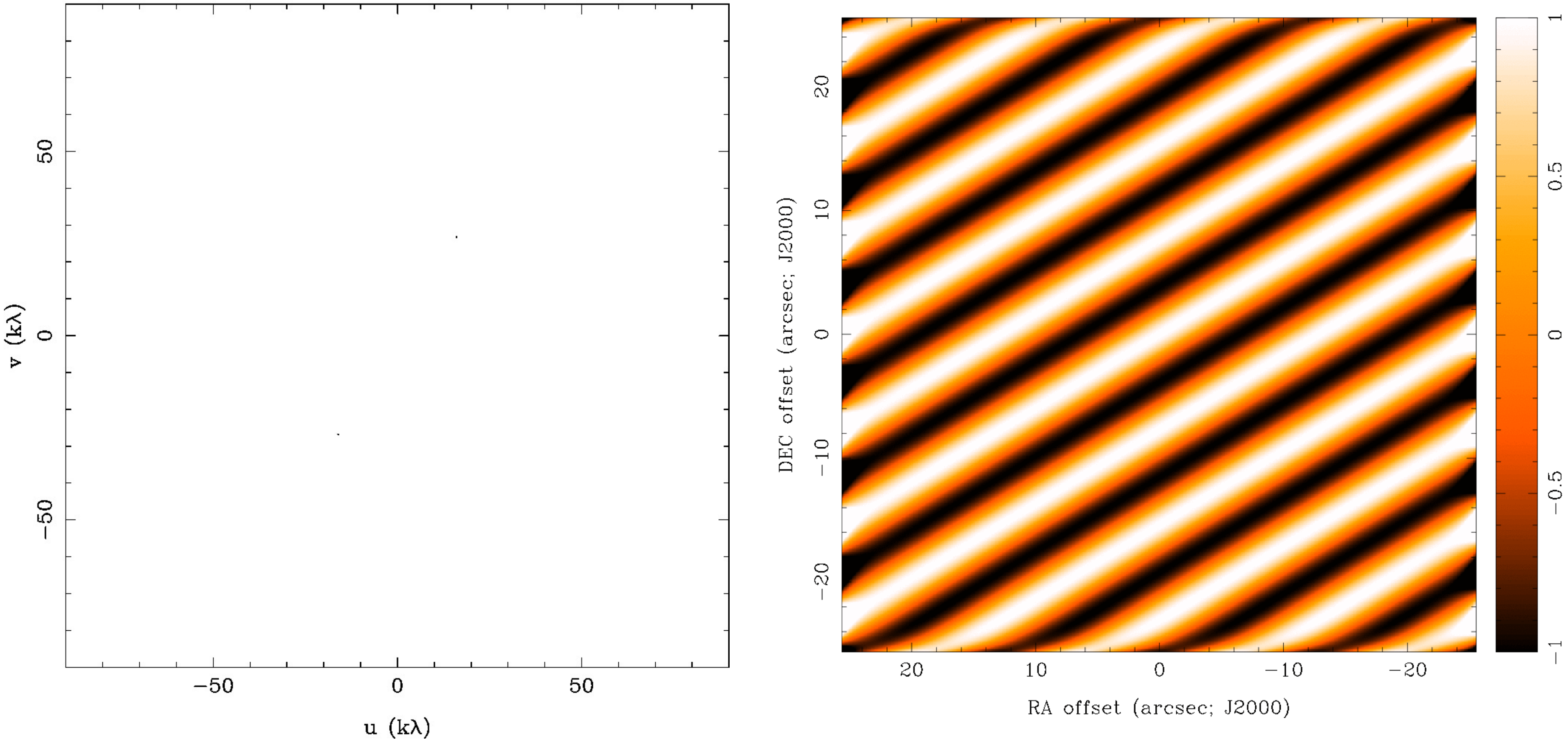
SMA antenna locations on October 23, 2009

ν = 345 GHz

source at dec = + 22 deg

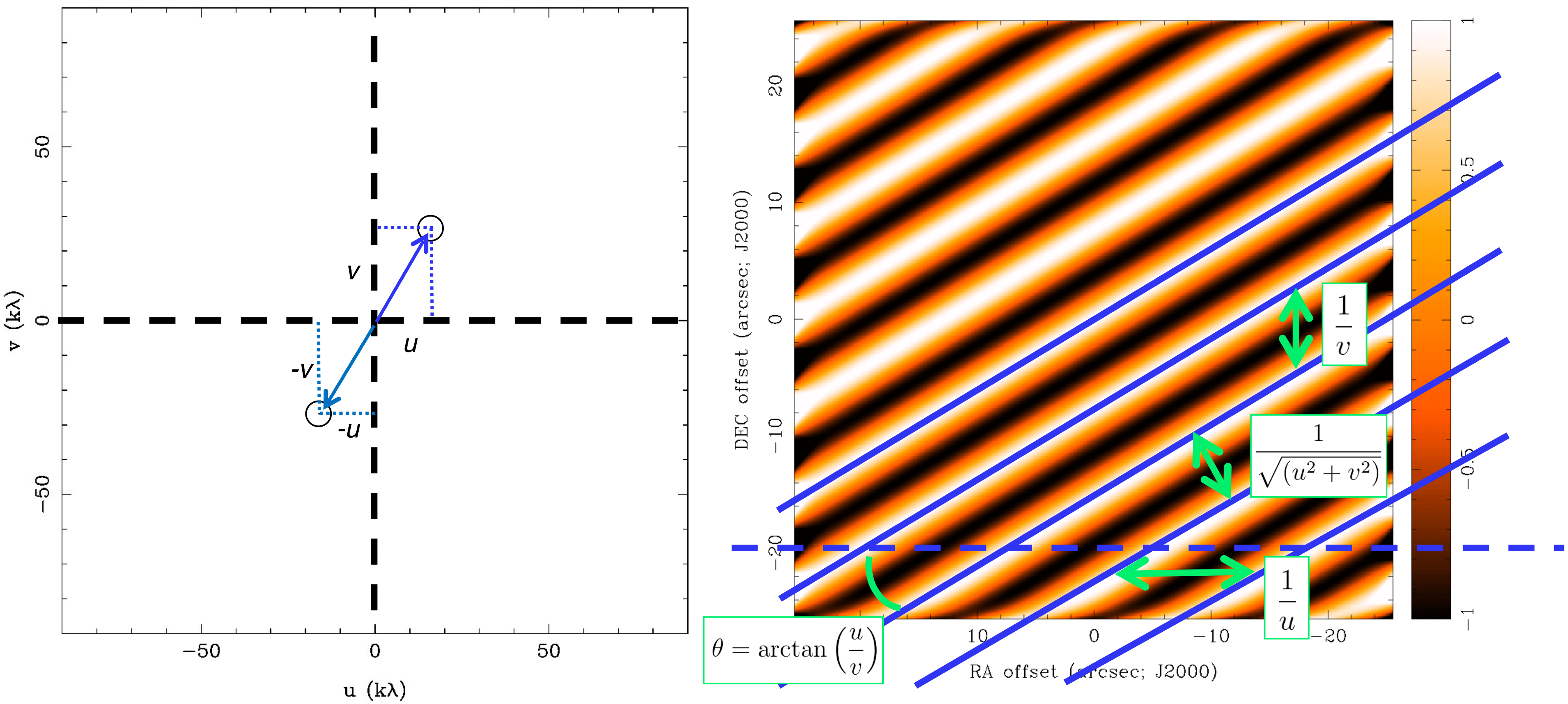
Example SMA (u,v) Plane Sampling

2 antennas, 1 x 30s sample



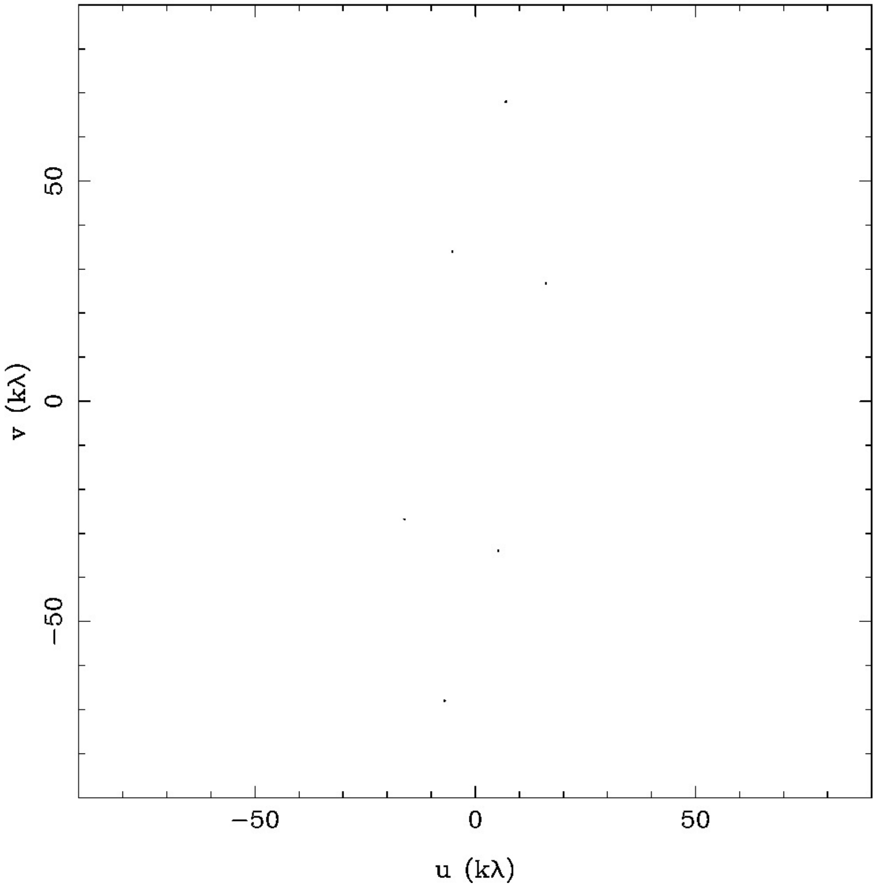
Example SMA (u,v) Plane Sampling

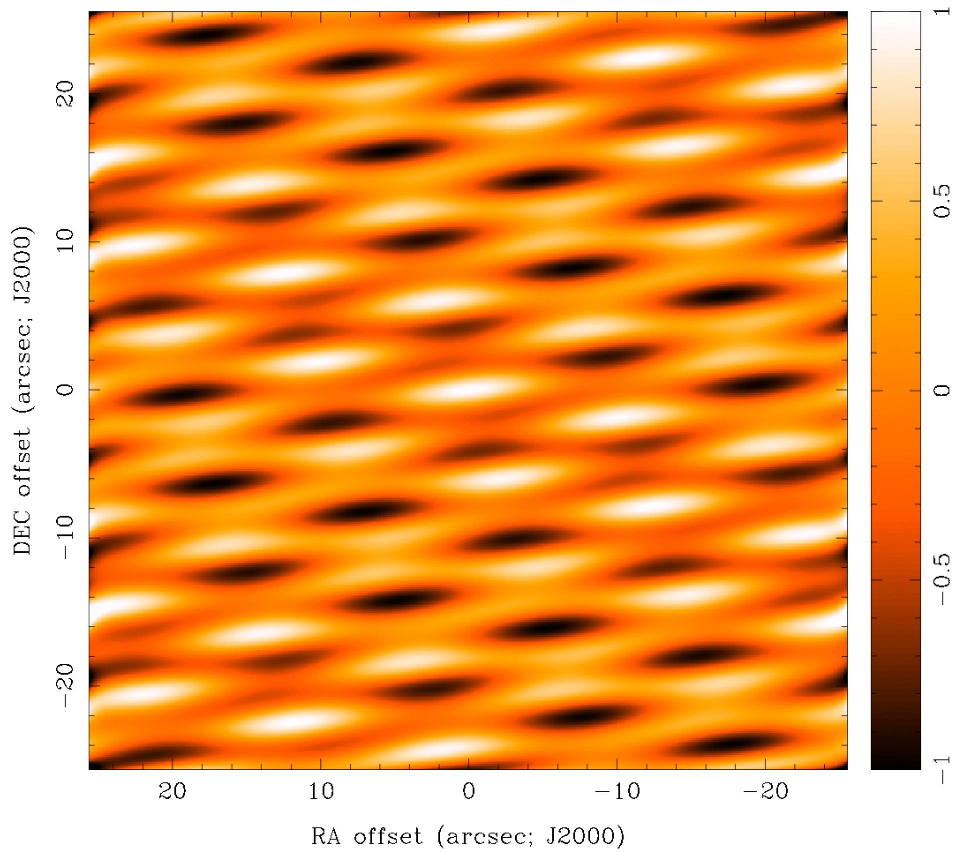
2 antennas, 1 x 30s sample



Example SMA (u,v) Plane Sampling

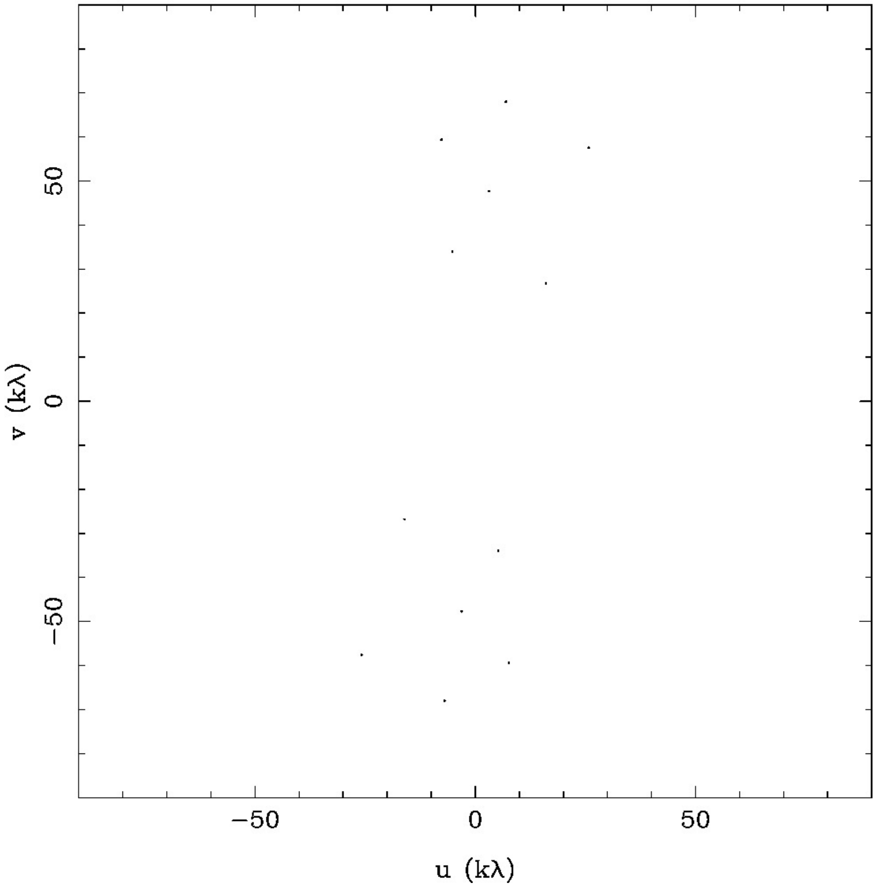
3 antennas, 1 x 30s sample

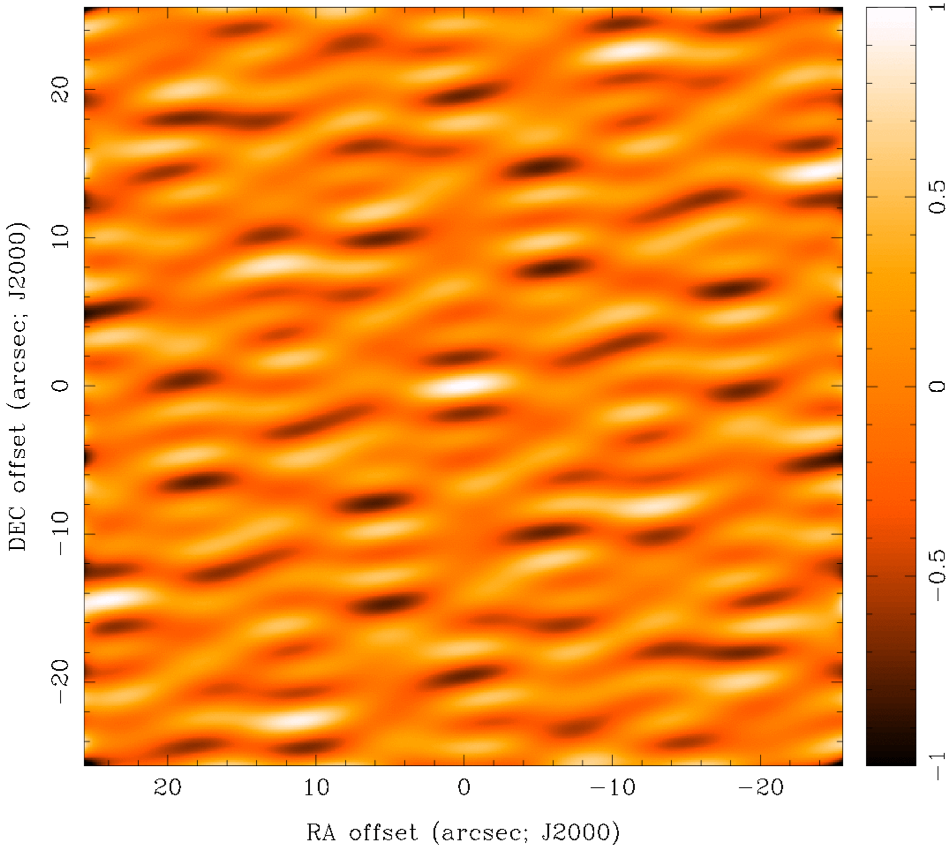




Example SMA (u,v) Plane Sampling

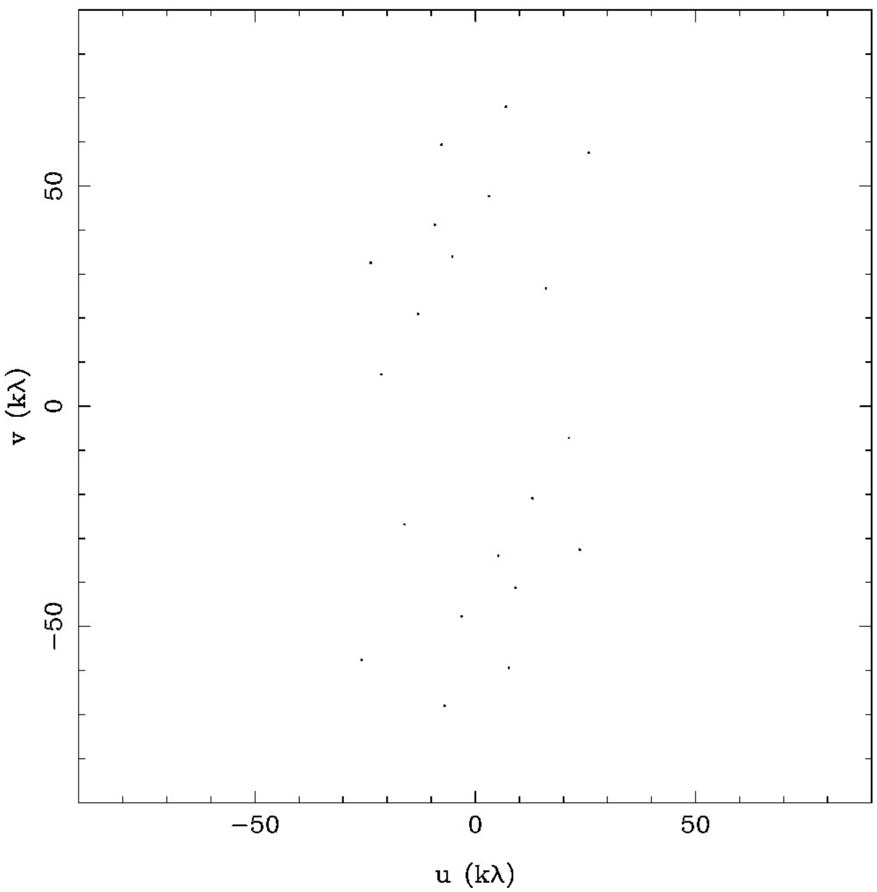
4 antennas, 1 x 30s sample

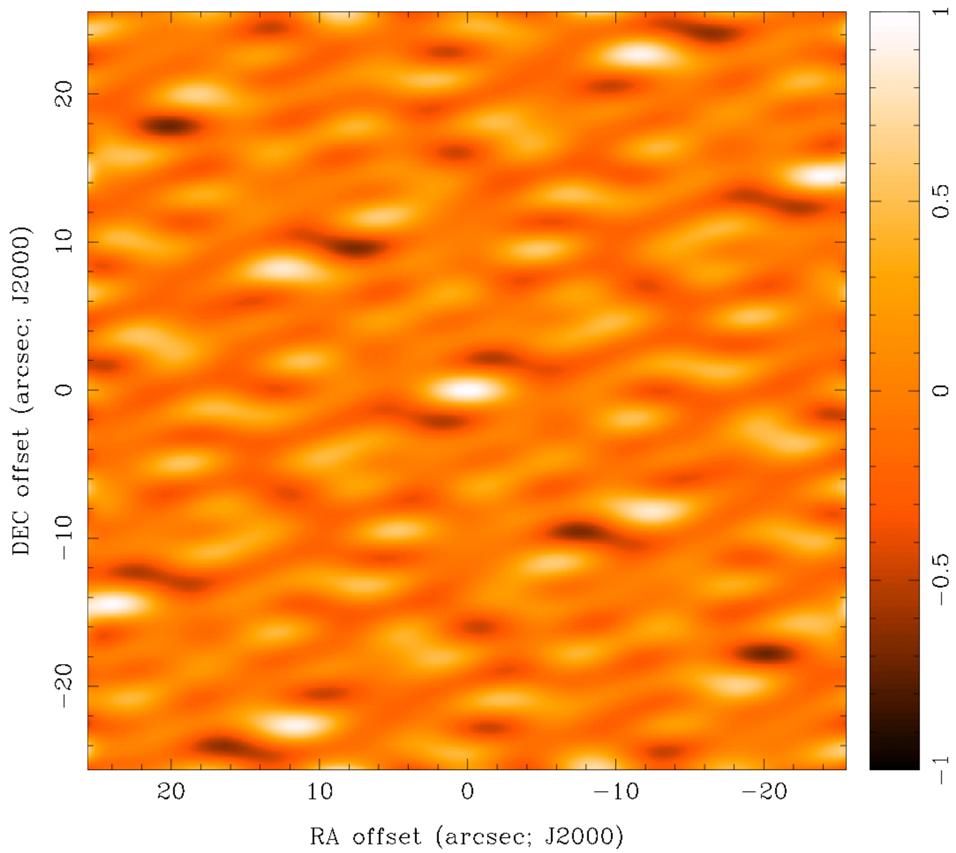




Example SMA (u,v) Plane Sampling

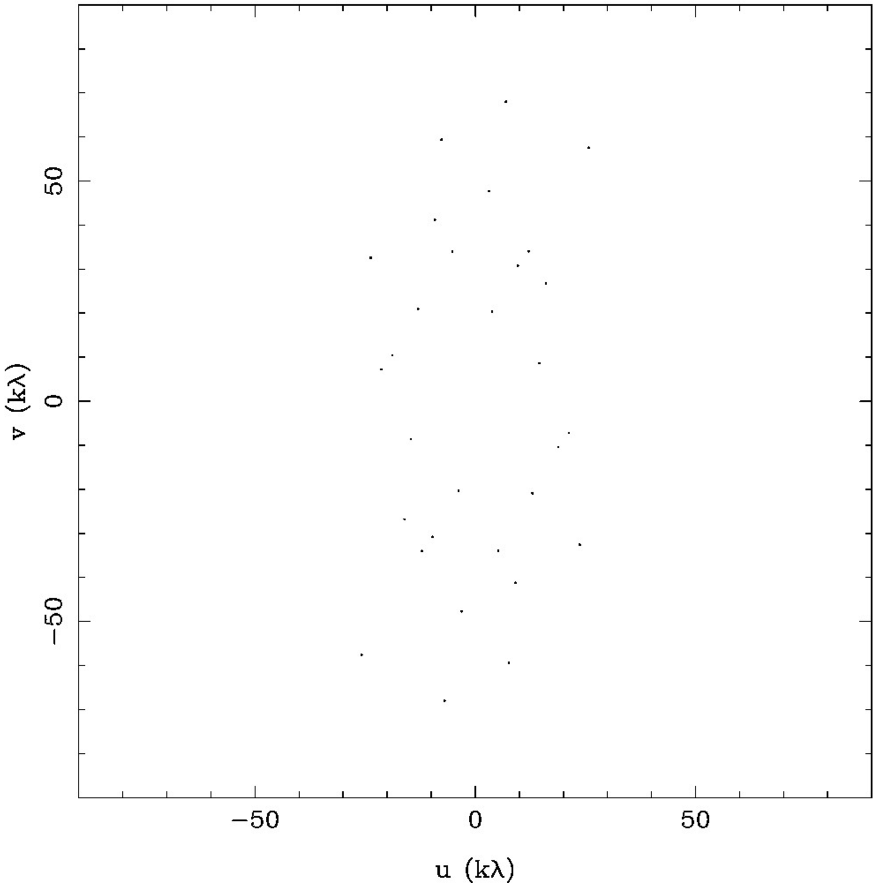
5 antennas, 1 x 30s sample

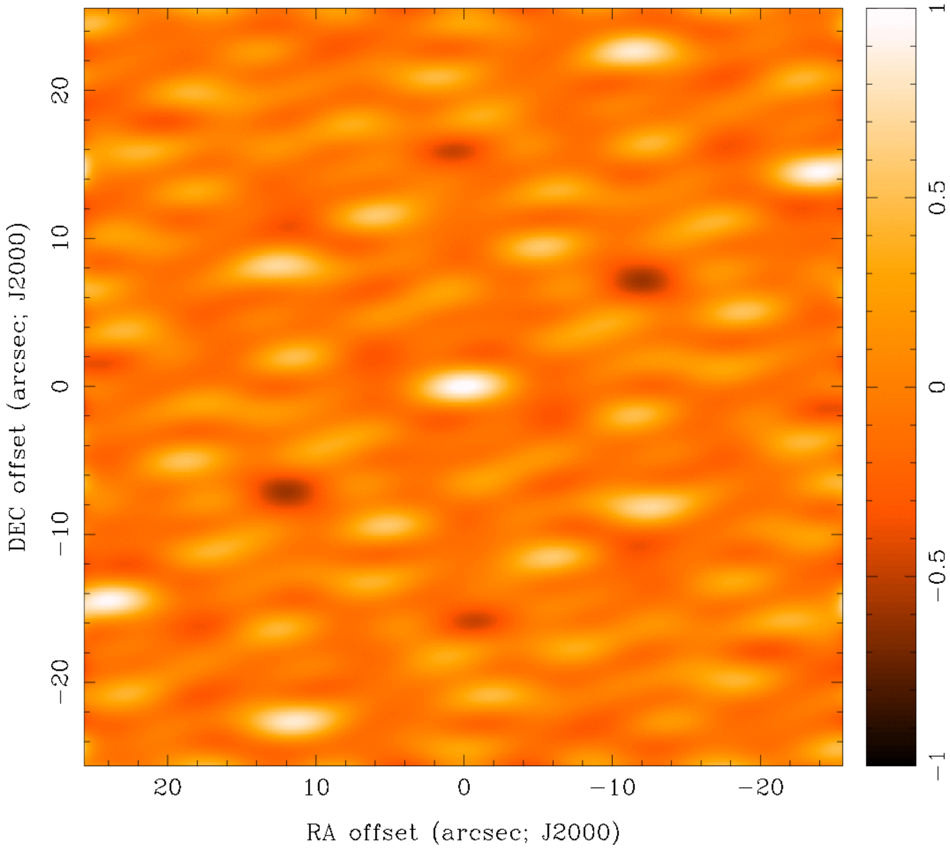




Example SMA (u,v) Plane Sampling

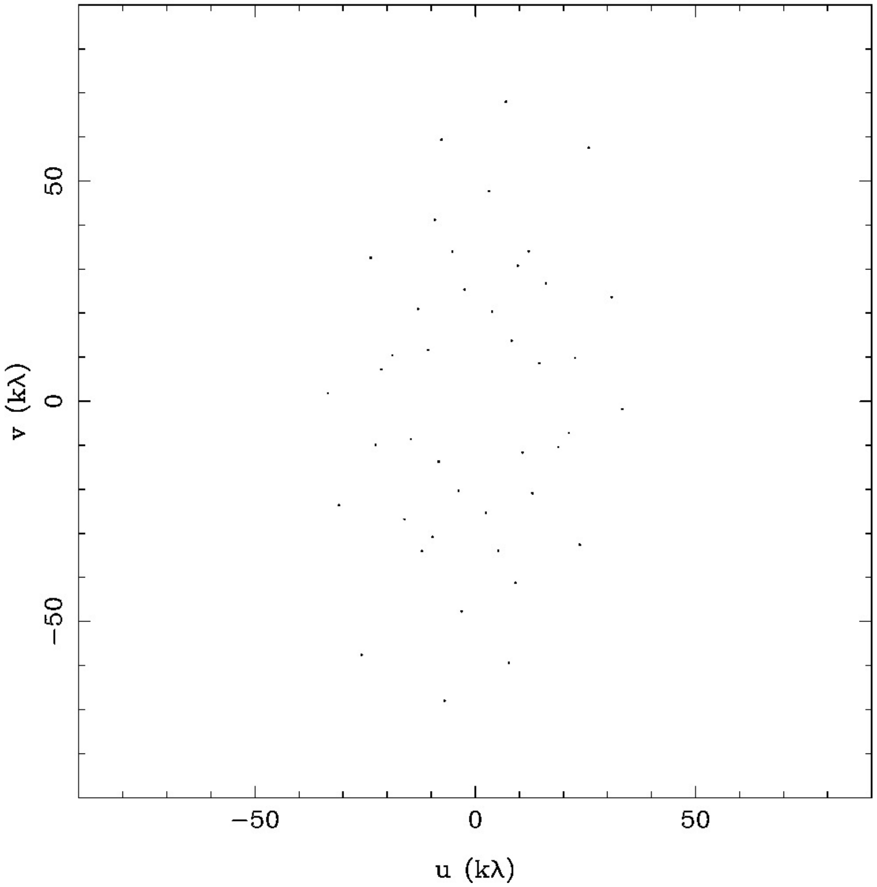
6 antennas, 1 x 30s sample

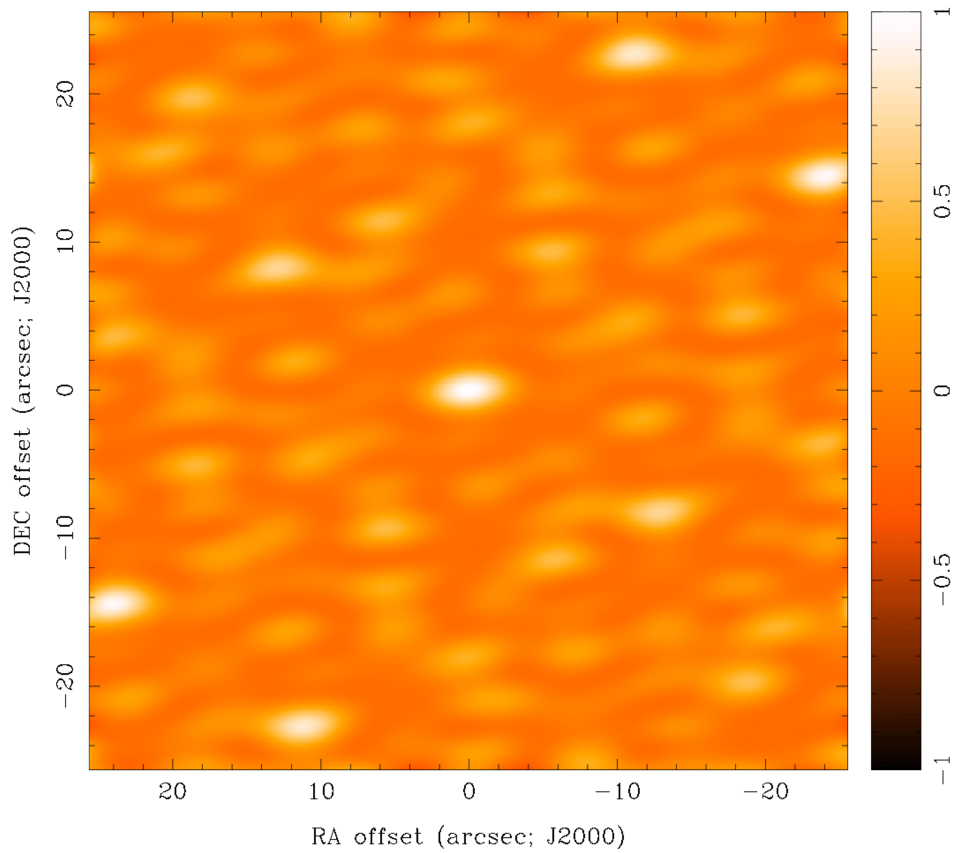




Example SMA (u,v) Plane Sampling

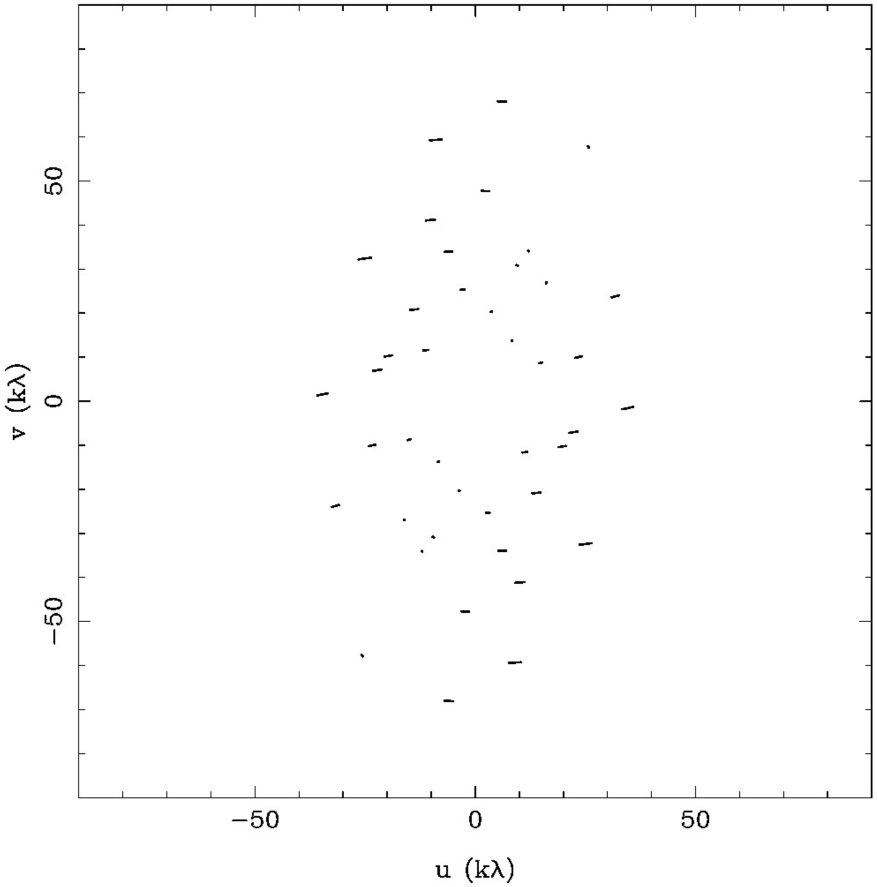
7 antennas, 1 x 30s sample

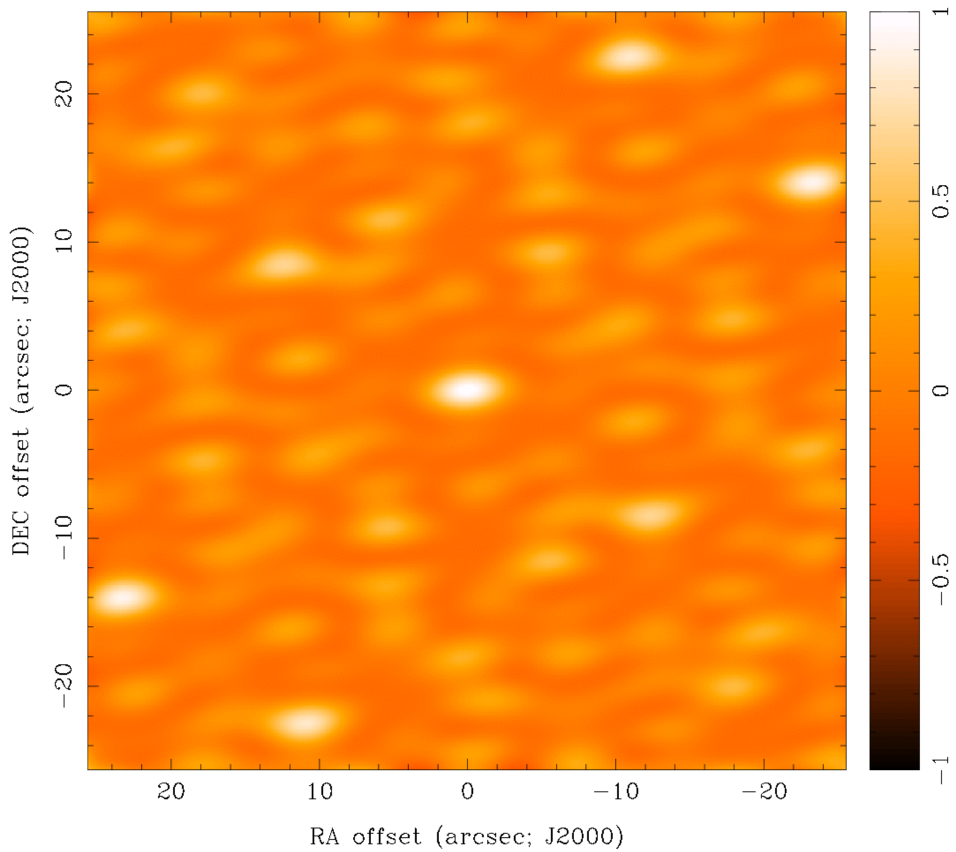




Example SMA (u,v) Plane Sampling

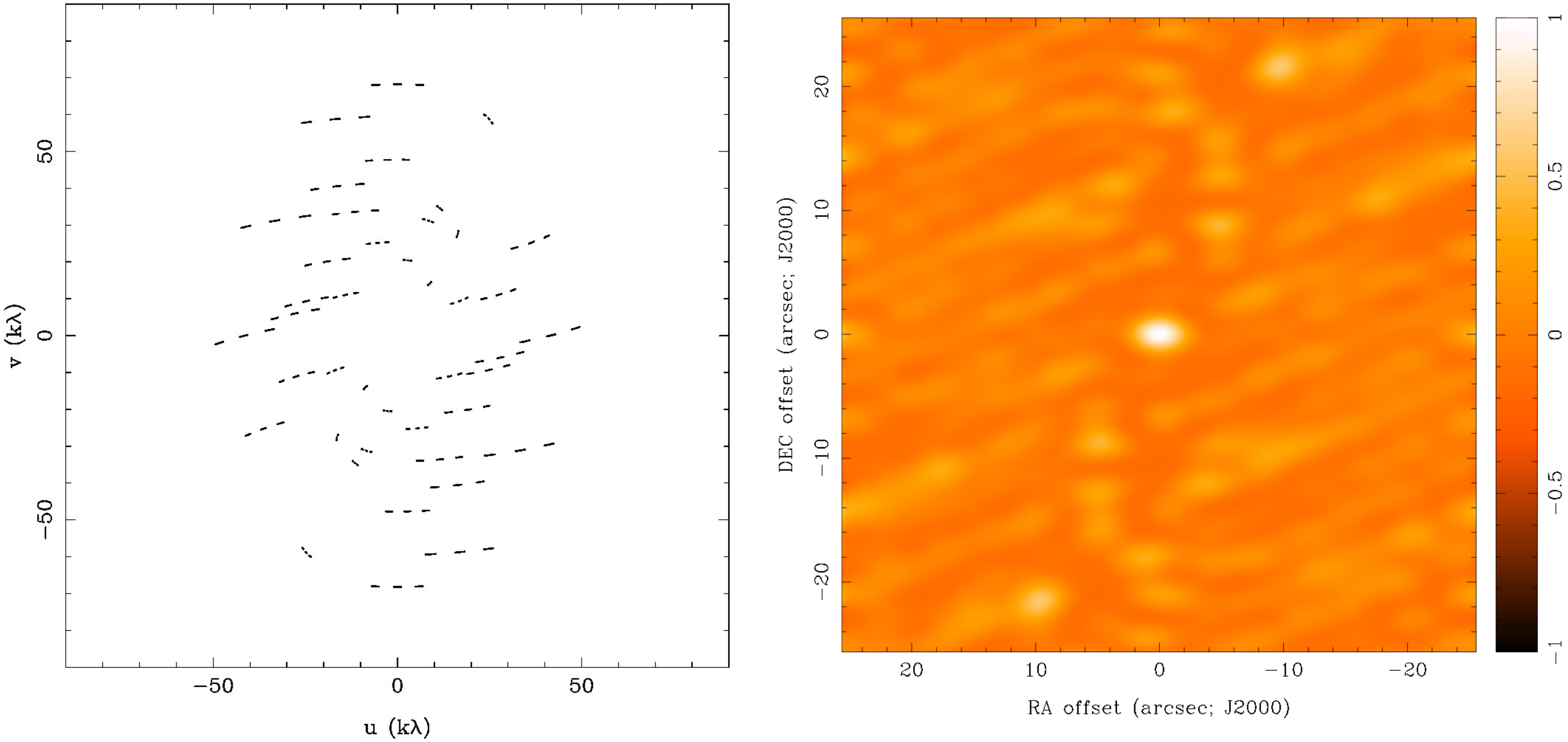
7 antennas, 10 x 30s samples





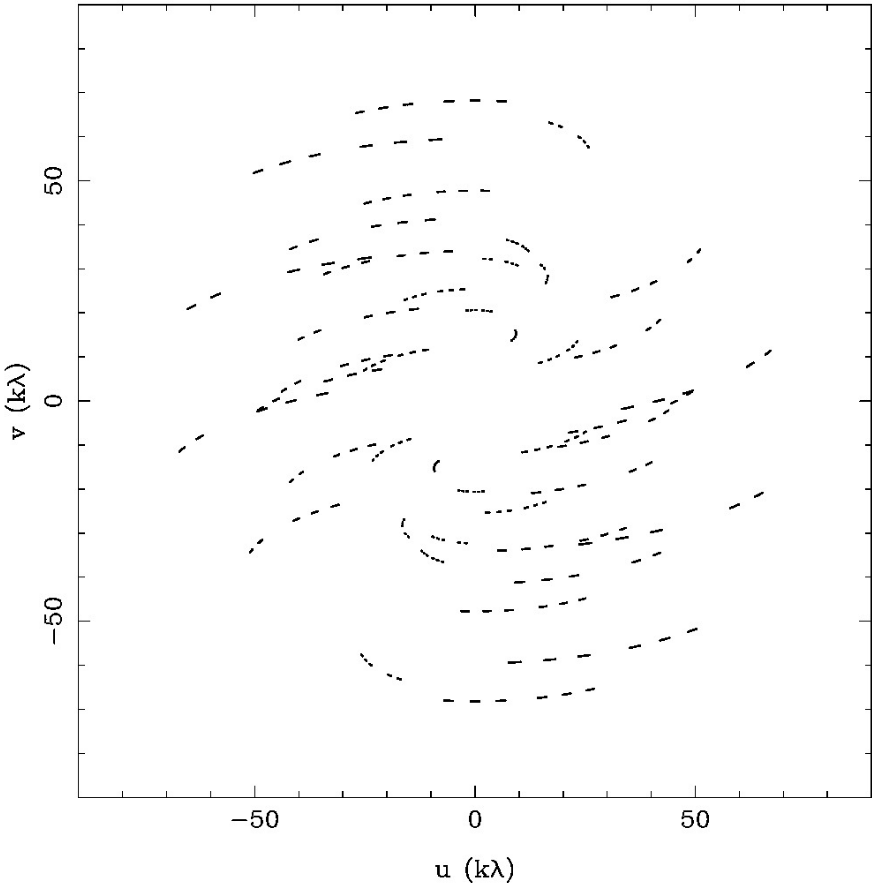
Example SMA (u,v) Plane Sampling

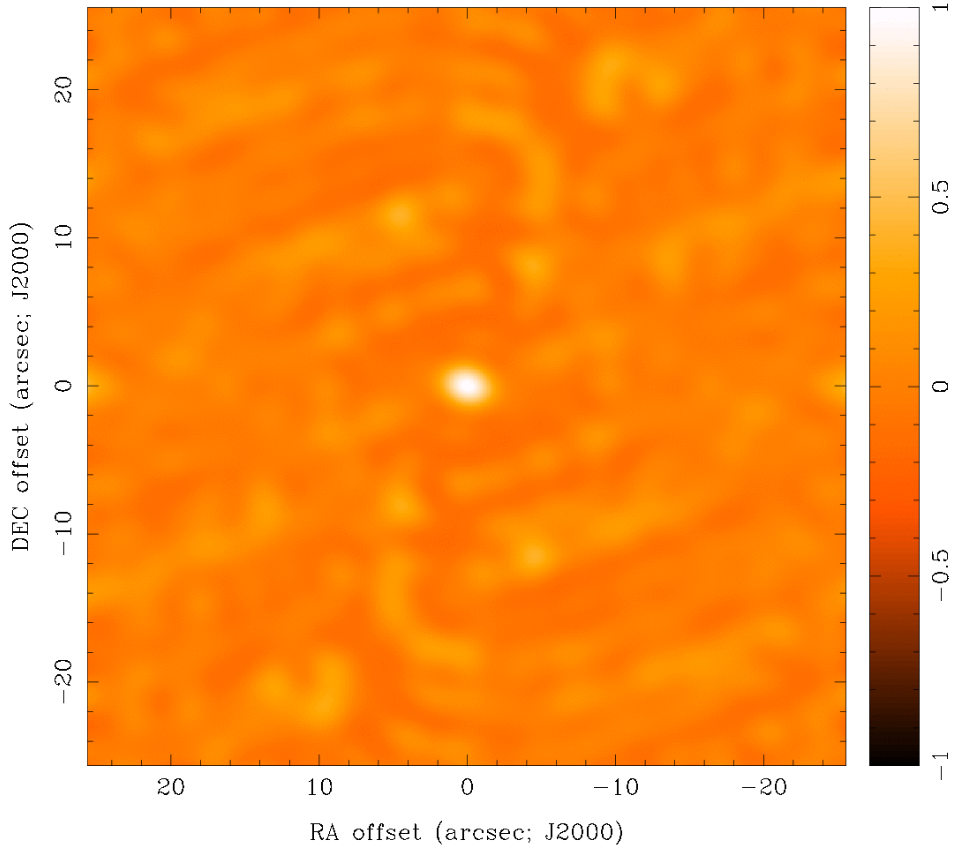
7 antennas, 1 hour



Example SMA (u,v) Plane Sampling

7 antennas, 3 hours





Example SMA (u,v) Plane Sampling

7 antennas, 7 hours