Essentials of Radio and (Sub-)Millimeter Astronomy

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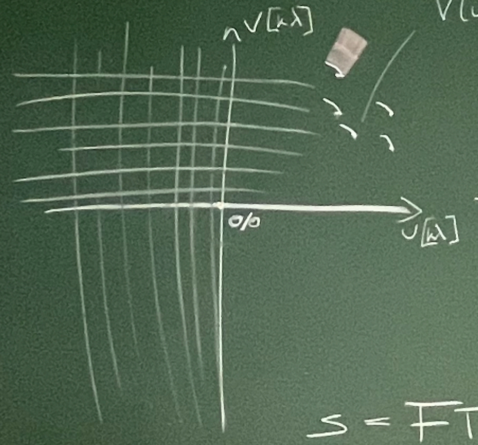
Lecture 8

Basics of Interferometry: Imaging II

slides: Essential Radio Astronomy by NRAO (Condon & Ransom)
Dr. Michael Wise (ASTRON)
Prof. David Wilner (Harvard)
see also SMA summer school 2021: https://lweb.cfa.harvard.edu/sma-school/program/



$V(u,v)$ A, ϕ

$$FT(V(u,v)) = I(l,m)$$

$$FT(V(u,v) \cdot S(u,v)) = FT(V(u,v)) * FT(S(u,v))$$

$$= I(l,m) * s(l,m) = I^D(l,m)$$

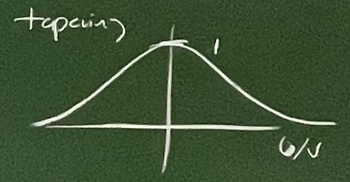
$W(u,v)$ {
- Na weighting : each V : equal weight
- Robust
- Un weighting : each cell : equal weight

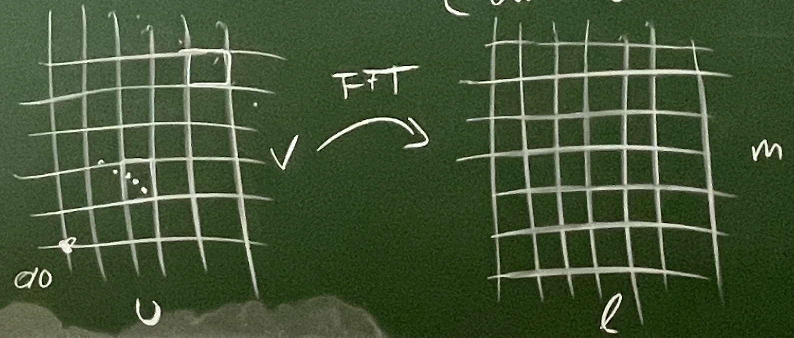
$$s = FT(S(u,v))$$

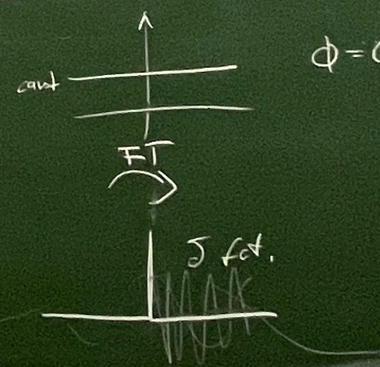
← $W(u,v)$

S : sampling fct.

s : dirty beam (PSF)

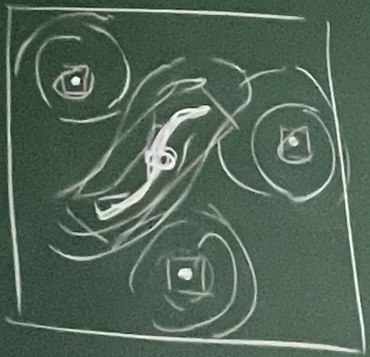






$\phi = 0$

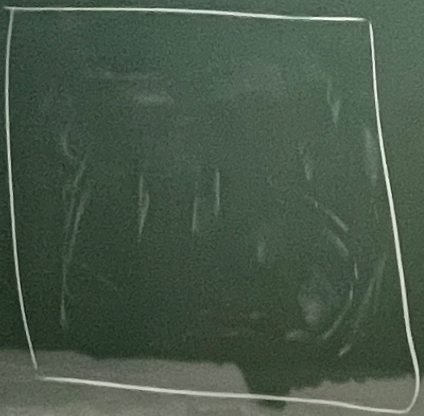
clean algorithm

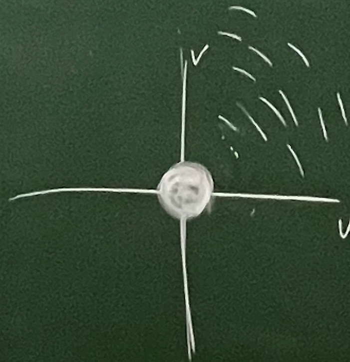


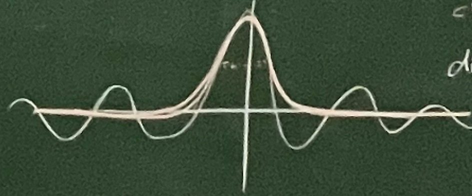
1) brightest pix
2) subtracts [PSF from (1)] × gain factor (0.1)
   → records pos [l,m], A → clean table
3) → 1)
   [until noise (3σ) is reached]

	l	m	A
1	:	:	:
2	:	:	:
3	:	↓	:

clean comp…







Beyond the Dirty Image

- to keep you awake at night…
  - ∃ an infinite number of *I(l,m)* compatible with sampled *V(u,v)*
  - also noise → undetected and corrupted structure in *I(l.m)*

Deconvolution

use non-linear techniques to interpolate/extrapolate *V(u,v)* samples into unsampled regions of *(u,v)* plane to find a *plausible* model of *I(l,m)*

- there is no unique prescription to extract an optimum model of *I(l,m)*
- requires *a priori* assumptions about *I(l,m)* to pick a plausible "invisible" distributions that fill the unsampled parts of *(u,v)* plane

Deconvolution in Radio Astronomy

Two most common deconvolution algorithms

- **clean** (Högbom, J.A 1974, A&AS, 15, 417)
  - a priori assumption: *I(l,m)* can be represented by point sources
  - many variants developed to improve computational efficiency and performance on extended structure

- based on this: **multi-scale clean**

- **maximum entropy** (Gull, S.F. & Daniell, G.J 1978, Nature, 272, 686)
  - special case of forward modeling that minimizes an objective function that includes the data and a regularization term
    (example of more general "regularized maximum likelihood" method)
  - a priori assumption for max entropy: *I(l,m)* is smooth and positive
  - vast literature about the deep meaning of entropy (Bayes theorem)

A very active research area! (see, e.g. EHT M87 imaging paper)

Clean Algorithm

- original version by Högbom is purely image based
  0. make dirty image and dirty beam (2x larger than the dirty image)
  1. find peak of dirty image
  2. subtract dirty beam centered at peak location, scaled by peak value and a loop gain factor (typically ~ 0.1) to make residual dirty image
  3. add subtracted peak to sky model (clean component list or image)
  4. if residual dirty image peak > stopping criterion, then goto step 1.

- Clark clean
  - use small patch of beam to improve speed, subtract clean components from gridded visibilities at once using FFT
- Cotton-Schwab clean
  - like Clark clean, but subtract clean components from ungridded visibilities and repeat entire image process to create residuals

Clean Algorithm Parameters

- stopping criterion
  - peak < threshold = multiple of theoretical rms noise
  - peak < threshold = fraction of dirty map maximum (useful if strong sources prevent a sensible noise threshold from being reached)
  - maximum number of clean components reached (no justification)
- loop gain parameter
  - values ~ 0.1 - 0.3 typically give good results
  - lower values can help with recovering smoother structures
- finite support
  - easy to include *a priori* information about where to search for clean components in the dirty map (image “masks” or clean “boxes”)
  - useful, often essential for best results, but potentially dangerous
  - can be an arduous manual process; automatic algorithms OK

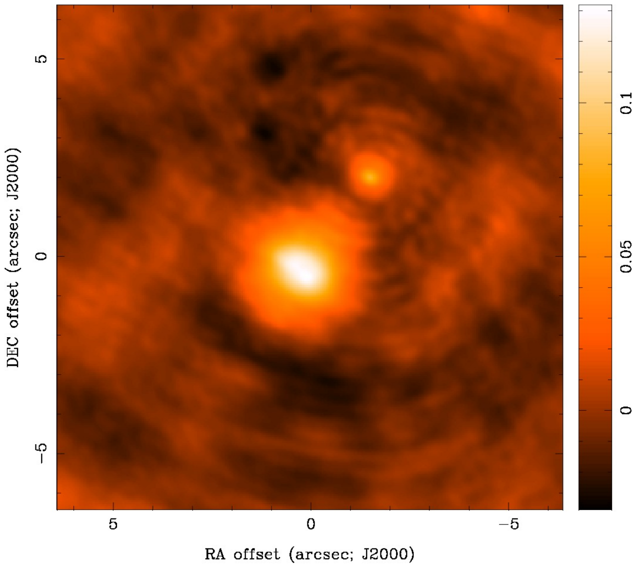
Clean: the restored image

- last step is to create a final "restored" image
  - make model image with all point source clean components
  - convolve point source model image with a "clean beam", an elliptical Gaussian fit to the main lobe of the dirty beam
  - add back residual dirty image with noise and structure below threshold
- the restored image is an estimate of the true sky brightness *I(l,m)* that includes a representation of thermal noise and flux not deconvolved
- **units of the restored image are (mostly) Jy per clean beam area**
  - a correction can be made to mitigate the problem that the clean model and residual image have mismatched units (Jorsater & can Morsel 1995)

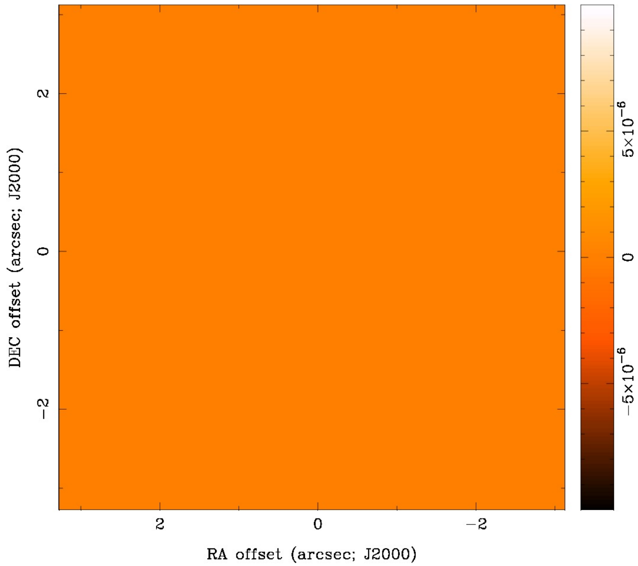
Clean example

initialize

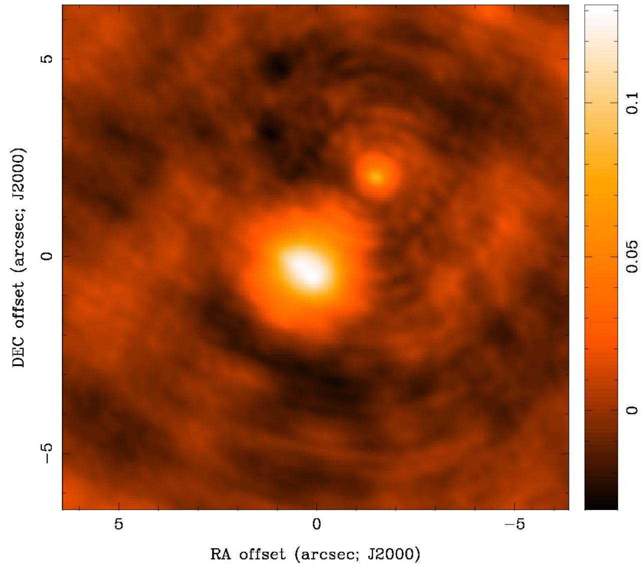
$T^D(l,m)$



0 clean components

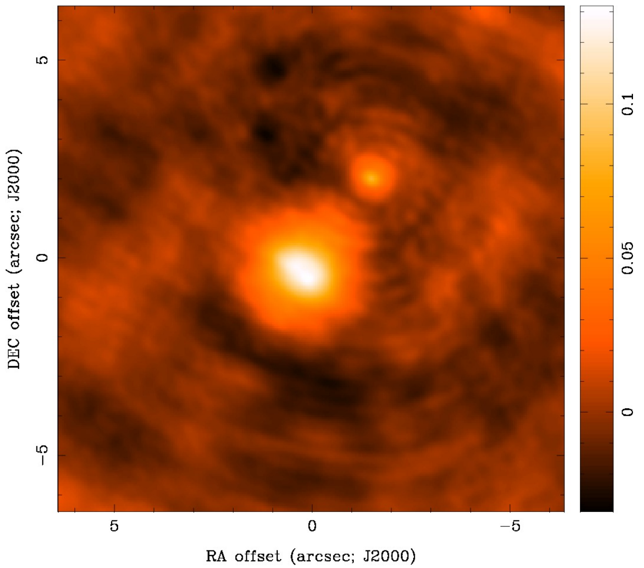


residual map

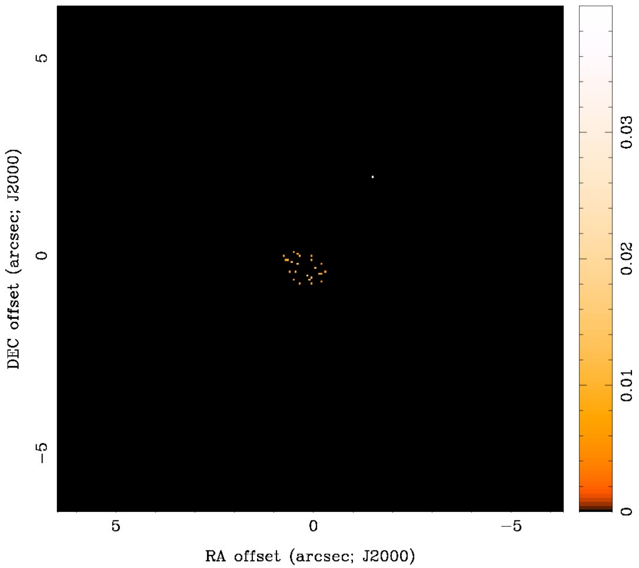


Clean example

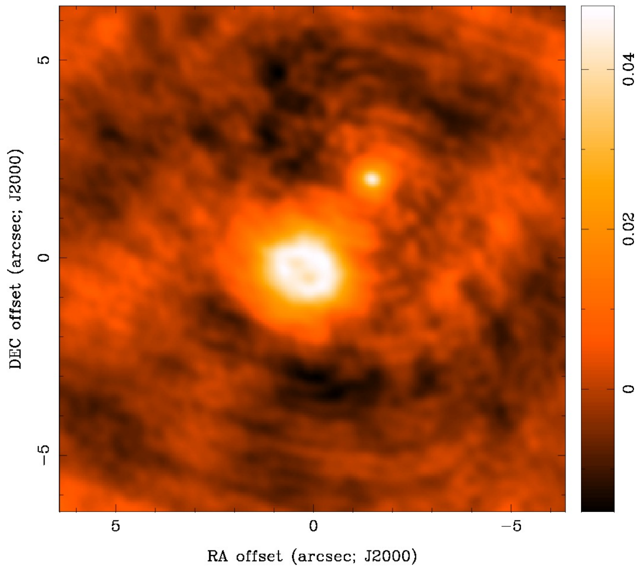
$T^D(l,m)$



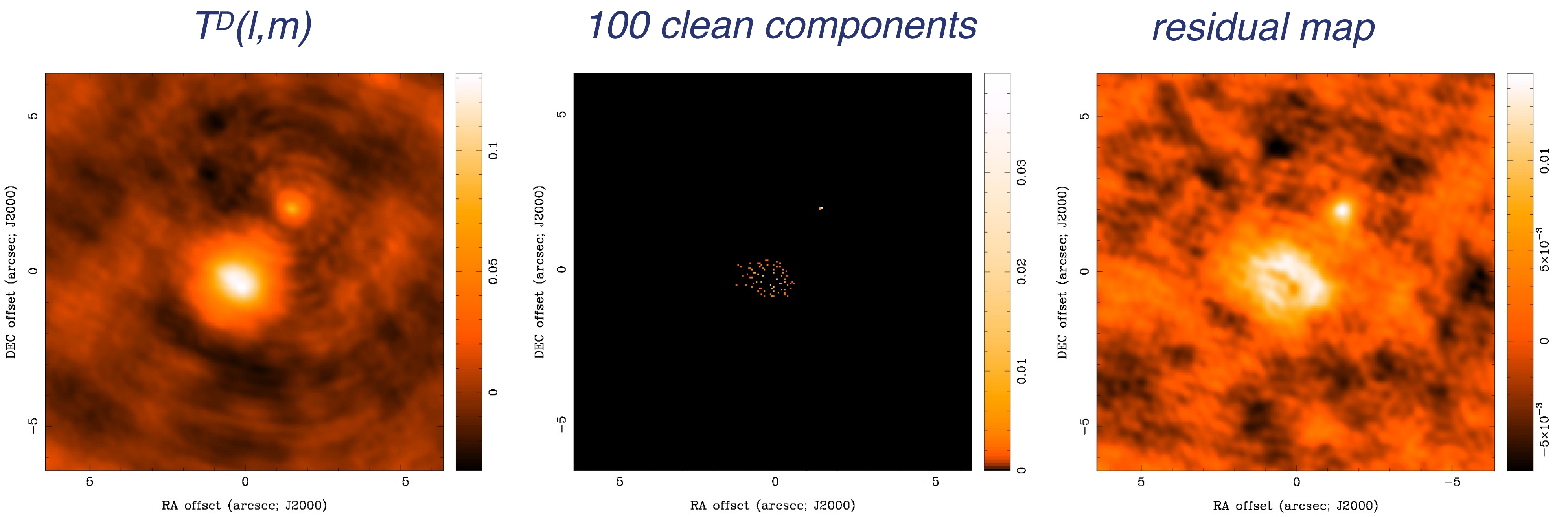
30 clean components



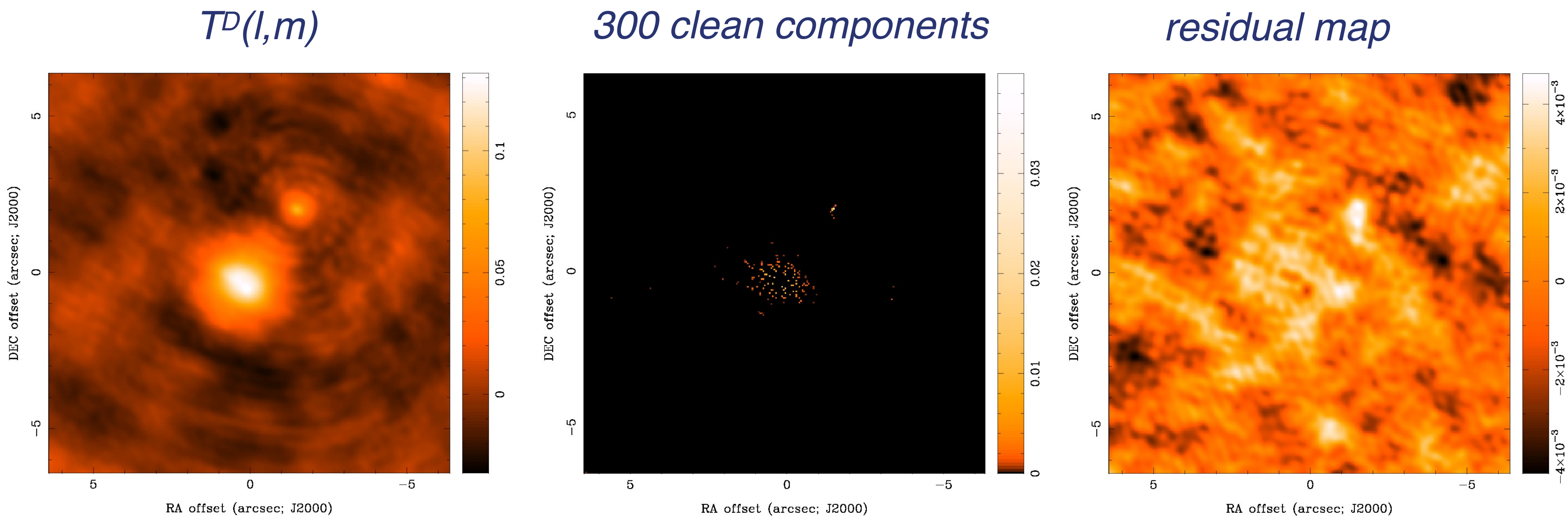
residual map



Clean example

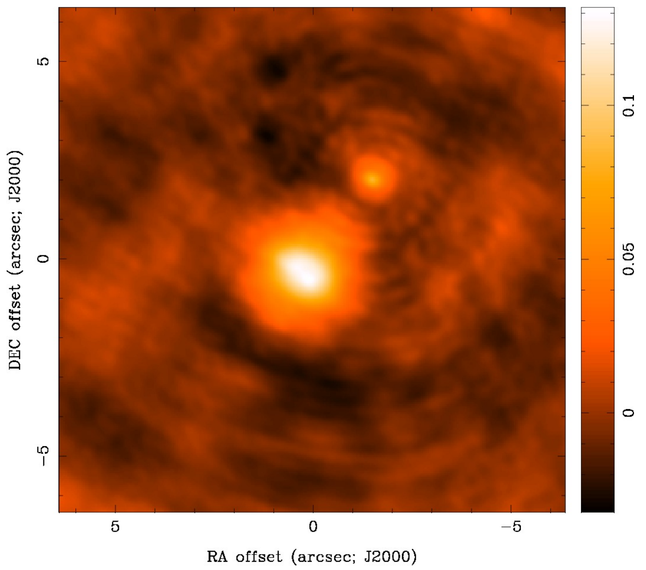


Clean example

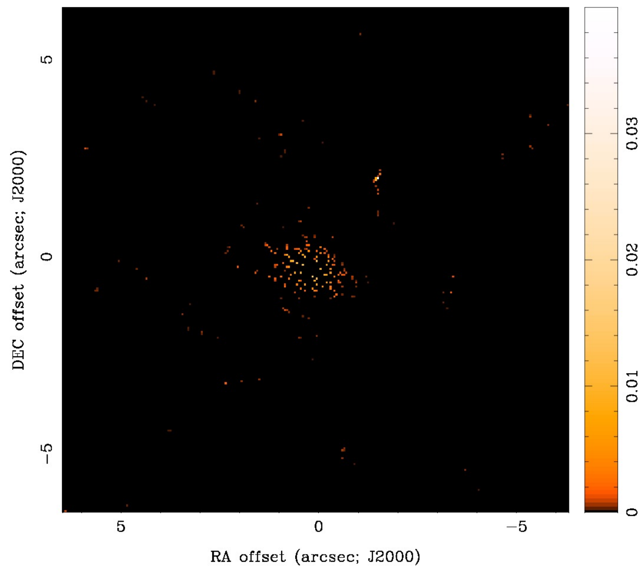


Clean example

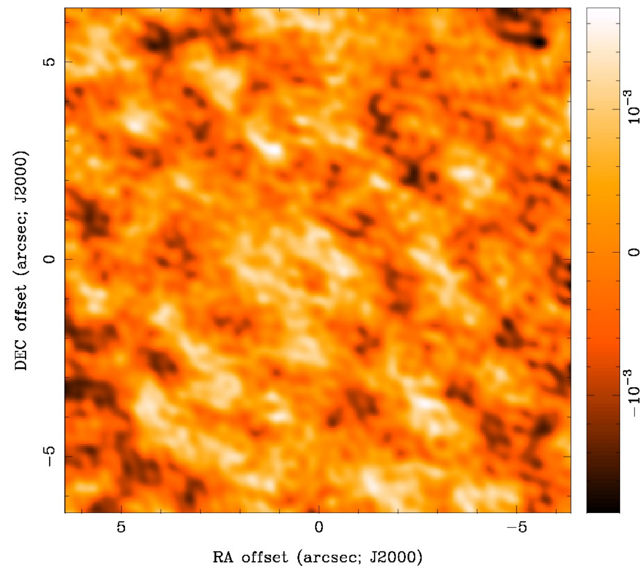
$T^D(l,m)$



585 clean components



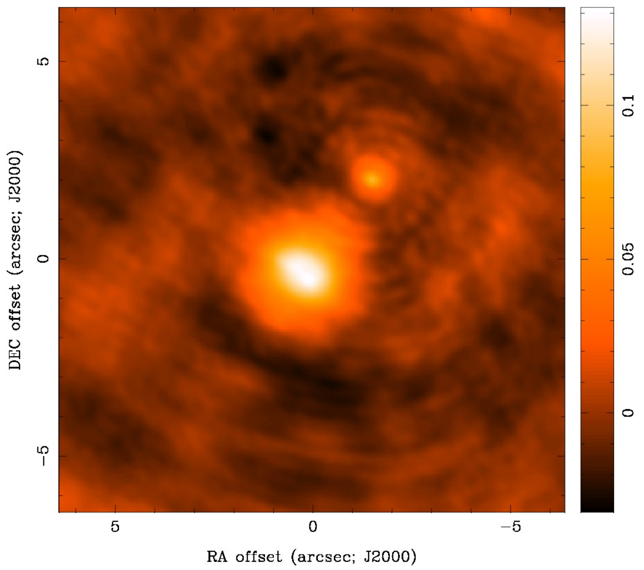
residual map



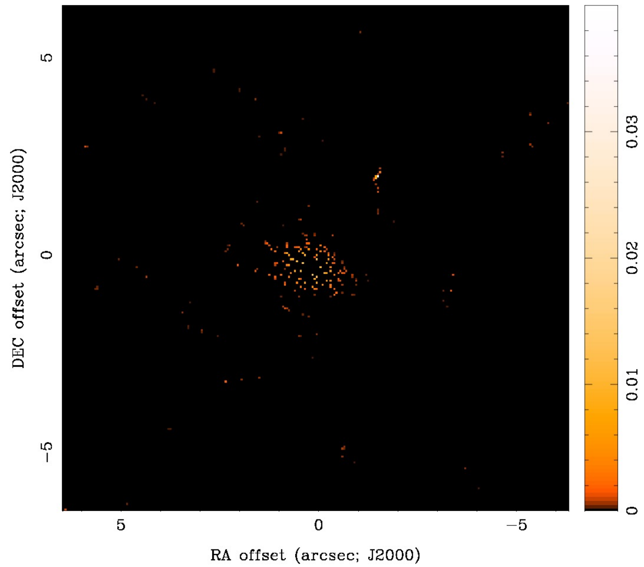
threshold reached

Clean example

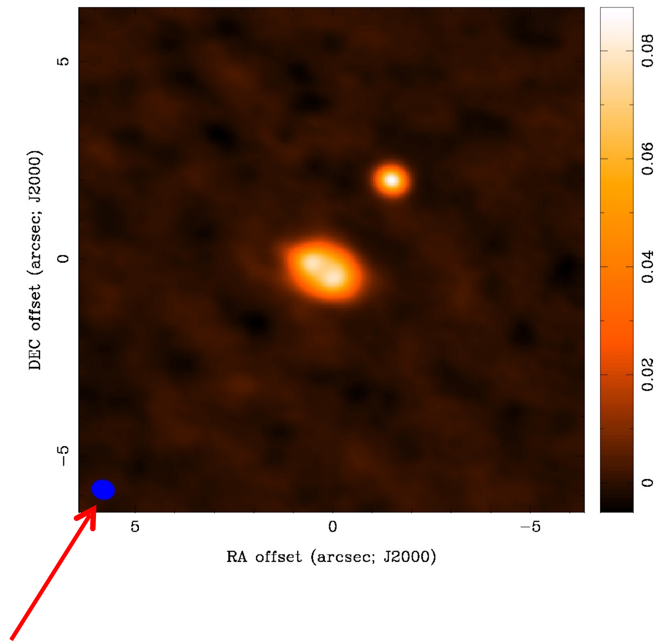
$T^D(l,m)$



585 clean components



restored image



ellipse = restoring beam fwhm

final image depends on

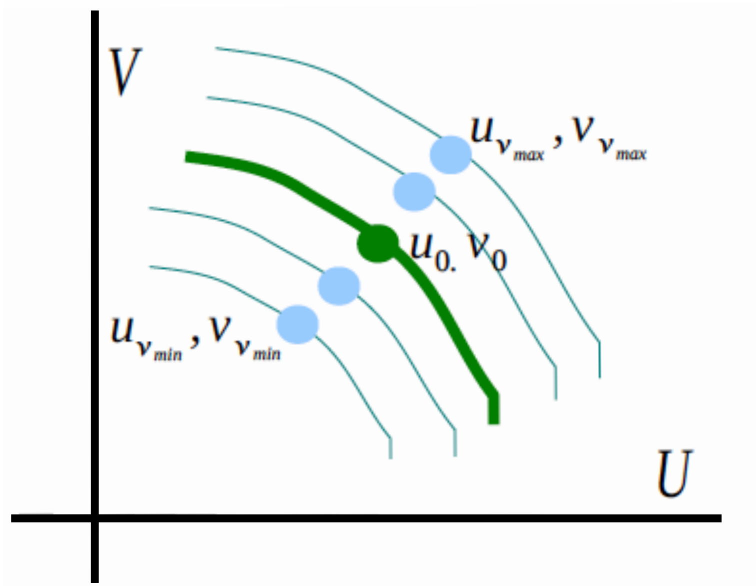
- imaging parameters: pixel size, visibility weighting scheme, gridding, …)
- deconvolution: algorithm, iterations, stopping criterion, …)

Scale Sensitive Methods

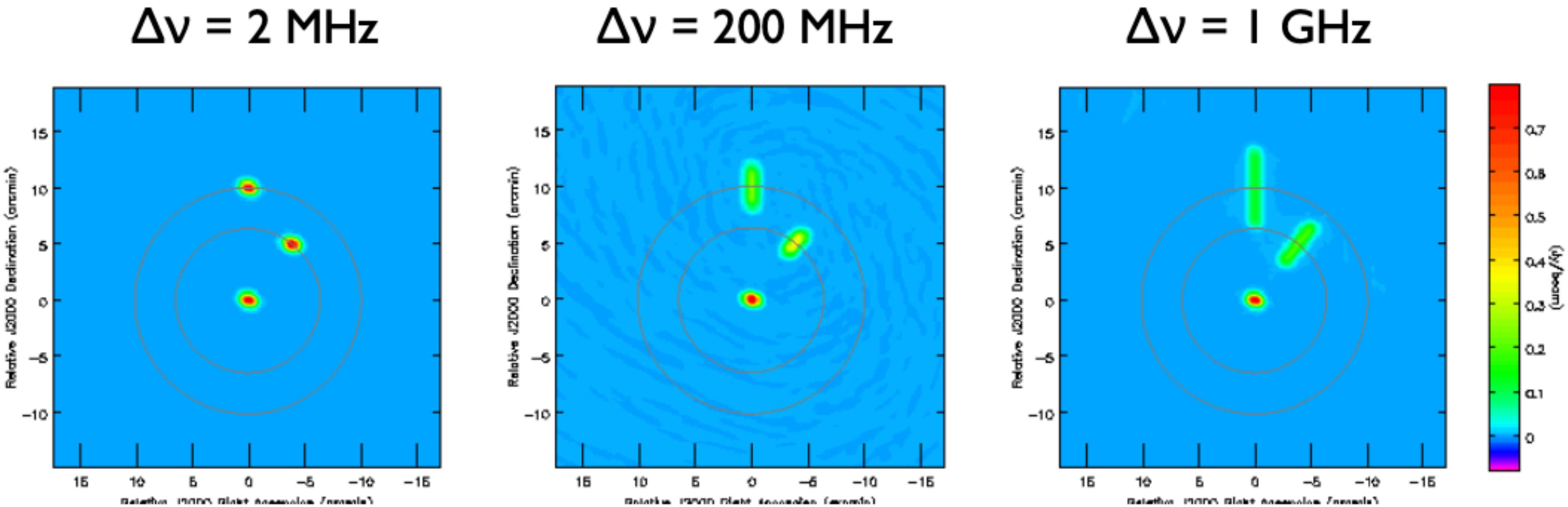
- standard clean often works poorly on very extended structure
  - and many point source components needed (slow)
- adjacent pixels in image are not independent
  - oversampled resolution limit
  - intrinsic source size: an extended source covering 1000 pixels might be better characterized by a few parameters than by 1000 parameters, e.g. 6 parameters for a 2D-Gaussian
- scale sensitive deconvolution algorithms employ fewer degrees of freedom in solution to model plausible sky brightness distributions
- e.g. multi-scale clean
  - make a collection of dirty images at different resolutions via convolution with input “scales” (e.g. 0,3,10,30 pixels)
  - find peak across all scales, remove fraction of peak at that scale from all dirty images, add corresponding blob to model, iterate..

Wide-band imaging

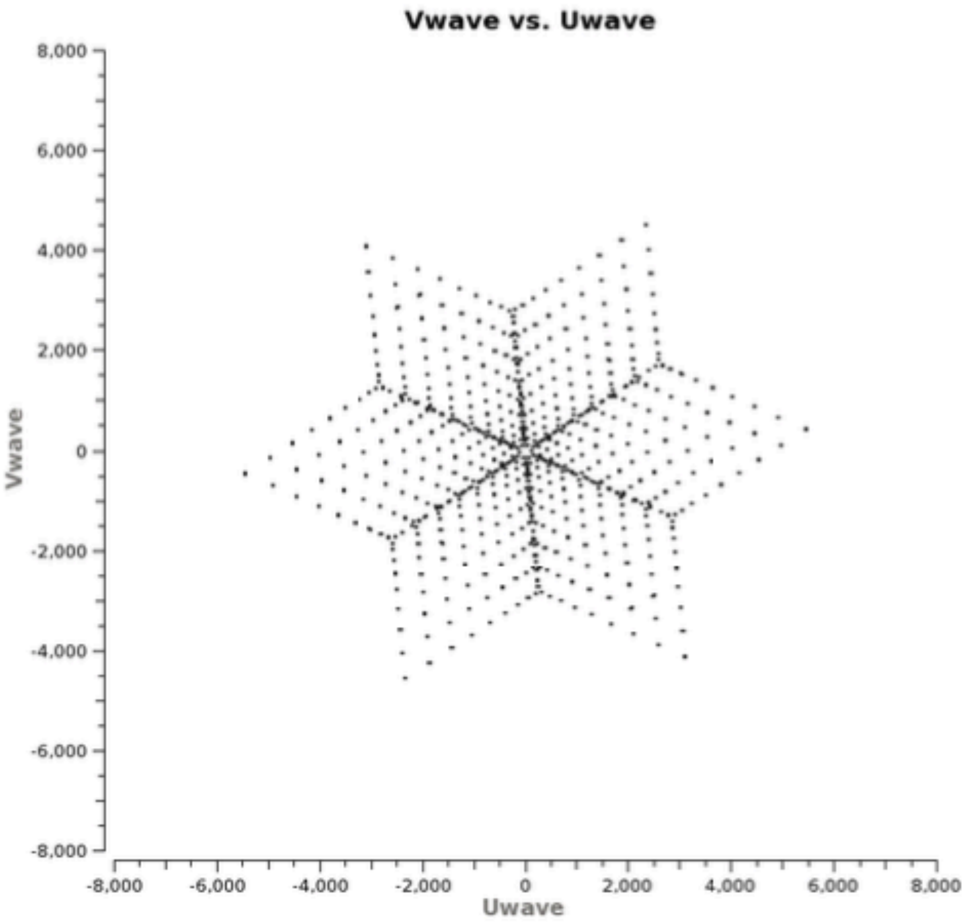
- Radio telescopes suffer from chromatic aberration ⇒ “bandwidth smearing”
- Measure visibilities in many narrowband channels to avoid bandwidth-smearing
- Construct visibilities for multiple narrowband channels
- Can use multi-frequency-synthesis to increase the uv-coverage used in deconvolution and image-fidelity
- Can make images at the angular-resolution allowed by the highest frequency
- Can take source spectrum into account

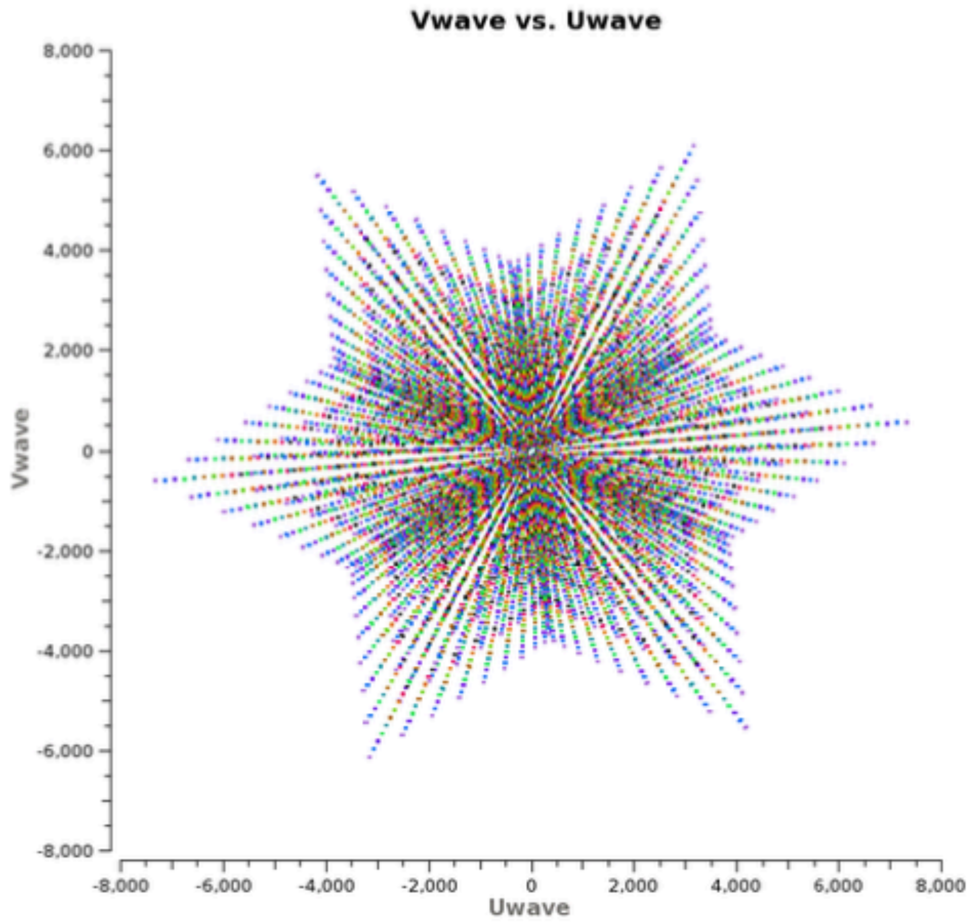


bandwidth smearing

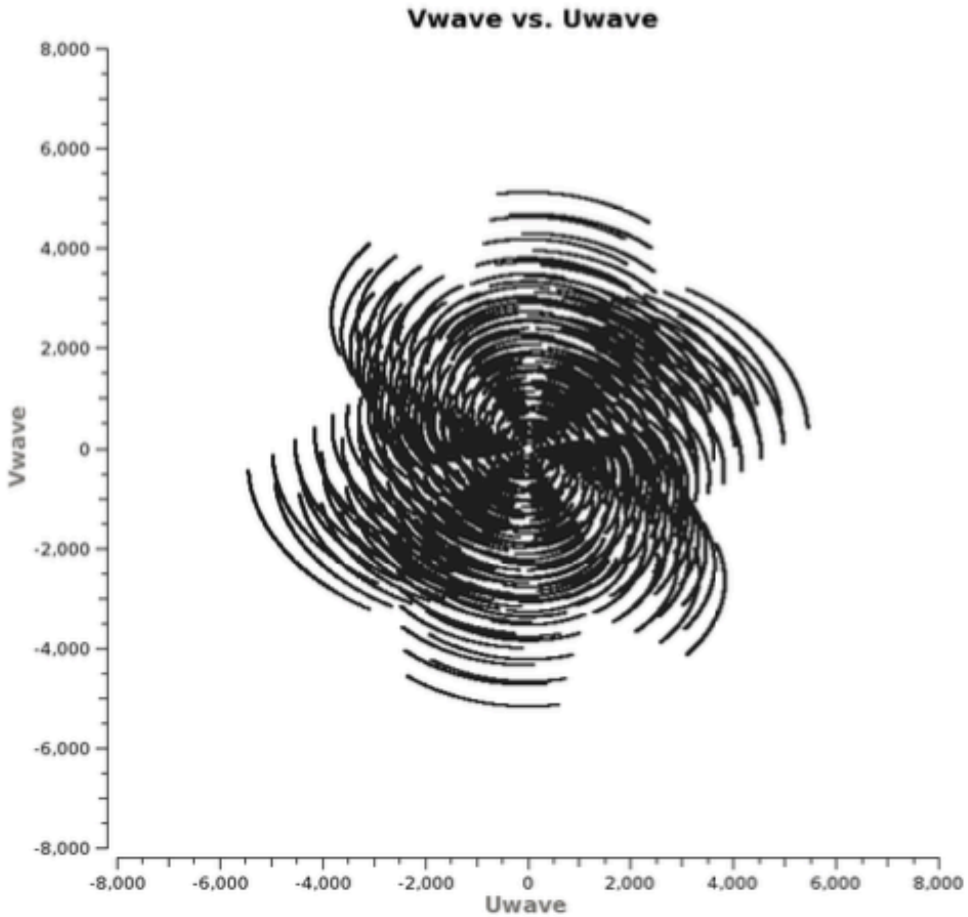


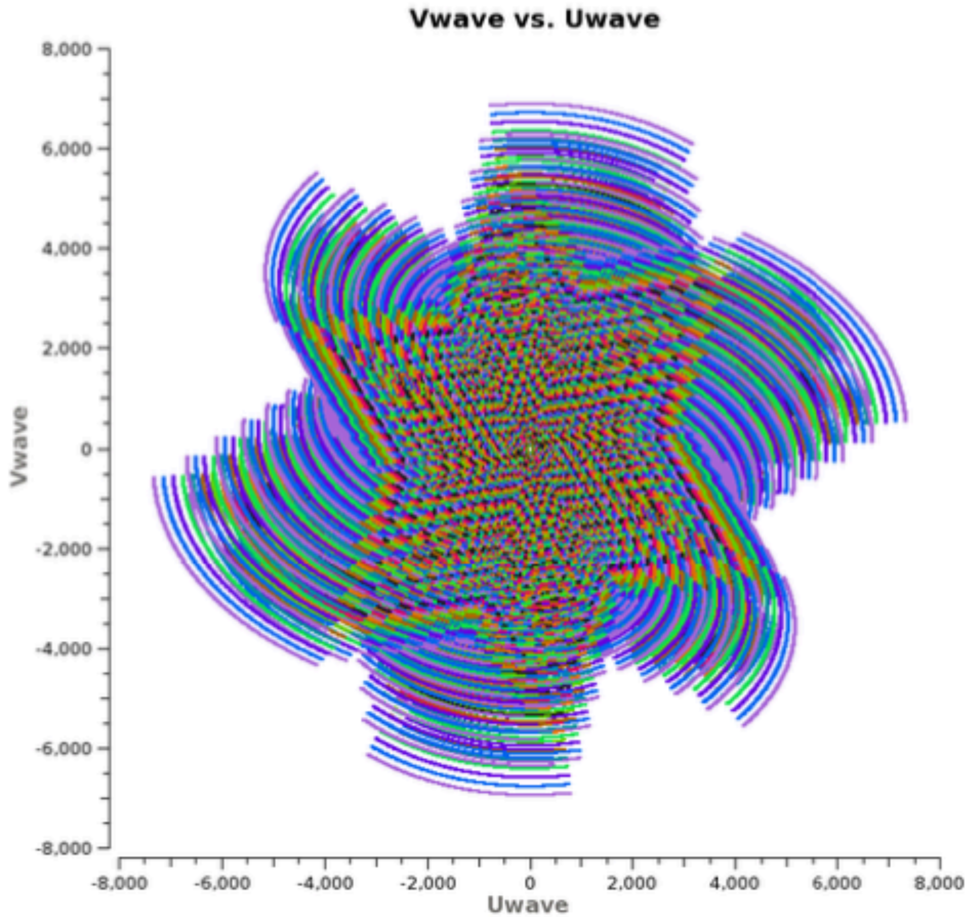
Multifrequency Synthesis





Multifrequency Synthesis





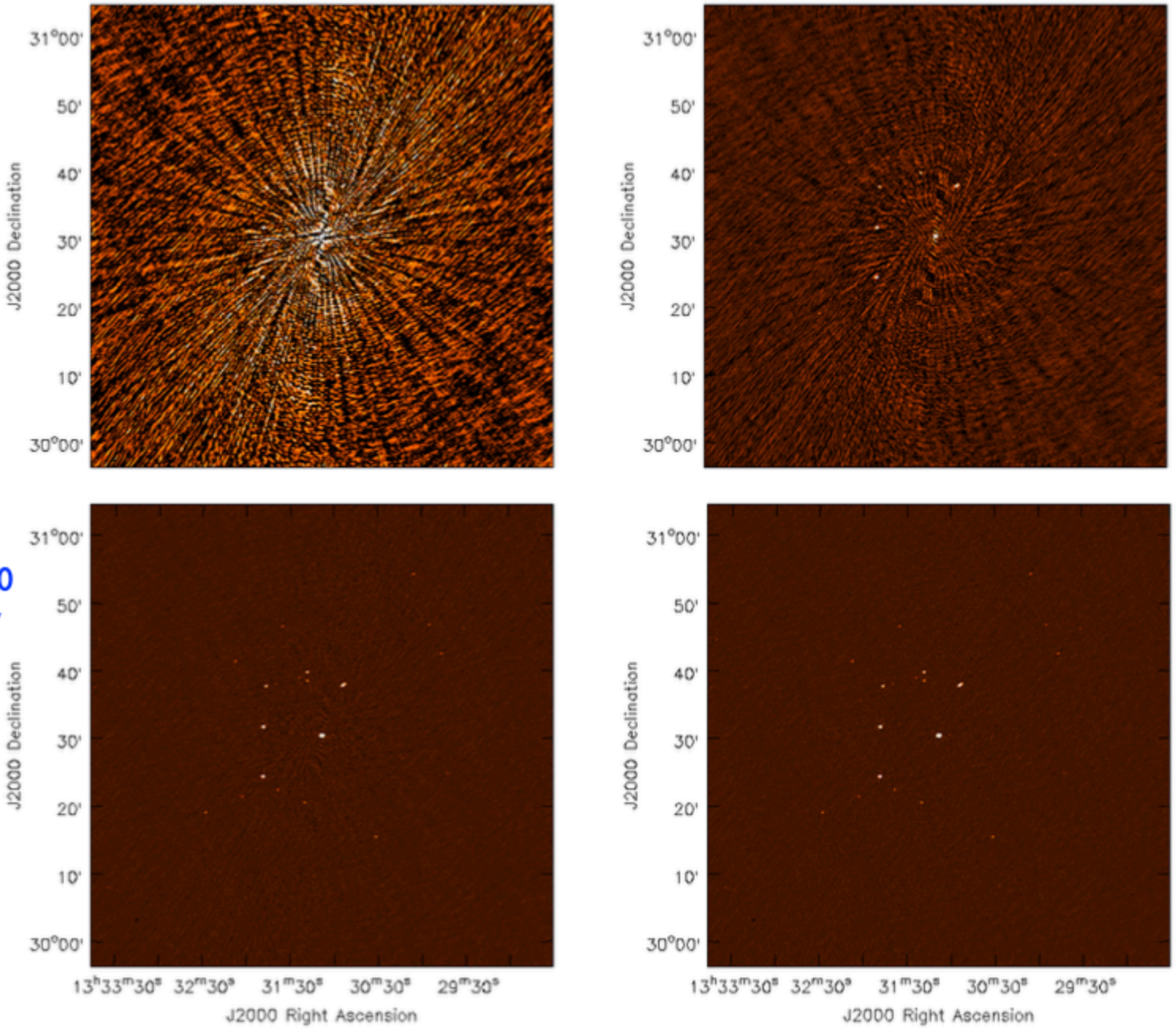
Add in time variation and you can get a much better sampling of the (u,v) plane.

This technique can produce much better images and is built into most standard analysis packages.

Multifrequency Synthesis: Example

DR =1600 -13000
σ=9 mJy -1 mJy
NTerms = 1

DR =10000 -17000
σ=1.0 mJy - 0.2 mJy
NTerms = 2



DR =65000 -170000
σ=0.2 mJy - 85 μJy
NTerms = 3

DR =110000 - 180000
σ=0.14 mJy - 80 μJy
NTerms = 4

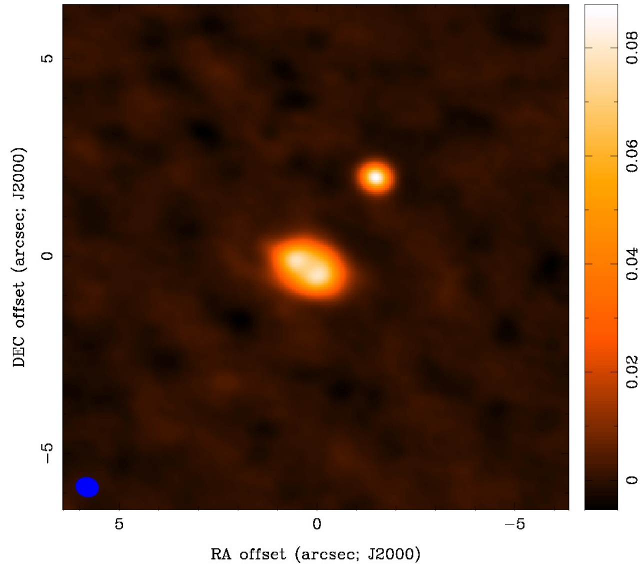
Measures of Image Quality

dynamic range

- ratio of peak brightness in image to rms noise in region devoid of emission
- easy way to calculate a *lower limit* to the error in brightness in a non-empty region

e.g. peak 88 and rms 1.0 mJy/beam

→ dynamic range = 88



fidelity

- difference between any reconstructed image and the correct image
- fidelity = input model/difference = inverse of the relative error

= model * beam / abs (model * beam – reconstructed image)

- generally much lower than implied by dynamic range

‘Dynamic Range'

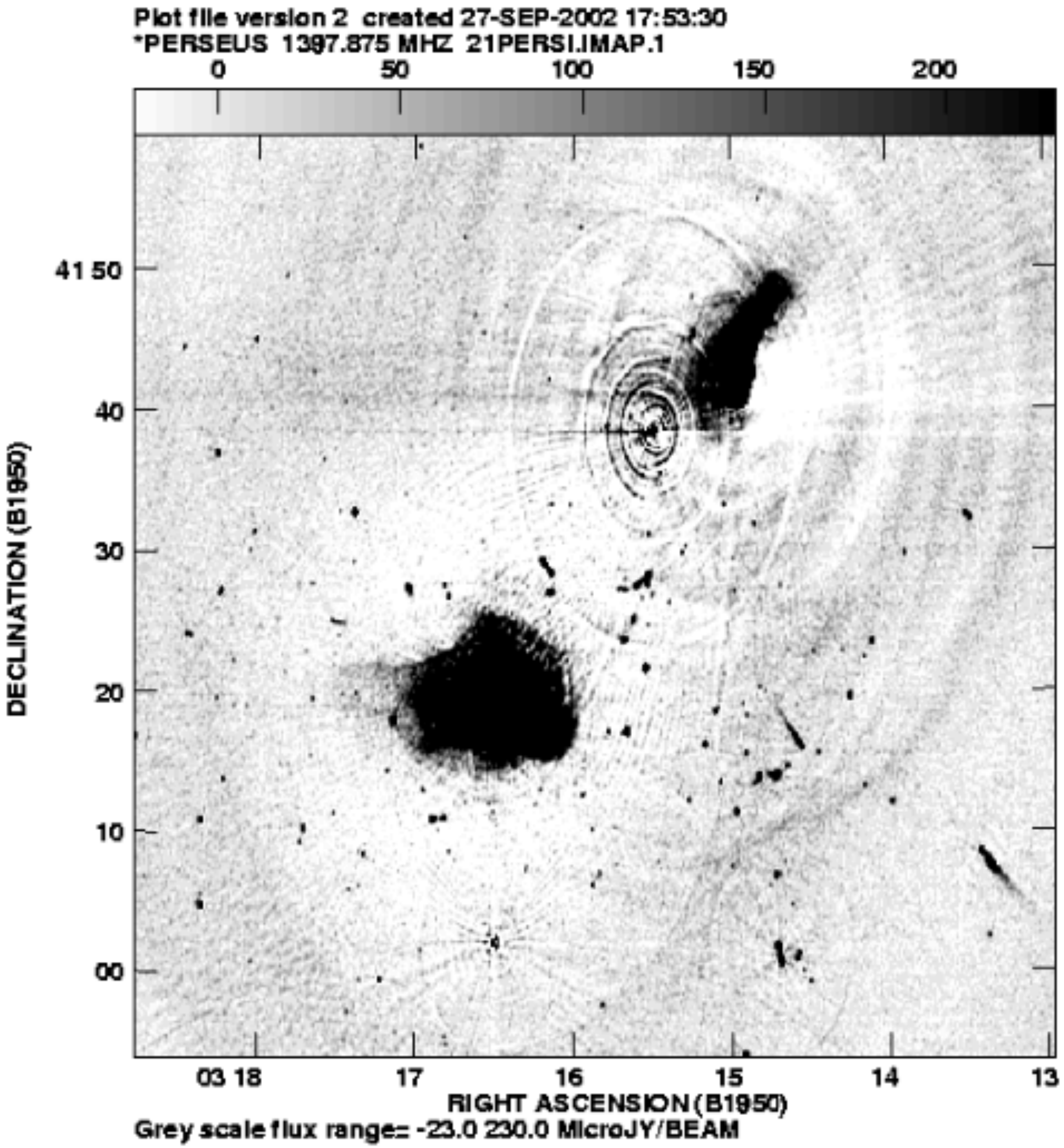


Image of the Perseus cluster showing details exposed at a dynamic range of 1,000,000:1

(de Bruyn & Brentjens 2010)

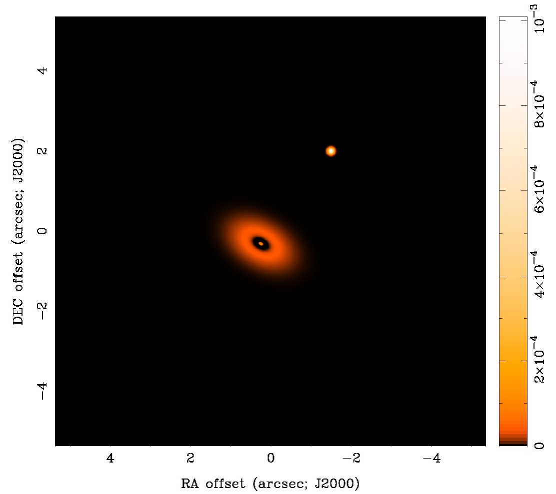
Invisible Large Scale Structure

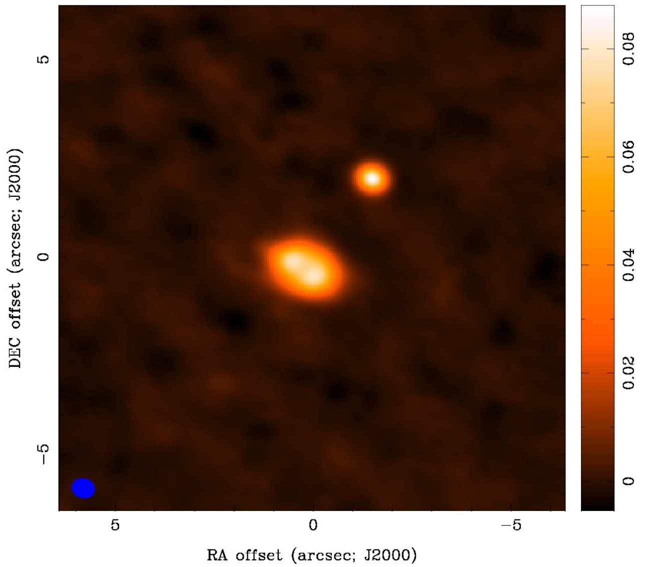
- inevitable central hole in *(u,v)* plane coverage
- extended structure may be missed, attenuated, or distorted
- to estimate if the lack of short baselines is a problem for science
  - simulate the observations using a model of the source

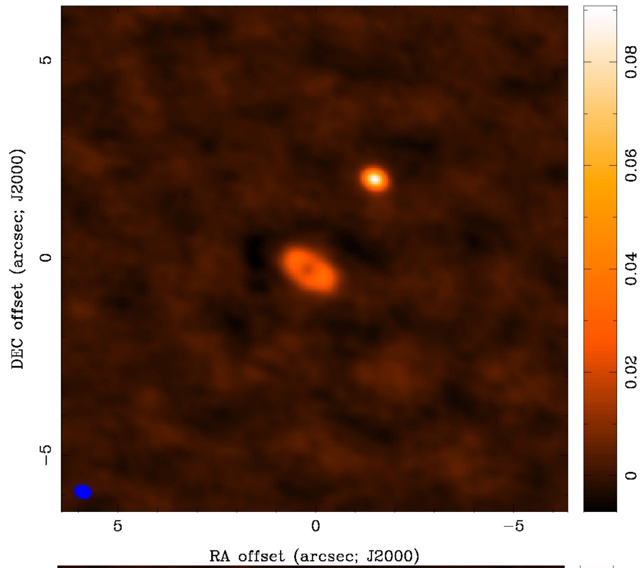
Missing Short Spacings: Example

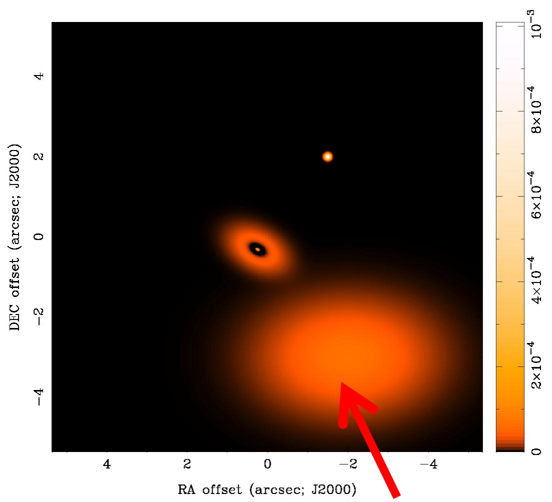
natural weight

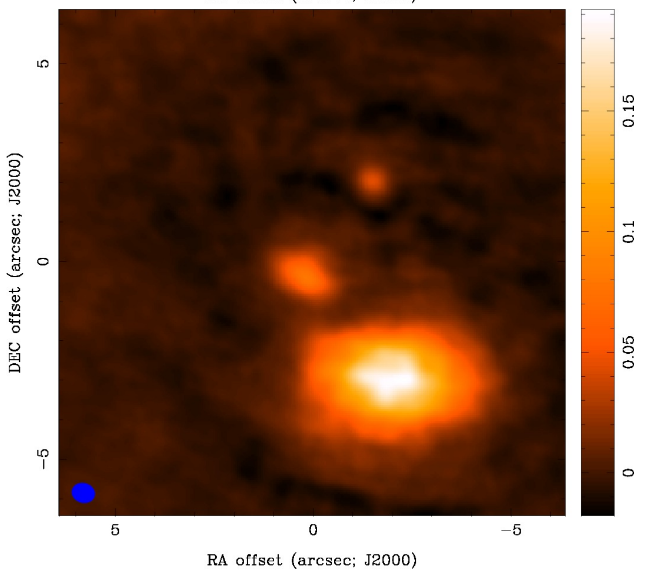
>75 kλ natural weight

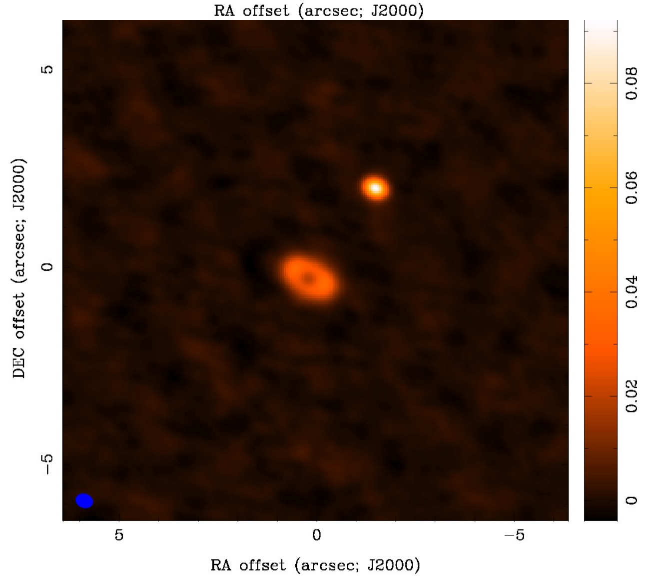










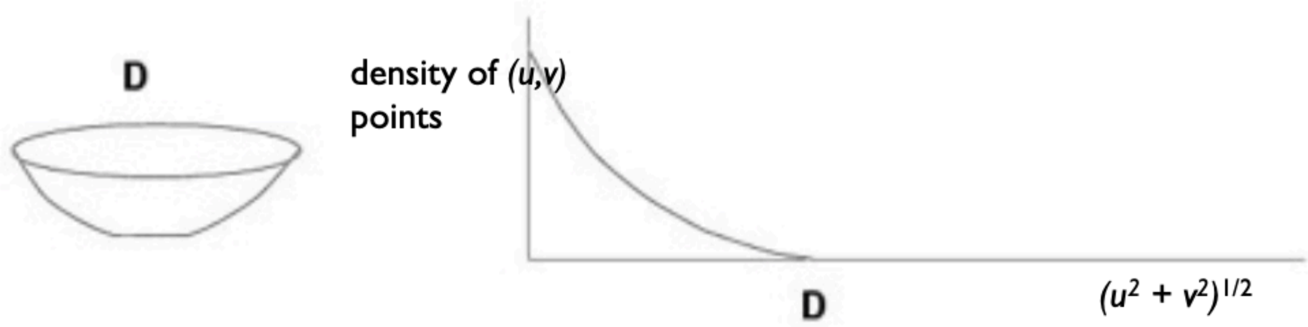


extended blob
(10x disk flux)

- important structure may be missed in central hole of (u,v) coverage!

Techniques to Obtain Short Spacings I

use a large single dish telescope — 'zero spacing correction'



- *all* Fourier components from 0 to D sampled, where D is dish diameter (weighting depends on illumination)
- scan single dish across sky to make an image $I(l,m) * A(l,m)$ where $A(l,m)$ is the single dish response pattern
- Fourier transform single dish image, $I(l, m) * A(l, m)$, to get $V(u, v)a(u, v)$ and then divide by $a(u,v)$ to estimate $V(u,v)$ for baselines < D
- choose D large enough to overlap interferometer samples of $V(u,v)$ and avoid using data where $a(u,v)$ becomes small

Techniques to Obtain Short Spacings II

use a separate array of smaller antennas

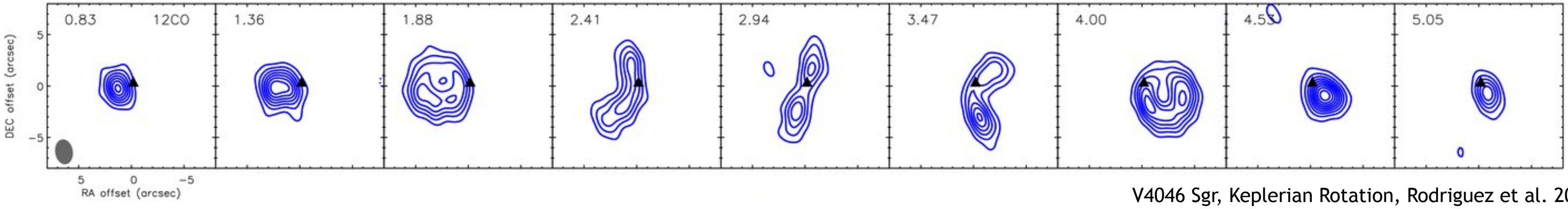
- *small antennas can observe short baselines inaccessible to larger ones*
- *the larger antennas can be used as single dish telescopes to make images with Fourier components not accessible to the smaller antennas*
- *example: ALMA main array + ACA*

main array
50 x 12m: 12m to 14+ km

ACA
12 x 7m: covers 7-12m
4 x 12m single dishes: 0-7m

Spectral Lines and Polarization

- most of discussion applies to a single spectral channel of width $\delta\nu$
- science may require many such channels across total bandwidth $\Delta\nu$
  - emission/absorption lines from molecular or atomic transitions



V4046 Sgr, Keplerian Rotation, Rodriguez et al. 2013

  - a significant continuum spectral slope or curvature
  - also helpful from a technical perspective: edit out interference "spikes", avoid "bandwidth smearing" (radial smearing in image plane that results from averaging over $\Delta\nu$ and limits useable field of view)

- science may require full Stokes parameters: I, Q, U, V
  - each Stokes parameter imaged (and deconvolved) independently, combined to form, e.g. fractional linear $\sqrt{(Q^2+U^2)}\,/I$